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Data Cloning Estimation and Identification of a Medium-Scale DSGE Model

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Abstract: We apply the data cloning method to estimate a medium-scale dynamic stochastic general equilibrium model. The data cloning algorithm is a numerical method that employs replicas of the original sample to approximate the maximum likelihood estimator as the limit of Bayesian simulation-based estimators. We also analyze the identification properties of the model. We measure the individual identification strength of each parameter by observing the posterior volatility of data cloning estimates and access the identification problem globally through the maximum eigenvalue of the posterior data cloning covariance matrix. Our results corroborate existing evidence suggesting that the DSGE model of Smeets and Wouters is only poorly identified. The model displays weak global identification properties, and many of its parameters seem locally ill-identified.

Keywords: data cloning; DSGE; identification; MCMC.

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1. Introduction

The data cloning (DC) methodology proposed by Lele et al. [1] is a numerical procedure for approximating maximum-likelihood estimates as the limit of Bayesian posterior simulation estimators when we repeat the original observed sample many times.

Because DC is applicable whenever we can employ Bayesian simulation techniques, such as Markov Chain Monte Carlo (MCMC), for estimating a model, the method has been adapted to many settings and applications, for example, Ponciano et al. [2], Baghishani and Mohammadzadeh [3], Torabi [4], Ponciano et al. [5], Torabi et al. [6], Picchini and Anderson [7], and Duan et al. [8]. We further discuss the DC method in Section 3.

Among econometricians, however, data cloning is not as popular. We only know of a few existing applications in the estimation of stochastic volatility models (Laurini [9], Marín et al. [10], de Zea Bermudez et al. [11]) A natural application of DC is in estimating Dynamic Stochastic General Equilibrium (DSGE) models. DSGE models take the form of nonlinear systems of expectational equations and play a central role in modern macroeconomic practice and research. Bayesian posterior simulation methods are often the preferred choice when estimating this wide class of models (Fernández-Villaverde [12]. Furlani et al. [13] use the DC method to estimate a small-scale DSGE model and report good properties with respect to sensitivity to initial values.

More complex DSGE models often display diverse problems regarding parameter identification (Canova and Sala [14]). Data cloning estimation has the convenient byproduct of uncovering parameter identification problems, which makes the method especially interesting to apply to DSGE models, since it becomes possible to, at the same time, estimate parameters and expose identification problems.

In this paper, we employ the DC method to estimate the DSGE model of Smets and Wouters [15] (SW) and then to assess parameter identification issues. SW is a medium-scale DSGE model with 36 free parameters, which are estimated from seven observed macroeconomic time series. Its general equilibrium structure is augmented with nominal rigidities and additional exogenous shocks. SW is a natural choice of subject for this analysis because it has been extensively studied and embodies all characteristic features of New Keynesian DSGE models. Indeed, much of the research on the identification of DSGE models, and even general critiques to modern macroeconomics, at times points to specific issues with the model of Smets and Wouters [15] (e.g., Canova and Sala [14], Chari et al. [16], Iskrev [17], Komunjer and Ng [18], Romer [19], and Chadha and Shibayama [20]).

The remainder of this paper is structured as follows. Section 2 discusses relevant estimation and identification issues with DSGE models, Section 3 overviews the data cloning idea and comments on some applications of the method, Section 4 details the setup of our exercise, Section 5 presents and discusses our results in relation to the literature, and Section 6 concludes.

2. Estimation and Identification of DSGE Models

A central focus of macroeconomic research effort, be it in academia or policymaker circles, is assessing the net effect of forces operating on different parts of the economy. DSGE models are the main tool employed for such task.

DSGE models are built up from microeconomic primitives, which makes them specially transparent with respect to underlying assumptions. Christiano et al. [21] argue that this openness is the main appeal of the DSGE approach, and what allowed diverse groups of researchers to make contributions to the field. While transparency is a great strength, it also makes DSGE models easy to criticize. Dubious and problematic assumptions can be singled out and scrutinized. Indeed, critics are plentiful, and we discuss some of them in relation to our results.

In this section, we follow some key steps when taking a DSGE model to data and extracting information about structural parameters; with each step, we highlight potential issues and discuss important results in the literature.

2.1. Estimation

As DeJong and Dave [22] put it, DSGE models start with a characterization of the environment in which decision makers reside, a set of decision rules that dictate their behavior, and a characterization of the uncertainty they face when making decisions. This structure takes the form of a nonlinear system of expectational equations, which are often linearized before solving.

Let $\bar{z}_t$ be an m-dimensional vector of stationary variables, and $z^\star$ be the steady state values of those variables such that $z_t = \bar{z}_t - z^\star$ is the present deviation from the steady-state value. Following Iskrev [17], most log-linearized DSGE models can be represented as

$$\Gamma_0(\theta)z_t = \Gamma_1(\theta)E_t z_{t+1} + \Gamma_2(\theta)z_{t-1} + \Gamma_3\epsilon_t. \tag{1}$$

Since (1) depends on expectational components, it must be solved in terms of rational expectations. There are several algorithms for solving linear rational expectation models (e.g., Blanchard and Kahn [23], Anderson and Moore [24], King and Watson [25], Sims [26]). Assuming that a unique solution exists, the linearized DSGE model can be written as

$$z_t = A(\theta)z_{t-1} + B(\theta)\epsilon_t. \tag{2}$$

Due to unobserved variables, the transition Equation (2) is augmented with the measurement equation

$$x_t = Cz_t + Du_t + \nu_t. \tag{3}$$

Equations (2) and (3) together form the state space representation of our DSGE model. Assuming that shocks ϵ_t are jointly Gaussian, we can recover the likelihood function $\mathcal{L}(\cdot|\theta)$ of the data X through Kalman filter recursions. Then, the maximum likelihood estimate (MLE) of θ is

$$\hat{\theta}^{ML} = \max_{\theta \in \Theta} \mathcal{L}(X|\theta). \tag{4}$$

The precision of the maximum-likelihood estimator $\hat{\theta}^{ML}$ is given by the inverse of the Fisher information matrix

$$\mathcal{I}(X|\theta) = \mathbb{E}\left[\left(\frac{\partial \mathcal{L}(X|\theta)}{\partial \theta'} \right)' \left(\frac{\partial \mathcal{L}(X|\theta)}{\partial \theta'} \right) \right]. \tag{5}$$

In practice, obtaining $\hat{\theta}^{ML}$ from the numerical optimization of $\mathcal{L}(X|\theta)$, as suggested in Equation (4), is not straightforward. The likelihood function of a typical DSGE model is an ill behaved object, with multiple local maxima/minima and flat surfaces. Due to those difficulties, Bayesian methods are often preferred when estimating DSGE models (Fernández-Villaverde [12]).

Let $\pi(\theta)$ denote the joint prior distribution of parameters θ. Bayes' theorem implies the posterior distribution of the parameters given data is

$$\pi(\theta|X) = \frac{\mathcal{L}(X|\theta)\pi(\theta)}{\int_{\Theta} \mathcal{L}(X|\theta)\pi(\theta) d\theta}. \tag{6}$$

The marginal likelihood of the denominator is constant over all parameter values, and in many applications it suffices to compute the nominator

$$\pi(\theta|X) \propto \mathcal{L}(X|\theta)\pi(\theta). \tag{7}$$

The common solution here is then to sample from $\pi(\theta|X)$ using a posterior-simulation Markov Chain Monte Carlo (MCMC) method [27]. Multiplying the likelihood by the prior distribution has the effect of attenuating the computational issues with optimizing the likelihood function that we alluded to before [28].

2.2. Identification

The classical result of identification in parametric models dates back to Rothenberg [29]. Local identifiability of a parameter vector is ensured by a non-singular Fisher information matrix. Komunjer and Ng [18] show this result does not apply to DSGE models. Because the reduced-form parameters of the state space representation are not generally identifiable, the non-singularity of the information matrix does not suffice to ensure local identification of the parameter vector.

Canova and Sala [14] frame the identification problem as having "to do with the ability to draw inference about the parameters of a theoretical model from an observed sample". This broad definition of the identification problem is well suited for the DSGE context, as issues with parameter identification may arise in all stages from model specification to estimation from data, or, as the authors put it, from different "mappings" between the estimated parameter vector $\hat{\theta}$ and data X.

The "solution mapping" regards the relationship between structural parameters θ and the coefficients of matrices $A(\theta)$ and $B(\theta)$. Common issues arise here when some individual parameter θ_1 disappears from the solution (under identification), or when two parameters θ_1 and θ_2 enter the solution only in relation to one another (partial identification). It is not uncommon for approximation methods to induce under-identification or partial identification problems. The SW model has two pairs of parameters that, due to log linear approximation, are only partially identified. Calvo [30] price reoptimization probabilities ξ_π, ξ_w feature in the solution only as functions of the Kimball curvature parameters ϵ_π, ϵ_w, respectively. Indeed, Smets and Wouters [15] fix $\epsilon_\pi = \epsilon_w = 10$ and estimate ξ_π and ξ_w accordingly.

Partial identification and under-identification are problems of the model, the objective function, and/or the approximation methods employed—therefore, a model that displays this sort of problem will display them no matter the data set at hand. Alternatively, we could say they involve only the "solution" and "objective function" mappings of Canova and Sala [14].

One fundamental identification problem arises when the likelihood function does not have a unique maximum. Thus, under the point of view of the objective function, two models, potentially having different economic interpretations, are indistinguishable, and information external to the model needs to be introduced in order to choose between structural forms. Observational equivalence is a well-known phenomenon in macroeconomics and has played part in many debates over the years. For example, Sargent [31] showed that it is difficult to distinguish between old Keynesian and neoclassical models using only parameters estimated from a single policy regime. Chari et al. [16] uses observationally equivalent, but structurally different, models to question to what extent shocks in the SW model are interpretable.

A fourth identification issue characteristic of the DSGE model is related to the accuracy with which some particular parameter θ can be estimated from finite samples. If the curvature of the likelihood function $\mathcal{L}(X|\theta)$ around $\hat{\theta}$ is small, then large changes in $\hat{\theta}$ will have little impact on the value of the likelihood function. Canova and Sala [14] dub this phenomenon "weak identification" in allusion to the literature on weak instruments.

Iskrev [17] tackle the weak identification problem as a model feature, in both a mathematical and economic sense. He mentions that although the non-singularity of $\mathcal{I}(X|\theta)$ ensures the model's expected likelihood is not flat and achieves a locally unique maximum, the precision with which the true parameter θ may be estimated in finite samples depends on the degree of curvature of the likelihood around an open neighborhood of $\hat{\theta}$, to which the rank condition provides no information. If the curvature of the log likelihood is nearly flat around $\hat{\theta}$, then large changes in the estimate $\hat{\theta}$ will induce small relative changes in $\mathcal{L}(X|\hat{\theta})$. The economic argument the author presents is that a weakly identified parameter is one that is either irrelevant, because it has only a negligible effect on the likelihood, or nearly redundant, because its effect on the likelihood may be closely approximated by other parameters, and thus the value of such parameters is hard to pin down on the basis of the information contained in the likelihood function alone.

Qu and Tkachenko [32] provide a characterization of necessary and sufficient conditions for the local identification of structural parameters in linearized DSGE models and propose a frequency domain quasi-maximum likelihood estimator. They show how in a quasi-Bayesian estimation scheme, the procedure can be used to access parameter identification properties. This method was employed by Tkachenko and Qu [33] to analyze the model of Smets and Wouters [15].

In a Bayesian estimation, we are interested in the posterior distribution $\pi(\theta|X)$, which is proportional to the likelihood function $\mathcal{L}(X|\theta)$ times the prior distribution $\pi(\theta)$. As long as there is a proper prior, the posterior distribution is well defined. The extreme case is inference without data, where the posterior is the prior itself. In fact, as An and Schorfheide [28] discuss, priors "might add curvature to a likelihood function that is (nearly) flat in some dimensions of the parameter space and therefore strongly influence the shape of the posterior distribution". In this sense, the presence of the prior distribution makes weak identification problems even harder to spot in real-world applications.

A naive Bayesian identification test used is to visually contrast prior and posterior shapes. Intuitively, if the posterior is very similar to the prior, then data have little to say about the parameter of interest. Canova and Sala [14] point out that this is only true if the parameter space is variation-free; i.e., there are no model-implied restrictions on the parameter space. DSGE models are seldom variation-free; they usually impose stability restrictions or economically motivated non-negativity constraints on some parameters.

If the parameter space is not variation-free, then beliefs updates about one parameter might affect the parameter space of another, ultimately affecting the shape of the posterior distribution even if data are not informative.

Koop et al. [34] introduce two Bayesian identification indicators. The first indicator looks to the expected posterior shape relative to the prior and provides a yes-or-no answer to the identification problem, much in line with Iskrev [17] and Komunjer and Ng [18]. The second indicator is especially relevant to us because it is similar to the data cloning method in that both use a scheme to create artificial data and tackle weak identification problems computationally. The Bayesian Learning Rate indicator of Koop et al. [34] consists of augmenting the observed sample with model simulated path and then observing the standard error of MCMC estimates as the simulated sample size increases.

Morris [35] discusses the sampling distribution of estimators for DSGE parameters tends to be non-normal and/or pile up on the boundary of the theoretical support. This phenomenon is showcased in our exercise below.

More recently, Ivashchenko and Mutschler [36] combine the "a priori" rank identification criteria of Iskrev [17], Komunjer and Ng [18], and Qu and Tkachenko [32], with the simulation-based Bayesian Learning Rate indicator of Koop et al. [34], to study the identification properties of two traditional DSGE models. Their focus is on how different investment adjustment cost specifications and output-gap definition affect individual parameter identification. Qu and Tkachenko [37] discuss global identification properties of DSGE models while taking into account indeterminacy of monetary policy rules that can be (nearly) observationally equivalent to each other. An alternative approach to verify identification conditions in DSGE models was proposed by Meenagh et al. [38], in the context of the estimation of DSGE models using indirect inference. They verified the identification through Monte Carlo procedures, where artificial restrictions were progressively imposed on the [15] model parameters, and checked the rejection rate over many samples from the true model. With this method, identification problems are verified through the power of the tests to reject invalid parameters in the model; in the absence of identification, the power of these tests is low. A procedure to verify global identification was proposed in Kocięcki and Kolasa [39], where the condition for identification was formulated through the restrictions linking the observationally equivalent state space representations of the DSGE model and on the constraints imposed on the deep parameters by the model solution. Kocięcki and Kolasa [40] refine this method by using the concept of a Gröbner basis and new algorithms to analytically compute the complete set of observationally equivalent parameter vectors.

3. The Data Cloning Method

The data cloning (DC)methodology was developed by Lele et al. [1] and Lele et al. [41] as a computational scheme to obtain maximum-likelihood (ML) parameter estimates as the limit of Bayesian estimates when the original data are pooled many times. It was first designed to estimate complex hierarchical ecological systems and has been applied to a wide range of Generalized Linear Mixed models. Furlani et al. [13] demonstrate the possibility of estimating DSGE models through the DC method.

Original data X are repeated K times to obtain an artificially augmented, cloned dataset $X^{(K)} = (X, \ldots, X)$. The new likelihood function based on the cloned data is given by $[L(X|\theta)]^K$. Then, the new posterior distribution is

$$\pi_K(\theta|X) = \frac{[\mathcal{L}(X|\theta)]^K \pi(\theta)}{\int_\Theta [\mathcal{L}(X|\theta)]^K \pi(\theta) d\theta}. \tag{8}$$

Lele et al. [1] and Lele [42] show that as the number of sample clones K grows larger, the DC posterior distribution converges to an approximately normal with mean $\hat{\theta}^{ML}$ and variance $\frac{1}{K}\mathcal{I}^{-1}(X|\hat{\theta}^{ML})$,

$$\pi_K(\theta|X) \underset{K\to\infty}{\longrightarrow} N\left(\hat{\theta}^{ML}, \frac{1}{K}\mathcal{I}^{-1}(X|\hat{\theta}^{ML})\right). \tag{9}$$

In other words, as the number of clones K increases, marginal distributions of individual parameters θ_0 should be nearly degenerated around the ML point estimate, and the variance of this distribution is K times the corresponding asymptotic variance of the ML estimator of θ.

Thus, the main result of the data cloning method is to guarantee that when we estimate the posterior distributions of model parameters using K clones of the original sample (Equation (8)), as K grows to infinity, the resulting estimator eliminates the influence of prior specification on parameter estimation by pinpointing maximum likelihood estimates as limiting estimates from the procedure. Intuitively, the influence of priors on posterior estimation is progressively eliminated by the sample cloning procedure because, since the contribution of the prior is fixed for any sample size, its influence is dominated by the information contained in an evaluated likelihood function from a K times replicated original sample. The factor $1/K$ multiplying the inverse of the information matrix in Equation (9) is necessary in order to correct the variance of the estimators. Since they are obtained using K-clones of the original sample, their posterior variance is artificially reduced by the increased sample size. The asymptotic normality of DC posterior estimates is demonstrated by [1]. Thus, the DC estimator can be taken as a computational method employed in order to exploit the standard asymptotic properties of Bayesian estimators given by the Bernstein–von Mises theorem, which states that under some regularity conditions, discussed, for example, in van der Vaart [43], Bayesian estimators converge to the maximum-likelihood estimator as sample size grows.

Data cloning estimators belong to the class of estimators denoted as quasi-Bayesian/Laplacian by Chernozhukov and Hong [44] and thus inherit the good computational properties that such methods exhibit when compared to directly maximizing the likelihood function. As the estimators are constructed through MCMC sampling, they are not obtained through numerical optimization and thus are robust to problems such as nonsmooth and discontinuous objective functions, do not suffer from the computational curse of dimensionality, and are as efficient as the extremum estimates. Chernozhukov and Hong [44] also provide evidence that these estimators are robust to multiple local maxima and initial values. However, these estimators can be affected by problems that affect the speed of convergence of the MCMC chains, for example, initial values in the tails of the posterior distribution, a problem that can be avoided through a choice of initial values based on economic considerations.

The implementation of the data cloning method is very straightforward. One has only to repeat the original sample K times to obtain an augmented sample with $K \times T$ observations and then sample from $\pi_K(\theta|X)$ using a traditional Metropolis–Hastings algorithm. As discussed by Lele et al. [1], one does not assume that the artificial samples are independent and the sample repetition is only a scheme to numerically obtain the maximum-likelihood estimator of parameter θ. As the number of clones increases, the algorithm better approximates the true location of the likelihood function and the true inverse of the Fisher information for the observed data. The statistical accuracy of the estimator is a function of sample size and is not enhanced by the data cloning procedure; increasing the number of clones will only contribute to enhancing the numerical approximation of the ML estimates.

An alluring theoretical advantage of the data cloning methodology is that since ML estimates are invariant to the choice of prior distributions, DC helps attenuate the impact of subjectivity that prior specification can have on posterior estimates. There is no guarantee, however, that the DC method will be successful in pinpointing ML estimates. As Lele [42] discuss, Bayesian hierarchical models are easy to construct and straightforward to analyze due to MCMC, and, as a general principle, the complexity of the model should not exceed the information content in the data. Data cloning can be helpful by alerting the researcher to potential issues with the model such as nonestimability. Data cloning is used by Ponciano et al. [5] to assess parameter identifiability in Bayesian phylogenetic models. Ponciano et al. [2] show how the data cloning can be used to construct hypothesis tests, confidence intervals, and perform model selection exercises.

Within the DC framework, an individual parameter is considered identifiable if its marginal posterior variance converges to 0 at the rate of $1/K$ as the number of K clones increases (Lele et al. [41], Ponciano et al. [5]). If, however, the likelihood function has insufficient curvature with respect to some individual parameter θ_0, then as the number of clones increases and the prior distribution has a diminished impact on the posterior, marginal variance will not converge to zero at the predicted rate because the flat likelihood dominates the DC marginal posterior distribution.

The computational properties of data cloning estimators in the context of estimating DSGE models are analyzed in Furlani et al. [13], which through Monte Carlo studies, show that data cloning estimators are robust to the choice of initial values and local maximums, and it has superior performance in finite samples terms of bias and root mean squared error in relation to several other numerical methods of maximizing the likelihood function. As discussed in Furlani et al. [13], data cloning estimators do not rely on the evaluation of derivatives and Hessians of the objective function, and in this respect, they are more robust to problems related to the evaluation of these functions, such as near-flat-likelihood functions, discontinuities, and problems in the numerical approximations of these functions through the use of methods such as finite differences.

Given its both practical and theoretical appeal, the idea of DC has been adapted to many settings and applications. We enumerate a few for the sake of context. Baghishani and Mohammadzadeh [3] employ the DC method to estimate generalized linear mixed models in a spatial setting. Picchini and Anderson [7] combine the DC idea with an approximate Bayesian computation (ABC) approach. More recently, Duan et al. [8] incorporated data cloning into a sequential Monte Carlo algorithm. They employed DC to help with convergence and argue that their method has good computational properties that take advantage of parallel computing.

The data cloning method allows the construction of information criteria such as the AIC and the BIC. These criteria can be constructed by evaluating the likelihood function with the parameters estimated through the limit of the Bayesian estimator, using the penalties associated with each information criterion. In the case of our study, we are only analyzing a single model, and therefore, there is no need to compare different specifications using these criteria.

4. Model and Estimation Details

The model of Smets and Wouters [15] is a medium-scale general equilibrium model augmented with nominal rigidities and additional exogenous shocks that embodies many key features of New Keynesian DSGE models.

There are 41 parameters in the SW model: 17 dictate the dynamics of exogenous AR and ARMA processes, the remainder characterize structural features of the economy. Five parameters are calibrated, leaving a total of 36 parameters to be estimated from seven observable series. The model is estimated from seven macroeconomic variables observed at a quarterly frequency: real output, worked hours, inflation rate, consumption, investment, and capital stock. We omit further details on the model structure due to space constraints, and point to the original paper for more information. Smets and Wouters [15] employ data from the third quarter of 1947 through the fourth quarter of 2004, totaling 230 observations. Estimation was carried out in Matlab and Dynare (version 4.3.3, Adjemian et al. [45]), using two MCMC chains composed of 250.000 iterations each.

First we replicate the baseline estimation exercise of Smets and Wouters [15], employing the same data and prior specification. Then, using gradually more sample repetitions (5, 10, and 25), we compute data cloning posterior estimates. As discussed in Section 3, parameter identification can be accessed by observing the behavior of data cloning posterior variance as the number of sample repetitions increases, and a global measure of identification comes from the largest eigenvalue of the DC posterior covariance matrix.

5. Results

The present application of the data cloning method consists in first replicating the baseline exercise of Smets and Wouters [15], then estimating the model using gradually more sample replications—we consider 5, 10, and 25. Local identification can be accessed by observing the posterior volatility of the data cloning estimated parameters. We also measured the global identification identification properties of the DSGE system obtained through the maximum eigenvalue of the data cloning posterior covariance matrix. In this section, we report our findings and place them in relation to established results in the literature.

Table 1 summarizes our estimation results. The first two columns display parameters and their economic definition. Prior specification, which mimics Smets and Wouters [15], is reported in columns three to five. Following standard practice in the DSGE literature, prior specification is reported in terms of distribution family (N for Normal, B for Beta, G for Gamma, and IG for Inverse Gamma), mean, and standard deviation. Columns three through eight display our single-sample MCMC estimation results: the mode obtained through numerical optimization (initial values for posterior sampling), the posterior mean, and standard deviation. Then we report posterior mean and standard deviations for data cloning estimations using 5, 10, and 25 sample replications. The last column reproduces posterior means reported by Smets and Wouters [15] in their Table 1A and 1B for easier comparison.

Table 1. Data cloning posterior estimates.

Parameter	Definition	Prior			Optimization	Single-Sample (MCMC)		5 Clones		10 Clones		25 Clones		Benchmark
		Density	Mean	Std	Mode	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean
ψ	Investment adj. cost.	N	4.00	1.50	5.49	5.588	2.241	6.851	1.091	6.004	0.694	5.974	1.044	5.74
σ_c	Inv. elats. intert. subst.	N	1.50	0.38	1.47	1.435	0.328	1.401	0.327	1.406	0.243	1.468	0.061	1.38
h	Consump. habit	B	0.70	0.10	0.70	0.705	0.102	0.794	0.071	0.771	0.032	0.759	0.054	0.71
ξ_w	Calvo wage	B	0.50	0.10	0.73	0.692	0.163	0.896	0.024	0.855	0.067	0.838	0.121	0.71
σ_l	Elast. labour supply	N	2.00	0.75	1.67	1.655	1.259	3.791	0.433	3.267	0.731	3.398	1.852	1.83
ξ_p	Calvo price	B	0.50	0.10	0.68	0.676	0.125	0.784	0.048	0.737	0.039	0.695	0.076	0.66
ι_w	Index. of wages	B	0.50	0.15	0.56	0.543	0.277	0.476	0.145	0.495	0.141	0.582	0.037	0.58
ι_p	Index. of prices	B	0.50	0.15	0.24	0.26	0.202	0.273	0.092	0.276	0.073	0.265	0.039	0.24
Ψ	Capital utilization	B	0.50	0.15	0.40	0.411	0.211	0.171	0.122	0.199	0.102	0.226	0.388	0.54
Φ	Fixed cost	N	1.25	0.12	1.65	1.643	0.171	1.581	0.095	1.591	0.065	1.575	0.078	1.6
r_π	Response to inflation	N	1.50	0.25	1.98	2.015	0.375	2.131	0.316	1.936	0.072	1.949	0.193	2.04
ρ	Interest rate smooth.	N	0.75	0.10	0.82	0.82	0.052	0.872	0.016	0.854	0.018	0.851	0.021	0.81
r_y	Response to output	N	0.13	0.05	0.09	0.095	0.052	0.138	0.069	0.123	0.029	0.118	0.007	0.08
$r_{\Delta y}$	Response to output gap	N	0.13	0.05	0.22	0.222	0.064	0.191	0.013	0.185	0.027	0.192	0.047	0.22
$\bar{\pi}$	Steady state inflation	G	0.63	0.10	0.67	0.679	0.157	0.534	0.121	0.604	0.107	0.604	0.065	0.78
$100\,(\beta^{-1}-1)$	Discount factor	G	0.25	0.10	0.21	0.241	0.203	0.232	0.167	0.186	0.041	0.195	0.038	0.16
$\bar{l}$	Steady state hours worked	N	0.00	2.00	0.40	0.296	2.249	2.351	1.921	0.876	1.135	1.038	1.222	0.53
$100\,(\gamma-1)$	Trend growth	N	0.40	0.10	0.44	0.435	0.035	0.451	0.011	0.456	0.025	0.455	0.011	0.43
α	Share of capital	N	0.30	0.05	0.32	0.314	0.092	0.354	0.106	0.371	0.031	0.356	0.067	0.19
δ	Depreciation rate	n.a.	0.025	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
g_y	Government/Output ratio	n.a.	0.18	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
ϕ_w	Wage mark-up	n.a.	1.5	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
ϵ_w	Kimball (wage)	n.a.	10	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
ϵ_p	Kimball (price)	n.a.	10	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
ρ_{ga}	Tech. shock to gov. spending	N	0.50	0.25	0.60	0.587	0.204	0.553	0.097	0.644	0.116	0.625	0.036	0.52
ρ_a	AR nonspecific technology	B	0.50	0.20	0.95	0.948	0.0354	0.974	0.027	0.968	0.014	0.967	0.005	0.95
ρ_b	AR risk premium	B	0.50	0.20	0.17	0.212	0.1957	0.219	0.183	0.217	0.034	0.209	0.014	0.22
ρ_g	AR government spending	B	0.50	0.20	0.97	0.973	0.0196	0.997	0.001	0.996	0.016	0.995	0.019	0.97
ρ_i	AR investment	B	0.50	0.20	0.74	0.747	0.134	0.647	0.083	0.671	0.101	0.681	0.045	0.71
ρ_m	AR monetary policy	B	0.50	0.20	0.12	0.142	0.1294	0.042	0.049	0.121	0.095	0.121	0.031	0.15
ρ_p	AR price mark-up	B	0.50	0.20	0.90	0.879	0.1326	0.999	0.001	0.999	0.075	0.999	0.016	0.89
ρ_w	AR wage mark-up	B	0.50	0.20	0.97	0.959	0.0366	0.972	0.029	0.969	0.004	0.967	0.011	0.96
μ_p	MA price mark-up	B	0.50	0.20	0.77	0.723	0.2217	0.966	0.013	0.952	0.115	0.927	0.081	0.69
μ_w	MA wage mark-up	B	0.50	0.20	0.87	0.809	0.1694	0.945	0.048	0.914	0.026	0.904	0.029	0.84
σ_a	Std. technology shock	IG	0.10	2.00	0.43	0.434	0.0506	0.472	0.029	0.445	0.011	0.446	0.026	0.45
σ_b	Std. risk premium shock	IG	0.10	2.00	0.24	0.237	0.0606	0.243	0.052	0.241	0.038	0.243	0.003	0.23
σ_g	Std. government spending	IG	0.10	2.00	0.51	0.516	0.0519	0.52	0.016	0.504	0.061	0.512	0.011	0.53
σ_i	Std. investment shock	IG	0.10	2.00	0.43	0.435	0.0688	0.469	0.079	0.458	0.085	0.461	0.011	0.56
σ_m	Std. monetary policy	IG	0.10	2.00	0.24	0.244	0.1014	0.231	0.011	0.233	0.006	0.228	0.009	0.24
σ_π	Std. price mark-up shock	IG	0.10	2.00	0.14	0.141	0.0328	0.145	0.016	0.139	0.014	0.133	0.004	0.14
σ_w	Std. wage mark-up shock	IG	0.10	2.00	0.24	0.235	0.0359	0.231	0.024	0.217	0.008	0.224	0.028	0.24

Note: table displays posterior estimates of the SW model. In the priors, specification N corresponds to a Normal distribution, B a Beta distribution, G a Gamma distribution, and IG an Inverse-Gamma distribution. The Single-Sample columns are the results of the usual estimation by MCMC. Calibrated parameters are represented by "n.a." values.

Single sample-point estimates are mostly in line with benchmark SW values and remain, with notable exceptions, stable as the number of sample replication increases. As we increase the number of clones, estimates of the autorregressive coefficient of the price-markup exogenous process ρ_p push the upper limit of unity. SW's estimate of ρ_p is 0.89, while our estimate with 20 sample clones is 0.99, implying qualitatively different dynamics for the exogenous price-markup process. The price-markup process is ARMA(1,1), and a similar issue seems to occur with the moving average parameter μ_p, whose benchmark estimate of 0.69 changes to 0.927 with 20 sample clones. This phenomenon seems to be specific to the price-markup process. For some other exogenous processes, we do have autorregressive coefficients estimated with relatively high magnitude, namely ρ_g and ρ_w, but those parameters have stable estimates not far from benchmark values. Furthermore, this issue seems to be of a different sort than the one discussed by Morris [35], as in his Monte Carlo experiment of SW's model, the sampling distribution of the estimator of ρ_p is indistinguishable from its Beta prior.

Following Furlani et al. [13], we compute normalized standard errors (proportional to the standard error in the single-sample estimation) to evaluate whether the data-cloning algorithm is successful in reducing posterior volatility. Table 2 displays the normalized standard errors as a function of the number of clones K, denoted by s^*_K. To enhance visualization, we list the parameters in decreasing order relative to the 25-clone standard deviation estimate. Parameters in Table 2 are listed from worse to best identified with respect to the normalized standard error metric.

Parameters related to the dynamics of exogenous stochastic processes, in general, seem to be better identified relative to structural parameters. In Table 1 shock parameters are presented below the calibrated parameters, while structural parameters are placed above. This is a recurring finding in the identification literature ([14,17,20]). Indeed, we find the best-identified parameters to be ρ_b and σ_b, which describe the risk-premium exogenous process. Even though ρ_p and σ_p are considered well identified according to the normalized standard deviation metric, from the previous discussion on the autorregressive coefficient, we know that there must be estimation problems related to those parameters.

Among shock parameters, the one whose posterior volatility is least diminished by the data cloning method is the standard deviation of shocks to wage markups σ_w. This is not unexpected, since this specific parameter is embroiled in the critique Chari et al. [16] made of the SW model, which is based on the implausible magnitude and interchangeable economic interpretation of exogenous wage markup shocks ε^w_t.

Several structural parameters that characterize the economic environment seem not to be identifiable. After 20 clones, the posterior standard deviation of Ψ, which determines costs to capital utilization readjustment, is almost twice the original standard deviation; in the case of wage elasticity of labor supply σ_l, s^*_{25} is almost one and a half times s^*_1, indicating the data cloning algorithm is unable to reduce posterior volatility and thus unable to accurately determine maximum likelihood point estimates.

Calvo reoptimization probabilities ξ_p and ξ_w are badly identified, while partial indexation degrees ι_p and ι_w display somewhat better identifiability. The fact that we seem to better identify partial indexation parameters could be, as suggested by Iskrev [17] and Canova and Sala [14], because ξ and ι are highly pairwise collinear. We believe it is important to remember that price rigidities are not primitive model features, and there is nothing that justifies the introduction of these rigidities besides a higher level of realism in the model. However, the empirical validity of the indexed-Calvo model is questionable (Dixon and Kara [46], Bils et al. [47], Bils and Klenow [48]), which adds to the catalogue of critiques directed towards the class of NK-DSGE models, of which SW's model is an important example.

Parameters that appear explicitly in the measurement equations are sometimes called "trend parameters". In SW's model, these are the growth rate trend γ, steady-state inflation $\bar{\pi}$, steady-state worked hours $\bar{l}$, and discount factor β. Most trend parameters seem to be badly identified, especially steady-state worked hours $\bar{l}$ and inflation $\bar{\pi}$. The parameter γ is featured explicitly as linear growth trend of output, consumption, investment, and worked hours. According to the the data cloning method, γ seems to be only weakly identified. Canova and Sala [14] and Chadha and Shibayama [20] show that these trend parameters mainly affect first moments, and having constant terms changes the identification in general. Interestingly, we find the intertemporal discount factor β to be relatively well identified—

the best performing among trend parameters. This result is unexpected since Iskrev [17] argues β has little expected impact over the likelihood function and should be especially hard to identify.

Table 2. Normalized data cloning posterior standard deviation.

Parameters	s_1^*	s_2^*	s_3^*	s_5^*	s_{10}^*	s_{25}^*
Ψ	1.000	0.389	0.373	0.582	0.445	1.841
σ_l	1.000	0.441	0.348	0.344	0.33	1.471
ρ_g	1.000	0.178	0.142	0.081	0.132	1.004
σ_w	1.000	0.406	0.434	0.685	0.328	0.799
ξ_w	1.000	0.176	0.174	0.151	0.318	0.738
α	1.000	0.428	0.545	1.149	0.495	0.733
$r_{\Delta y}$	1.000	0.338	0.361	0.207	0.515	0.733
ξ_p	1.000	0.339	0.343	0.382	0.409	0.61
$\bar{l}$	1.000	0.621	0.575	0.853	1.046	0.543
h	1.000	0.383	0.294	0.7	0.127	0.534
σ_a	1.000	0.284	0.533	0.588	0.792	0.519
r_π	1.000	0.352	0.571	0.843	0.225	0.516
ψ	1.000	0.429	0.423	0.486	0.206	0.466
Φ	1.000	0.488	0.611	0.556	0.441	0.456
$\bar{\pi}$	1.000	0.529	0.375	0.763	1.059	0.413
ρ	1.000	0.299	0.329	0.316	0.193	0.397
μ_p	1.000	0.097	0.086	0.059	0.087	0.362
ρ_i	1.000	0.426	0.452	0.621	0.555	0.341
$100\ (\gamma - 1)$	1.000	0.314	0.257	0.311	0.226	0.311
ρ_w	1.000	0.265	0.295	0.803	0.532	0.306
ρ_m	1.000	0.326	0.333	0.383	0.912	0.238
σ_g	1.000	0.425	0.483	0.312	0.676	0.204
ι_p	1.000	0.485	0.288	0.455	0.433	0.194
σ_c	1.000	0.425	0.465	0.999	0.536	0.187
$100\ (\beta^{-1} - 1)$	1.000	0.705	0.525	0.825	0.226	0.186
ρ_{ga}	1.000	0.357	0.497	0.475	0.328	0.178
μ_w	1.000	0.108	0.109	0.285	0.324	0.171
σ_i	1.000	0.302	0.773	1.156	1.315	0.175
ρ_a	1.000	0.209	0.333	0.779	0.581	0.163
r_y	1.000	0.316	0.728	1.324	0.277	0.148
ι_w	1.000	0.503	0.384	0.523	0.628	0.135
σ_p	1.000	0.307	0.371	0.496	0.368	0.128
ρ_p	1.000	0.027	0.017	0.007	0.002	0.122
σ_m	1.000	0.359	0.116	0.116	0.146	0.088
ρ_b	1.000	0.392	0.386	0.937	0.325	0.072
σ_b	1.000	0.402	0.419	0.871	0.712	0.061

Note: Table reports the posterior standard deviation s_K^* of parameters of the SW model obtained from DC estimates as a proportion of the single sample standard deviation s_1^*.

Figure 1 plots the standardized maximum eigenvalue of the data cloning covariance matrix λ_K^S against its expected value of $1/K$. We find λ_K^S above its reverence level for all samples considered. From this evidence, we cannot say the SW model is globally identified, which is to be expected given the widespread poor individual parameter identification performance.

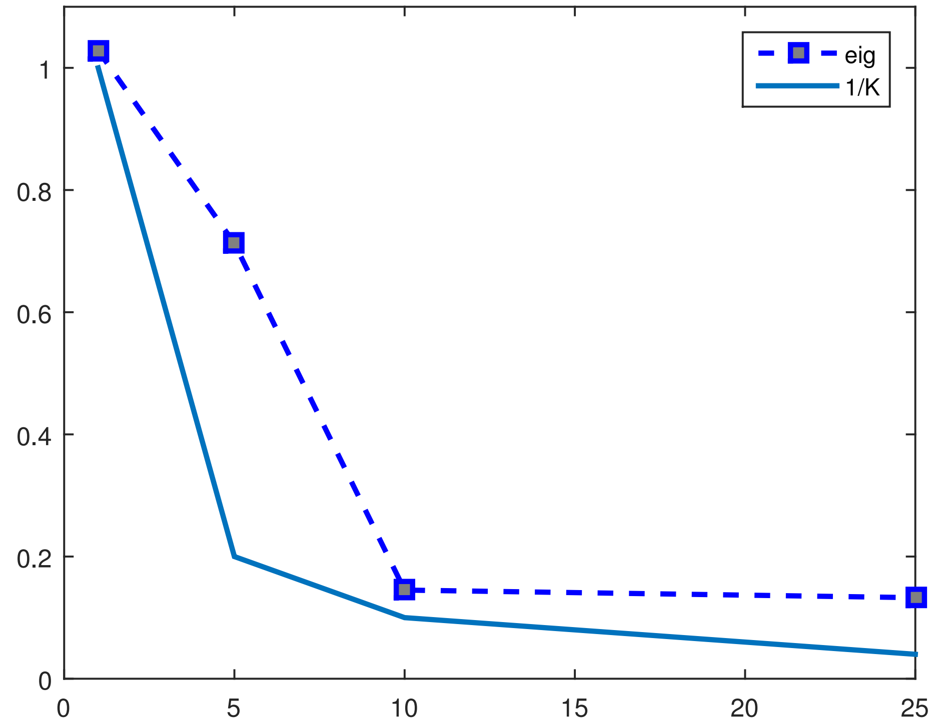

Figure 1. Standardized maximum eigenvalue of the posterior covariance matrix. Note: figure plots the standardized maximum eigenvalue of data cloning posterior the covariance matrix estimates against the $1/K$ expected value for a well-identified model.

6. Conclusions

In this paper, we applied the data cloning methodology of Lele et al. [1] to the flagship NK-DSGE model of Smets and Wouters [15]. The data-cloning method is a numerical scheme to obtain maximum-likelihood parameter estimates by artificially replicating the original sample. A byproduct of this method is an identification metric related to the posterior variability of the parameter as the number of sample clones increases.

The data cloning method is a simple way of obtaining maximum likelihood estimators, and in this respect, it allows verifying the impact of the prior structure on the estimation results of DSGE models with some simple modifications in the Bayesian methods traditionally used in the estimation of these models.

In addition, this method produces measures of local and global identification in this class of models as direct by-products. This result is particularly useful since identification diagnoses are notably more complex due to the non-linear structure and the use of the approximations and linearizations required for estimation. We believe that the data cloning method is an effective contribution to the estimation and analysis of DSGE models and can be built from the Bayesian estimation methods that are the standard in this literature.

Regarding the identification diagnostics obtained by the data cloning method for the Smets and Wouters [15] model analyzed in this work, shock-related parameters seem better identified relative to structural parameters, which has become a stylized fact. The former group displays identification problems nonetheless. Point estimates for the autoregressive coefficient of the exogenous price markup process ρ_p become larger as the number of sample clones increases, pushing the upper limit of the unit. The standard deviation of shocks to the wage-markup process σ_w does not seem well identified, which relates to the critique that Chari et al. [16] make of the NK-DSGE model of Smets and Wouters [15].

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