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H_∞ and Passive Fuzzy Control for Non-Linear Descriptor Systems with Time-Varying Delay and Sensor Faults

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Abstract: In this paper, the problem of reliable control design with mixed H_∞ /passive performance is discussed for a class of Takagi–Sugeno TS fuzzy descriptor systems with time-varying delay, sensor failure, and randomly occurred non-linearity. Based on the Lyapunov theory, firstly, a less conservative admissible criterion is established by combining the delay decomposition and reciprocally convex approaches. Then, the attention is focused on the design of a reliable static output feedback (SOF) controller with mixed H_∞ /passive performance requirements. The key merit of the paper is to propose a simple method to design such a controller since the system output is subject to probabilistic missing data and noise. Using the output vector as a state component, an augmented model is introduced, and sufficient conditions are derived to achieve the desired performance of the closed-loop system. In addition, the cone complementarity linearization (CCL) algorithm is provided to calculate the controller gains. At last, three numerical examples, including computer-simulated truck-trailer and ball and beam systems are given to show the efficacy of our proposed approach, compared with existing ones in the literature.

Keywords: descriptor systems; TS fuzzy model; sensor failure; randomly occurred non-linearity; CCL

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1. Introduction

Descriptor systems are recognized as a powerful mathematical model able to describe the dynamic behavior as well as the interconnection properties of many practical plants [1,2]; also referred to as singular systems, descriptor systems have known many applications in the fields related to electrical circuits, large-scale interconnected systems, electromechanical and constrained mechanical systems, economy, and biological systems. Due to its impulse behavior, this class of systems has received many research works [3,4]. Particularly, the discrete descriptor systems have received a great deal of attention and many research value in asymptotic stability, regularity and causality have been published in the literature [5–11].

On another hand, it is well known that time delay is ubiquitous in various engineering systems and it is the main cause of instability and performance degradation of dynamic systems. Consequently, the study of descriptor systems with delays has attracted lots of researchers' attention [12–14]. Based on the LMI approach, the direct Lyapunov method is generally used for investigating the admissibility condition for discrete-time descriptor systems with time-varying delay. Many works have been done on this topic. By taking advantage of the delay partitioning technique, a delay-dependent admissibility criterion was developed in [8]. Recently, the reciprocally convex combination method was investigated in [10] to derive a less conservative admissibility criterion. In [11], a new summation inequality, which is less conservative than the usual Jensen inequality, was introduced and utilized to analyze the admissibility problem. Usually, employing the reciprocally convex combination approach or the delay decomposition approach yields less conservative results. This motivates us to combine both approaches to revisit the admissibility condition of discrete-time descriptor delay systems.

On another research front, the analysis and design of non-linear systems is one of the most challenging issues in systems and control theory. Recently, Takagi–Sugeno TS fuzzy model was proved to be a powerful and promising method to approximate non-linear plants in applications [15,16]. In this context, the sector non-linearity approach was extensively used as a systematic method to derive an equivalent TS fuzzy model of the original non-linear system [17]. A shortcoming of this approach is that the fuzzy rules number increases exponentially with the number of non-linearities arising from the original systems, leading to increased computational costs. Benefiting from these models, many results are dedicated for non-linear descriptor systems with time delay. In the quoted papers, the admissibility analysis for discrete and continuous fuzzy descriptor systems were addressed [6,18]. For discrete-time fuzzy singular systems subject to actuator saturation, passive and H_∞ control schemes were addressed in [19,20], respectively. A second motivation of this paper is to benefit from this model to describe the system under consideration.

To keep the controlled systems less sensitive to external disturbances, a great deal of attention has been dedicated to the H_∞ control theory. Furthermore, the passivity, which is defined in terms of energy dissipation and transformation, was investigated in control engineering to deal with robust stability problems for different classes of systems, such as networked control systems [21], fuzzy systems [22], signal processing systems [23], and stochastic systems [24]. Very recently, the mixed H_∞ /passive problem was introduced as a new criterion of performance, and some studies on control were launched in [25,26]. However, to the best of our knowledge, no result on mixed H_∞ /passive control for nonlinear discrete-time descriptor systems with delay is available. On another hand, due to the environmental circumstances, many practical systems are affected by additive nonlinear exogenous disturbances, which may occur in the probabilistic way. For example, as an important class of network-induced phenomena, the randomly occurring non-linearities were largely overlooked [27,28].

Undoubtedly, it is not true to assume that the control systems are fully reliable. In fact, the failures which generally originate from the aging of sensors and actuators, the abrupt changes of working conditions, the erosion, the internal components, etc., may cause performance degradation and even instability of the system. To maintain the critical functionality and survivability of the system, it becomes of paramount importance to design reliable control systems in order to tolerate sensor failures while still retaining desired properties. Accordingly, a great work of literature has appeared on the reliable control problem with various schemes ranging from active to passive control methods. The authors in [29] provided an excellent literature review on fault-tolerant control. Recently the problem of mixed H_∞ and passivity-based reliable control for a class of stochastic TS fuzzy systems with Markovian switching and probabilistic time varying delays was studied in [30]. In [3], the sliding mode approach was explored for reliable control design of discrete-time uncertain singular Markovian jump systems with sensor fault and randomly occurring non-linearities. Unfortunately, up to now, the problem of mixed H_∞ and passive reliable control and filtering for discrete-time descriptor systems with the simultaneous presence of randomly occurred non-linearities and sensor failures remains open and unsolved, despite their engineering importance in networked control systems. This constitutes further motivation to carry out the present study. In practical applications, it is not always possible to have access to all state variables, and only partial information is available via measured outputs. The static output-feedback control problem plays a central role in control theory and application. It can be easily implemented with low cost and using augmented plants; the dynamic output-feedback control design can be formulated by the structure of the SOF controller. The SOF controller design problems were extensively discussed in recent years for TS fuzzy systems, using LMI-based convex conditions. In [31], the robust SOF H_∞ control problem for discrete-time TS fuzzy systems was studied. In [11], a new admissibility criterion was developed, and a fuzzy static output feedback controller was designed for a class of non-linear discrete-time systems with time-varying delay. In [32,33], the problem of networked fuzzy static output feedback control for discrete-time TS fuzzy systems was developed. However, except [32], the above-mentioned results are mainly focused on the case where the measured output vector is not influenced by the noise. To handle this real case, the main objective of this paper is to pave the way for dealing with the problem of reliable fuzzy SOF controller design for TS fuzzy descriptor systems subject to time-varying delay, stochastic non-linearities and sensor failures. Based on the above considerations, the main contributions of this paper are summarized as follows:

- The system under consideration is subject to real factors, such as time-varying delay, uncertainties, and random non-linear external disturbances. Moreover, by employing the delay decomposition and reciprocally convex approaches, a new admissible criterion is established to improve the existing ones;
- Design a new reliable SOF controller for a descriptor system subject to stochastic non-linearities and sensors failures;
- Provide a simple method of the controller design based on introducing appropriate augmented closed-loop systems that decouple the output matrices and controller gain matrices;
- Different from the works of [19,20], a new admissibility criterion with mixed H_∞ /passive performance is derived for the closed-loop system, and with the help of the CCL algorithm, the effective reliable controllers gains are obtained..

The rest of the paper is organized as follows. In Section 2, some preliminaries are introduced. The admissibility analysis is conducted in Section 3. In Section 4, the design procedure of the fuzzy reliable static output feedback is presented, and a computational algorithm is given to characterize the design of the mixed H_∞ /passive controller. Section 5 is dedicated to show the effectiveness of the theoretical results through three illustrative examples. A conclusion is given in Section 6.

Notations. The notations in this paper are quite standard, except where otherwise stated. The superscript 'T' stands for matrix transposition; $X \in \mathbb{R}^n$ denotes the n-dimensional Euclidean space, while $X \in \mathbb{R}^{n\times m}$ refers to the set of all $n \times m$ real matrices; $X > 0$ (respectively, $X \geq 0$) means that the matrix X is real symmetric positive definite (respectively, positive semi-definite); $l_2[0,\ \infty)$ is the space of square summable vectors; I and 0 represent the identity matrix and a zero matrix with appropriate dimensions, respectively; $diag\{\cdots\}$ stands for a block-diagonal matrix; $\mathrm{sym}(X)$ stands for $X + X^T$; $\|.\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix; and $\mathbb{E}[.]$ stands for the mathematical expectation. In symmetric block matrices, we use a star $*$ to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. $\lambda_{max}()$ means the largest eigenvalue of a matrix. To avoid clutter, in what follows, h_i denotes $h_i(\theta)$, X_θ represents the convex combination $\sum_{i=1}^{r} h_i X_i$, and $X_{\theta\theta}$ will denote a convex combination of the form $\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j X_{ij}$.

2. Preliminaries

In this paper, the following TS fuzzy descriptor system is considered to describe the dynamic of a non-linear plant with time-varying delay:

$$\begin{cases} Ex(k+1) = \sum_{i=1}^{r} h_i(\theta)\Big\{A_i(k)x(k) + A_{di}(k)x(k-d(k)) + B_{1i}w(k) + B_i u(k) + \zeta(k)H_i f(x(k))\Big\}, \\ z(k) = \sum_{i=1}^{r} h_i(\theta)\Big\{C_{1i}x(k) + D_{1i}w(k)\Big\}, \\ y(k) = \sum_{i=1}^{r} h_i(\theta)\Big\{C_{2i}x(k) + C_{2di}(k)x(k-d(k)) + \beta(k)\varphi(x(k)) + D_{2i}\nu(k))\Big\}, \\ x(k) = \phi_0(l),\ l \in [-d_M\ 0] \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$ is the control input, $f(x(k))$ and $\varphi(x(k))$ are randomly occurred non-linear functions, $y(k) \in \mathbb{R}^p$ is the measured output, $z(k) \in \mathbb{R}^{nz}$ is the controlled output, and $w(k) \in \mathbb{R}^w$ and $\nu(k) \in \mathbb{R}^\nu$ are the the exogenous disturbance signals, which are assumed to belong to $l_2[0,\infty)$. $\phi_0(l)$ is a vector-valued function on $[-d_M\ 0]$. Delay $d(k)$ is time-varying and satisfies the following:

$$d_m \le d(k) \le d_M \tag{2}$$

where d_m and d_M are positive integers.
In this paper, we assume that matrix $E \in \mathbb{R}^{n\times n}$ may be singular and assume that $\text{rank}(E) = q < n$. $A_i(k)$ and $A_{di}(k)$ are matrices with time-varying uncertainties, that is, the following:

$$A_i(k) = A_i + \Delta A_i(k), \quad A_{di}(k) = A_{di} + \Delta A_{di}(k) \tag{3}$$

Matrices A_i, A_{di}, B_{1i}, B_i, H_i, C_{1i}, D_{1i}, C_{2i}, C_{2di} and D_{2i} are known with appropriate dimensions. $\Delta A_i(k)$ and $\Delta A_{di}(k)$ are unknown matrices representing time-varying parameter uncertainties.
In dealing with this study, the following assumptions are necessary for further development:

1. $\Delta A_i(k)$ and $\Delta A_{di}(k)$ are assumed to satisfy the following admissible conditions:

$$\begin{bmatrix}\Delta A_i(k) & \Delta A_{di}(k)\end{bmatrix} = M_i F(k)\begin{bmatrix}N_i & N_{di}\end{bmatrix},\ i = 1,2,\ldots,r \tag{4}$$

   where M_i, N_i and N_{di} are known real constant matrices of appropriate dimensions, and $F(k) \in \mathbb{R}^l$ is an unknown time-varying matrix function subject to $F^T(k)F(k) \le I \quad \forall k$.
2. The non-linear functions $f(x(k))$ and $\varphi(x(k))$ are assumed to be continuous and satisfies the following conditions:

$$\begin{aligned} & f(0) = 0, \quad \varphi(0) = 0 \\ & [f(x(k)) - \Psi_1 x(k)]^T[f(x(k)) - \Omega_1 x(k)] \le 0 \\ & [\varphi(x(k)) - \Psi_2 x(k)]^T[\varphi(x(k)) - \Omega_2 x(k)] \le 0 \end{aligned} \tag{5}$$

   where Ψ_1, Ω_1, Ψ_2 and Ω_2 are real matrices with compatible dimensions.

As is well known, for many practical processes, such as networked control systems, the system output has a probabilistic aspect, due to the randomly missing data. In this study, we assume that the stochastic variables $\zeta(k) \in \mathbb{R}$ and $\beta(k) \in \mathbb{R}$ are Bernoulli stochastic white sequences with the following distribution laws:

$$\begin{cases} \text{Prob}\{\zeta(k)=1\} &= \mathbb{E}\{\zeta(k)\} = \bar{\zeta}, \quad 0 \le \bar{\zeta} \le 1, \quad \text{Prob}\{\zeta(k)=0\} = 1-\bar{\zeta} \\ \text{Prob}\{\beta(k)=1\} &= \mathbb{E}\{\beta(k)\} = \bar{\beta}, \quad 0 \le \bar{\beta} \le 1, \quad \text{Prob}\{\beta(k)=0\} = 1-\bar{\beta} \end{cases} \tag{6}$$

Obviously, for the stochastic variables ζ_k and β_k, we show the following:

$$\mathbb{E}\{(\zeta(k)-\bar{\zeta})^2\}=\bar{\zeta}(1-\bar{\zeta}), \quad \mathbb{E}\{(\beta(k)-\bar{\beta})^2\}=\bar{\beta}(1-\bar{\beta}) \tag{7}$$

Remark 1.

- *In order to obtain a TS fuzzy model with a few rules, we can perform the sector non-linearity approach [17] for a restrictive number of non-linear terms included in the system under consideration. In addition, due to the environmental circumstances, such as random failures of the system components, sudden environment changes and unexpected change in the subsystem interconnections, etc., the processes are probably influenced by additive randomly occurred non-linear disturbances. Consequently, the term $\zeta_k H_i f(x(k))$ in (1) involves both model uncertainties and random occurred non-linearities.*
- *It is worth mentioning that sensors may not always produce ideal signals, due mainly to environmental constraints. The system's output in (1) reflects tightly the reality; however, it turns out that the controller design is more difficult.*
- *In this study, it is assumed that the non-linear functions belong to sectors. This description, suggested in [34], is more general, and includes the usual Lipschitz conditions as a special case.*

Consider the following autonomous discrete-time descriptor system:

$$\begin{cases} Ex(k+1)=Ax(k)+A_d x(k-d(k))+B_1 w(k) \\ \quad z(k)=Cx(k)+Dw(k) \end{cases} \tag{8}$$

Throughout the paper, the following definitions are adopted.

Definition 1 ([1,35])**.**

1. *pair (E,A) is said to be regular if* $\det(zE-A)$ *is not identically zero;*
2. *pair (E,A) is said to be causal, if it is regular and $deg\Big(det(zE-A)\Big)=rank(E)$;*
3. *Pair (E,A) is said to be admissible, if it is regular, causal and stable;*

Definition 2 ([22])**.** *For $\gamma>0$ and $0\le\phi\le 1$, descriptor system (8) is said to be mean-square admissible with a mixed H_∞ /passive performance γ, if under zero initial condition, the following inequality holds:*

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}\Big(\gamma^{-1}\phi z^T(k)z(k)-2\gamma(1-\phi)z^T(k)w(k)\Big)\right\}<\gamma\mathbb{E}\left\{\sum_{k=0}^{\infty}w^T(k)w(k)\right\} \tag{9}$$

for all $0\neq w(k)\in L_2[0,\infty)$.

Remark 2. *Definition 2 includes both H_∞ and strict passivity performances as special cases by choosing different values for ϕ.*

- *If $\phi=1$, inequality (9) reduces to an H_∞ performance requirement.*
- *If $\phi=0$, inequality (9) corresponds to the passivity performance index.*

We recall the following lemmas to be used in the proof of our main results.

Lemma 1 ([36])**.** *Given matrices M, N and $P=P^T$ of appropriate dimensions, then*

$$P+MF(k)N+N^TF^T(k)M^T<0$$

for any $F(k)$ satisfying $F^T(k)F(k) \le I$, if and only if there exists a scalar $\varepsilon > 0$ such that the following holds:

$$P + \varepsilon MM^T + \varepsilon^{-1}N^TN < 0 \tag{10}$$

Lemma 2 ([37]). *For any vectors ψ_1, ψ_2, matrices S, R and real scalars $\alpha_1 \ge 0, \alpha_2 \ge 0$, satisfying the following:*

$$\begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \ge 0, \quad \alpha_1 + \alpha_2 = 1,\ \psi_i = 0 \text{ if } \alpha_i = 0\ (i = 1,2)$$

we have the following:

$$-\frac{1}{\alpha_1}\psi_1^T R\psi_1 - \frac{1}{\alpha_2}\psi_2^T R\psi_2 \le -\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}^T \begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}$$

3. Admissibility Analysis

In this section, a new sufficient delay-dependent condition is established to deal with causality and stability for the nominal system of (1) (i.e., $u(k) = 0$, $w(k) = 0$ and $\zeta(k) = 0$). By invoking the delay bi-partitioning and improved reciprocally convex combination approaches, an LMI-based criterion is derived so that the descriptor system with delay is admissible.

Consider the following nominal compact presentation of (1).

$$Ex(k+1) = A_\theta x(k) + A_{d\theta}x(k - d(k)) \tag{11}$$

As in [38], we denote

$$\delta = \frac{d_M + d_m}{2} + \frac{1}{2}\min\{(-1)^{d_M+d_m}, 0\}, \quad \tau_M = d_M - \delta, \quad \tau_m = \delta - d_m \quad d_r = d_M - d_m \tag{12}$$

$$\begin{aligned}
&\eta(k) = x(k+1) - x(k), \quad \Gamma(k) = \mathrm{col}\left\{Ex(k), E\eta(k)\right\}, \\
&\xi(k) = \mathrm{col}\left\{x(k), x(k-d_m), x(k-d(k)), x(k-d_M), x(k-\delta), \sum_{s=k-d_m}^{k-1} Ex(s), \sum_{s=k-d_M}^{k-d(k)-1} Ex(s), \right. \\
&\left. \sum_{s=k-d(k)}^{k-d_m-1} Ex(s) \right\}
\end{aligned} \tag{13}$$

$$\begin{aligned}
&\Gamma_1 = \mathrm{col}\left\{e_6, E(e_1 - e_2)\right\}, \quad \Gamma_{21} = \mathrm{col}\left\{e_7, E(e_3 - e_4)\right\}, \quad \Gamma_{22} = \mathrm{col}\left\{e_8, E(e_2 - e_3)\right\}, \\
&\Gamma_2 = \mathrm{col}\left\{\Gamma_{21}, \Gamma_{22}\right\}, \quad \Gamma_3 = \mathrm{col}\left\{E(e_5 - e_4), E(e_3 - e_5), E(e_2 - e_3)\right\}, \\
&\Gamma_4 = \mathrm{col}\left\{E(e_2 - e_5), E(e_4 - e_3), E(e_3 - e_5)\right\} \\
&e_i = \begin{bmatrix} 0_{n,(i-1)n} & I_n & 0_{n,(8-i)n} \end{bmatrix}, \quad i = 1, 2, \cdots, 8.
\end{aligned} \tag{14}$$

Theorem 1. *Given positive integers d_m and d_M. If there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Q_4 > 0$, $Z_1 > 0$, $Z_2 > 0$, $W_1 > 0$, $W_2 > 0$, R_1, R_{21}, R_{22}, S_{11}, S_{12}, S_{21}, S_{22}, S, X and Y satisfying the following:*

$$\begin{cases} \Phi_{1i} = \Phi_0 + \Phi(E, \mathbb{A}_i, \mathbb{A}_{Ei}, \mathbb{A}_{Si}) - \Gamma_3^T \mathbb{W}_1 \Gamma_3 < 0 & (15) \\ \Phi_{2i} = \Phi_0 + \Phi(E, \mathbb{A}_i, \mathbb{A}_{Ei}, \mathbb{A}_{Si}) - \Gamma_4^T \mathbb{W}_2 \Gamma_4 < 0 & (16) \\ \mathbb{Z}_1 > 0, \quad \mathbb{Z}_2 > 0, \quad \mathbb{W}_1 > 0, \quad \mathbb{W}_2 > 0 & (17) \end{cases}$$

where

$$\begin{aligned} \Phi_0 &= e_1^T \Big(Q_1 + Q_2 + Q_3 + (d_r + 1) Q_4 - E^T P E + d_m E^T R_1 E \Big) e_1 \\ &- e_2^T \Big(Q_1 + d_m E^T R_1 E + d_r E^T R_{22} E \Big) e_2 + e_3^T (-Q_4 + d_r E^T (R_{21} - R_{22}) E) e_3 \\ &- e_4^T \left(+ Q_2 + d_r E^T R_{21} E \right) e_4 - e_5^T Q_3 e_5 - \Gamma_1^T \mathbb{Z}_1 \Gamma_1 - \Gamma_2^T \mathbb{Z}_2 \Gamma_2 \end{aligned} \quad (18)$$

$$\Phi(E, \mathbb{A}_i, \mathbb{A}_{Ei}, \mathbb{A}_{Si}) = \mathbb{A}_i^T P \mathbb{A}_i + \mathbb{A}_{Ei}^T \Big(\tau_m^2 W_1 + \tau_M^2 W_2 \Big) \mathbb{A}_{Ei} + \Upsilon_i^T \Big(d_m^2 Z_1 + d_r^2 Z_2 \Big) \Upsilon_i \quad (19)$$

$$+ \operatorname{sym} \Big(e_1^T S R_0^T \mathbb{A}_{Si} \Big) \quad (20)$$

$$\begin{aligned} & Z_1 = \begin{bmatrix} Z_1^{11} & Z_1^{12} \\ * & Z_1^{22} \end{bmatrix} \quad Z_2 = \begin{bmatrix} Z_2^{11} & Z_2^{12} \\ * & Z_2^{22} \end{bmatrix} \quad \mathbb{Z}_1 = \begin{bmatrix} Z_1^{11} & Z_1^{12} + R_1 \\ * & Z_1^{22} + R_1 \end{bmatrix} \quad \mathbb{Z}_{21} = \begin{bmatrix} Z_2^{11} & Z_2^{12} + R_{21} \\ * & Z_2^{22} + R_{21} \end{bmatrix} \\ & \mathbb{Z}_{22} = \begin{bmatrix} Z_2^{11} & Z_2^{12} + R_{22} \\ * & Z_2^{22} + R_{22} \end{bmatrix} \quad \mathbb{W}_1 = \begin{bmatrix} W_2 & 0 & 0 \\ * & W_1 & X \\ * & * & W_1 \end{bmatrix} \quad \mathbb{W}_2 = \begin{bmatrix} W_1 & 0 & 0 \\ * & W_2 & Y \\ * & * & W_2 \end{bmatrix} \\ & \mathbb{Z}_2 = \begin{bmatrix} \mathbb{Z}_{21} & \mathbb{S} \\ * & \mathbb{Z}_{22} \end{bmatrix} \quad \mathbb{S} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}, \\ & \mathbb{A}_i = A_i e_1 + A_{di} e_3, \quad \mathbb{A}_{Ei} = (A_i - E) e_1 + A_{di} e_3, \quad \mathbb{A}_{Si} = A_i e_1 + A_{di} e_3, \quad \Upsilon_i = \operatorname{col}\{E e_1, \mathbb{A}_{Ei}\} \end{aligned} \quad (21)$$

then, system (11) is admissible. $R_0 \in \mathbb{R}^{n \times n-r}$ *is any matrix with full column rank satisfying* $E^T R_0 = 0$.

Proof. First, we prove the regularity and causality properties of the system.
Since $\operatorname{rank}(E) = q < n$, there always exist two non-singular matrices $\hat{M}$ and $\hat{N} \in \mathbb{R}^{n \times n}$ such that the following holds:

$$\hat{E} = \hat{M} E \hat{N} = \begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix} \quad (22)$$

Then, R_0 can be characterized as the following: $R_0 = \hat{M}^T \begin{bmatrix} 0 \\ \hat{R}_0 \end{bmatrix}$, where $\hat{R}_0 \in \mathbb{R}^{(n-q) \times (n-q)}$ is any non-singular matrix. We define also the following:

$$\hat{A}_i = \hat{M} A_i \hat{N} = \begin{bmatrix} \hat{A}_{11i} & \hat{A}_{12i} \\ \hat{A}_{21i} & \hat{A}_{22i} \end{bmatrix}, \quad \hat{S} = \hat{N}^T S = \begin{bmatrix} \hat{S}_{11} \\ \hat{S}_{21} \end{bmatrix}, \quad \hat{A}_{di} = \hat{M} A_{di} \hat{N} = \begin{bmatrix} \hat{A}_{d11i} & \hat{A}_{d12i} \\ \hat{A}_{d21i} & \hat{A}_{d22i} \end{bmatrix}. \quad (23)$$

From (15) and (16) the following inequality holds:

$$\begin{aligned} & \operatorname{sym}(S R_0^T A_i) + d_m^2 \operatorname{sym}(E^T Z_1^{12} (A_i - E)) + d_r^2 \operatorname{sym}(E^T Z_2^{12} (A_i - E)) \\ & + d_m E^T R_1 E - E^T (Z_1^{22} + R_1) E + d_m^2 E^T Z_1^{11} E + d_r^2 E^T Z_2^{11} E - E^T P E < 0 \end{aligned} \quad (24)$$

Since $h_i(\theta) \geq 0$ and $\sum_{i=1}^{r} h_i(\theta) = 1$, it yields the following:

$$\begin{aligned} & \operatorname{sym}(S R_0^T A_\theta) + d_m^2 \operatorname{sym}(E^T Z_1^{12} (A_\theta - E)) + d_r^2 \operatorname{sym}(E^T Z_2^{12} (A_\theta - E)) \\ & + d_m E^T R_1 E - E^T (Z_1^{22} + R_1) E + d_m^2 E^T Z_1^{11} E + d_r^2 E^T Z_2^{11} E - E^T P E < 0 \end{aligned} \quad (25)$$

Pre- and post-multiplying (25) by $\hat{M}^T$ and $\hat{M}$, respectively, and then using expressions (22) and (23) gives the following:

$$sym(\hat{S}_{21}\hat{R}_0^T\hat{A}_{22\theta}) < 0 \tag{26}$$

and $\hat{A}_{22\theta}$ is, thus, non-singular.
For the stability analysis of system (11), we construct the following Lyapunov–Krasovskii functional:

$$\begin{aligned}
&V(k) = V_1(k) + V_3(k) + V_3(k) + V_4(k)\\
&V_1(k) = x^T(k)E^TPEx(k)\\
&V_2(k) = \sum_{s=k-d_m}^{k-1} x^T(s)Q_1x(s) + \sum_{s=k-d_M}^{k-1} x^T(s)Q_2x(s)\\
&+ \sum_{s=k-\delta}^{k-1} x^T(s)Q_3x(s) + \sum_{\theta=-d_M}^{-d_m}\sum_{s=k+\theta}^{k-1} x^T(s)Q_4x(s)\\
&V_3(k) = \tau_m \sum_{\theta=-\delta}^{-d_m-1}\sum_{s=k+\theta}^{k-1} \eta^T(s)E^TW_1E\eta(s) + \tau_M \sum_{\theta=-d_M}^{-\delta-1}\sum_{s=k+\theta}^{k-1} \eta^T(s)E^TW_2E\eta(s)\\
&V_4(k) = d_m \sum_{\theta=-d_m}^{-1}\sum_{s=k+\theta}^{k-1} \Gamma^T(s)Z_1\Gamma(s) + d_r \sum_{\theta=-d_M}^{-d_m-1}\sum_{s=k+\theta}^{k-1} \Gamma^T(s)Z_2\Gamma(s)
\end{aligned} \tag{27}$$

Calculating the difference of $V(k)$ along the system trajectories, one obtains the following:

$$\begin{aligned}
&\Delta V_1(k) = \Big(A_\theta x(k) + A_{d\theta}x(k-d(k))\Big)^T P\Big(A_\theta x(k) + A_{d\theta}x(k-d(k))\Big) - E^TPE\\
&= \xi^T(k)\Big(\mathbb{A}_\theta^T P\mathbb{A}_\theta - e_1^TE^TPEe_1\Big)\xi(k)\\
&\Delta V_2(k) \le \xi^T(k)\Big(e_1^T(Q_1+Q_2+Q_3+(d_r+1)Q_4)e_1 - e_2^TQ_1e_2 - e_4^TQ_2e_4 - e_5^TQ_3e_5\\
&- e_3^TQ_4e_3\Big)\xi(k)\\
&\Delta V_3(k) = \xi^T(k)\mathbb{A}_{E\theta}^T\Big(\tau_m^2W_1 + \tau_M^2W_2\Big)\mathbb{A}_{E\theta}\xi(k) - \tau_m \sum_{s=k-\delta}^{k-d_m-1} \eta^T(s)E^TW_1E\eta(s)\\
&- \tau_M \sum_{s=k-d_M}^{k-\delta-1} \eta^T(s)E^TW_2E\eta(s)\\
&\Delta V_4(k) = \Gamma^T(k)\Big(d_m^2Z_1 + d_r^2Z_2\Big)\Gamma(k) - d_m \sum_{s=k-d_m}^{k-1} \Gamma^T(s)Z_1\Gamma(s) - d_r \sum_{s=k-d_M}^{k-d(k)-1} \Gamma^T(s)Z_2\Gamma(s)\\
&- d_r \sum_{s=k-d(k)}^{k-d_m-1} \Gamma^T(s)Z_2\Gamma(s)
\end{aligned} \tag{28}$$

For any symmetrical matrix R_1, one can verify the following:

$$\begin{aligned}
&\sum_{s=k-d_m}^{k-1}\Big(x^T(s+1)E^TR_1Ex(s+1) - x^T(s)E^TR_1Ex(s)\Big)\\
&= \sum_{s=k-d_m}^{k-1} \eta^T(s)E^TR_1E\eta(s) + 2x^T(s)E^TR_1E\eta(s) = \sum_{s=k-d_m}^{k-1} \Gamma^T(s)\begin{bmatrix} 0 & R_1 \\ R_1^T & R_1 \end{bmatrix}\Gamma(s)\\
&= x^T(k)E^TR_1Ex(k) - x^T(k-d_m)E^TR_1Ex(k-d_m)
\end{aligned} \tag{29}$$

From Equation (29), the following null equation holds:

$$x^T(k)E^TR_1Ex(k) - x^T(k-d_m)E^TR_1Ex(k-d_m) - \sum_{s=k-d_m}^{k-1}\Gamma^T(s)\begin{bmatrix} 0 & R_1 \\ R_1^T & R_1 \end{bmatrix}\Gamma(s) = 0 \tag{30}$$

Then, by adding the previous zero-value equation and using the Jensen inequality, algebraic manipulation yields the following:

$$\begin{aligned} -d_m\sum_{s=k-d_m}^{k-1}\Gamma^T(s)Z_1\Gamma(s) = {}& \xi^T(k)\Big(d_me_1^TE^TR_1Ee_1 \\ & - d_me_2^TE^TR_1Ee_2\Big)\xi(k) - d_m\sum_{s=k-d_m}^{k-1}\Gamma^T(s)\mathbb{Z}_1\Gamma(s) \\ & \leq \xi^T(k)\Big(d_me_1^TE^TR_1Ee_1 - d_me_2^TE^TR_1Ee_2\Big)\xi(k) - \Gamma_1^T\mathbb{Z}_1\Gamma_1 \end{aligned} \tag{31}$$

Following the same procedure as above, the following inequalities can be established:

$$\begin{aligned} & - d_r\sum_{s=k-d_M}^{k-d(k)-1}\Gamma^T(s)Z_2\Gamma(s) \leq \xi^T(k)\Big(d_re_3^TE^TR_{21}Ee_3 - d_re_4^TE^TR_{21}Ee_4\Big)\xi(k) \\ & - \frac{d_r}{d_M - d(k)}\Gamma_{21}^T\mathbb{Z}_{21}\Gamma_{21} \\ & - d_r\sum_{s=k-d(k)}^{k-d_m-1}\Gamma^T(s)Z_2\Gamma(s) \leq \xi^T(k)\Big(d_re_2^TE^TR_{22}Ee_2 - d_re_3^TE^TR_{22}Ee_3\Big)\xi(k) \\ & - \frac{d_r}{d(k) - d_m}\Gamma_{22}^T\mathbb{Z}_{22}\Gamma_{22} \end{aligned} \tag{32}$$

According to Lemma 2, one obtains the following:

$$\begin{aligned} & - d_r\sum_{s=k-d_M}^{k-d_m-1}\Gamma^T(s)Z_2\Gamma(s) \leq \xi^T(k)\Big(d_re_2^TE^TR_{22}Ee_2 + d_re_3^TE^T(R_{21} - R_{22})Ee_3 \\ & - d_re_4^TE^TR_{21}Ee_4\Big)\xi(k) - \Gamma_2^T\mathbb{Z}_2\Gamma_2 \end{aligned} \tag{33}$$

On the other hand, when $d_m \leq d(k) \leq \delta$, we derive the following:

$$\begin{aligned} -\tau_m\sum_{s=k-\delta}^{k-d_m-1}\eta^T(s)E^TW_1E\eta(s) = {}& -\tau_m\sum_{s=k-\delta}^{k-d(k)-1}\eta^T(s)E^TW_1E\eta(s) \\ & - \tau_m\sum_{s=k-d(k)}^{k-d_m-1}\eta^T(s)E^TW_1E\eta(s) \\ & \leq -\frac{\tau_m}{\delta - d(k)}\xi^T(k)(e_3 - e_5)^TE^TW_1E(e_3 - e_5)\xi(k) \\ & - \frac{\tau_m}{d(k) - d_m}\xi^T(k)(e_2 - e_3)^TE^TW_1E(e_2 - e_3)\xi(k) \end{aligned} \tag{34}$$

and

$$-\tau_M\sum_{s=k-d_M}^{k-\delta-1}\eta^T(s)E^TW_2E\eta(s) \leq -\xi^T(k)(e_5 - e_4)^TE^TW_2E(e_5 - e_4)\xi(k) \tag{35}$$

Then, the following condition holds using Lemma 2:

$$-\tau_m \sum_{s=k-\delta}^{k-d_m-1} \eta^T(s)E^T W_1 E\eta(s) - \tau_M \sum_{s=k-d_M}^{k-\delta-1} \eta^T(s)E^T W_2 E\eta(s) \le -\Gamma_3^T \mathbb{W}_1 \Gamma_3 \quad (36)$$

Note that $E^T R_0 = 0$, for any matrix S with appropriate dimensions yields the following:

$$2x(k)SR_0^T Ex(k+1) = 2\xi^T(k)e_1^T SR_0^T \mathbb{A}_{S\theta} = 0 \quad (37)$$

Combining (28)–(37), we have the following:

$$\Delta V(k) \le \xi^T(k)\Phi_{1\theta}\xi(k) \quad (38)$$

When $\delta \le d(k) \le d_M$, by proceeding as before, it is readily seen that the following condition is verified:

$$-\tau_m \sum_{s=k-\delta}^{k-d_m-1} \eta^T(s)E^T W_1 E\eta(s) - \tau_M \sum_{s=k-d_M}^{k-\delta-1} \eta^T(s)E^T W_2 E\eta(s) \le -\Gamma_4^T \mathbb{W}_2 \Gamma_4 \quad (39)$$

and then, we obtain the following:

$$\Delta V(k) \le \xi^T(k)\Phi_{2\theta}\xi(k) \quad (40)$$

Hence, under the conditions of Theorem 1, we can deduce from (38) and (40) the following:

$$V(k+1) \le V(k) - \alpha\|\xi(k)\|^2,\ \alpha = \max\left\{\lambda_{max}\left\{\Phi_{1i}\right\}, \lambda_{max}\left\{\Phi_{2i}\right\}\right\} \quad (41)$$

From (41), one can obtain the following:

$$V(k+1) \le V(0) - \alpha \sum_{l=0}^{k} \|\xi(l)\|^2 \quad (42)$$

and therefore, we obtain the following:

$$\sum_{l=0}^{k} \|\xi(l)\|^2 \le \frac{1}{\alpha} V(0) < \infty \quad (43)$$

which implies $\lim_{k \longrightarrow \infty} \xi(k) = 0$. Thus, system (11) is stable. This concludes the proof. □

Remark 3.

- *We would like to stress that in order to obtain less conservative stability conditions for discrete-time singular systems, the authors in [10] proposed a new Lyapunov functional with triple sum, and the reciprocally convex combination approach is extended to bound the double summable term. The authors in [11] developed a new summation inequality as a less conservative extension of the Jensen inequality. However, the key merit of the obtained less conservative criterion lies in the application of the delay partitioning method combined with the improved reciprocally convex combination approach.*

4. Reliable SOF Controller Design

In this section, we shall focus on the reliable output feedback control design problem whose purpose is to design a reliable mixed H_∞ /passive controller for system (1) via a fuzzy static output controller. Assume that the sensors suffer from failures. The following model of failure is adopted in this paper to describe the measured signal sent from the sensors as follows:

$$y^F(k) = R(k)y(k) \tag{44}$$

where $R(k)$ is the sensor fault matrix defined as follows:

$$\begin{cases} R(k) = \operatorname{diag}\left(r_1(k), r_2(k), \cdots, r_p(k)\right) \\ \underline{r}_s \leq r_s(k) \leq \bar{r}_s,\ s = 1, 2 \cdots, p \end{cases} \tag{45}$$

where $r_s(k)$ is the degradation level of the 's'th sensor.
Let the following hold:

$$\begin{cases} Q = \operatorname{diag}\left(q_1, q_2, \cdots, q_p\right), \quad q_s = \dfrac{\bar{r}_s - \underline{r}_s}{\underline{r}_s + \bar{r}_s}, \\ R_0 = \operatorname{diag}\left(r_{01}, r_{02}, \cdots, r_{0p}\right), \quad r_{0s} = \dfrac{\underline{r}_s + \bar{r}_s}{2}, \ (s = 1, 2 \cdots, p) \\ G(k) = \operatorname{diag}\left(g_1(k), g_2(k), \cdots, g_p(k)\right), \quad g_s(k) = \dfrac{r_s(k) - r_{0s}}{r_{0s}} \end{cases} \tag{46}$$

Matrix $R(k)$ can be rewritten as the following:

$$R(k) = R_0(I + G(k)) \tag{47}$$

It can be verified that the following holds:

$$\|G(k)\| \leq \|Q\| \tag{48}$$

The problem of mixed H_∞ /passive reliable SOF control could be phrased as follows: given the fuzzy system in (1), determine a reliable SOF control law as follows:

$$u(k) = \sum_{i=1}^{r} h_i K_i R_0^{-1} y^F(k) = K_\theta R_0^{-1} y^F(k) \tag{49}$$

such that, for all admissible sensor failures and exogenous disturbances, the following requirements are ensured:

- The resulting closed-loop system is robustly mean-square admissible,
- Under zero-initial condition, the mixed H_∞ /passive performance is satisfied in the sense of Definition 2.

K_i is compatible with the dimensional control gain.
Since the system output contains probabilistic missing data and noise, the design of the SOF controller using the standard approach is made difficult, even impossible. To overcome this problem, we propose the transformation that considers $y(k)$ as a state component. The closed-loop system has the following structure:

$$(\Sigma_c) : \begin{cases} \bar{E}\bar{x}(k+1) & = \bar{A}_{\theta\theta}(k)\bar{x}(k) + \bar{A}_{d\theta}(k)\bar{x}(k - d(k)) + \bar{B}_{1\theta}\bar{w}(k) \\ & +\bar{\Pi}\bar{H}_\theta \bar{f} + (\Pi(k) - \bar{\Pi})\bar{H}_\theta \bar{f}, \\ z(k) & = \bar{C}_{1\theta}\bar{x}(k) + \bar{D}_{1\theta}\bar{w}(k), \end{cases} \tag{50}$$

where $\bar{x}(k) = \begin{bmatrix} x^T(k) & y^T(k) \end{bmatrix}^T$, $\bar{w}(k) = \begin{bmatrix} w^T(k) & \nu^T(k) \end{bmatrix}^T$, $\bar{f} = \begin{bmatrix} f^T(x(k)) & \varphi^T(x(k)) \end{bmatrix}^T$, $\bar{A}_{\theta\theta}(k) = \bar{A}_{\theta\theta} + \bar{M}_\theta F(k)\bar{N}_\theta$, $\bar{A}_{d\theta}(k) = \bar{A}_{d\theta} + \bar{M}_\theta F(k)\bar{N}_{d\theta}$,

$$\begin{aligned} &\bar{E} = \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{A}_{\theta\theta} = \begin{bmatrix} A_\theta & B_\theta K_\theta(k) \\ C_{2\theta} & -I \end{bmatrix}, \quad \bar{A}_{d\theta} = \begin{bmatrix} A_{d\theta} & 0 \\ C_{2d\theta} & 0 \end{bmatrix}, \\ &\bar{B}_{1\theta} = \begin{bmatrix} B_{1\theta} & 0 \\ 0 & D_{2\theta} \end{bmatrix}, \quad \bar{C}_{1\theta} = \begin{bmatrix} C_{1\theta} & 0 \end{bmatrix}, \quad \bar{D}_{1\theta} = \begin{bmatrix} D_{1\theta} & 0 \end{bmatrix}, \quad \bar{H}_\theta = \begin{bmatrix} H_\theta & 0 \\ 0 & I \end{bmatrix}, \\ &\bar{M}_\theta = \begin{bmatrix} M_\theta^T & 0 \end{bmatrix}^T, \quad \bar{N}_\theta = \begin{bmatrix} N_\theta & 0 \end{bmatrix}, \quad \bar{N}_{d\theta} = \begin{bmatrix} N_{d\theta} & 0 \end{bmatrix}, \quad \Pi(k) = \begin{bmatrix} \zeta(k) & 0 \\ 0 & \beta(k) \end{bmatrix}, \end{aligned} \tag{51}$$

Easily, we can verify the following: $\bar{A}_{\theta\theta} = \bar{A}_\theta^0 + \bar{B}_\theta \bar{K}_\theta (I + G(k))$. where

$$\bar{A}_\theta^0 = \begin{bmatrix} A_\theta & 0 \\ C_{2\theta} & -I \end{bmatrix}, \qquad \bar{B}_\theta = \begin{bmatrix} B_\theta \\ 0 \end{bmatrix}, \qquad \bar{K}_\theta = \begin{bmatrix} 0 & K_\theta \end{bmatrix}, \tag{52}$$

4.1. Mixed H_∞ /Passive Analysis

Our goal here is to establish a tractable condition satisfying system (50) to be mean-square admissible with a mixed weighted H_∞ /passive performance index γ.

Theorem 2. *Given positive integers d_m, d_M and a positive scalar $0 \le \phi \le 1$. The closed-loop system (Σ_c) without uncertainties is admissible in the mean square for $w(k) = 0$ and satisfies the mixed weighted H_∞ /passive performance index (9) under zero initial condition for any non-zero $w \in l_2[0,\infty)$, if there exist a positive scalar τ and matrices $\bar{P} > 0$, $\bar{Q}_1 > 0$, $\bar{Q}_2 > 0$, $\bar{Q}_3 > 0$, $\bar{Q}_4 > 0$, $\bar{W}_1 > 0$, $\bar{W}_2 > 0$, $\bar{V} > 0$, $\bar{S}$, $\bar{X}$ and $\bar{Y}$ satisfying the following:*

$$\begin{cases} \bar{\Phi}_{1ij}\left(\bar{\mathcal{A}}_{ij}, \bar{\mathcal{A}}_{Eij}, \bar{\mathcal{A}}_{Sij}^0\right) < 0 \ (53) \\ \bar{\Phi}_{2ij}\left(\bar{\mathcal{A}}_{ij}, \bar{\mathcal{A}}_{Eij}, \bar{\mathcal{A}}_{Sij}^0\right) < 0 \ (54) \end{cases}$$

where

$$\bar{\Phi}_{lij}\left(\bar{\mathcal{A}}_{ij}, \bar{\mathcal{A}}_{Eij}, \bar{\mathcal{A}}_{Sij}^0\right) = \begin{bmatrix} \bar{\Phi}_{11lij} \sqrt{\phi}\bar{\mathcal{C}}_i^T & \bar{\mathcal{A}}_{ij}^T & \tau_m \bar{\mathcal{A}}_{Eij}^T & \tau_M \bar{\mathcal{A}}_{Eij}^T & \bar{\mathcal{B}}_{Si} & \bar{\mathcal{K}}_j^T \bar{\mathcal{K}}_j^{0T} & \\ * & -\gamma I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\bar{P}^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{W}_1^{-1} & 0 & 0 & 0 \\ * & * & * & * & -\bar{W}_2^{-1} & 0 & 0 \\ * & * & * & * & * & -\bar{V}^{-1} & 0 \\ * & * & * & * & * & * & -\bar{V} \end{bmatrix}, \tag{55}$$

$$l = 1,2 \tag{56}$$

$$\begin{aligned} &\bar{\Phi}_{11lij} = \bar{\Phi}_0 - \Gamma_{2+l}^T \mathbb{W}_l \Gamma_{2+l} + \mathrm{sym}\left(e_1^T S R_0^T \bar{\mathcal{A}}_{Sij}^0\right) + \bar{\Phi}_{ci} \\ &\bar{\Phi}_0 = e_1^T \left(\bar{Q}_1 + \bar{Q}_2 + \bar{Q}_3 + (d_r + 1)\bar{Q}_4 - E^T P E - \tau \hat{\Psi}\right) e_1 \\ &- e_2^T \bar{Q}_1 e_2 - e_3^T \bar{Q}_4 e_3 - e_4^T \bar{Q}_2 e_4 - \tau\, \mathrm{sym}\left(e_1^T \hat{\Omega} e_6\right) - \tau e_6^T e_6 \\ &\bar{\mathcal{A}}_{ij} = \bar{A}_{ij} e_1 + \bar{A}_{di} e_3 + \left(\bar{\Pi} + \sqrt{\bar{\Pi}(1 - \bar{\Pi})}\right) \bar{H}_i e_6 + \bar{B}_{1i} e_7, \\ &\bar{\mathcal{A}}_{Eij} = (\bar{A}_{ij} - \bar{E}) e_1 + \bar{A}_{di} e_3 + \left(\bar{\Pi} + \sqrt{\bar{\Pi}(1 - \bar{\Pi})}\right) \bar{H}_i e_6 + \bar{B}_{1i} e_7, \\ &\bar{\mathcal{A}}_{Sij}^0 = \bar{A}_{ij}^0 e_1 + \bar{A}_{di} e_3 + \bar{\Pi} \bar{H}_i e_6 + \bar{B}_{1i} e_7, \quad \bar{A}_{ij}^0 = \bar{A}_i^0 + \bar{B}_i \bar{K}_i^0 \\ &\bar{\mathcal{C}}_i = \bar{C}_{1i} e_1 + \bar{D}_{1i} e_7\ \bar{\mathcal{K}}_i = \bar{K}_i e_1, \quad \bar{\mathcal{K}}_i^0 = \bar{K}_i^0 e_1, \ \bar{\mathcal{B}}_{Si} = e_1^T S R_0^T \bar{B}_i \\ &\hat{\Psi} = \frac{\Psi_1^T \Omega_1 + \Omega_1^T \Psi_1}{2}, \quad \hat{\Omega} = \frac{\Psi_1^T + \Omega_1^T}{2} \end{aligned} \tag{57}$$

$\bar{R}_0$ is any matrix with full column rank satisfying $\bar{E}^T\bar{R}_0 = 0$ and K_i^0 are given matrices such that $A_i + B_{2i}K_i^0C_{2i}$ is Hurwitz.

Proof. To improve the computational efficiency, we select the Lyapunov–Krasovskii functional (27) without the $V_4(k)$ term.
Calculating the difference of $V(k)$ along the system (50) and taking the mathematical expectation, we have the following:

$$\begin{aligned}
&\mathbb{E}\Big\{\Delta V_1(k)\Big\}\\
&= \mathbb{E}\Big\{\Big(\bar{A}_{\theta\theta}(k)\bar{x}(k) + \bar{A}_{d\theta}(k)\bar{x}(k-d(k)) + \bar{B}_{1\theta}\bar{w}(k) + \bar{\Pi}\bar{H}_\theta\bar{f} + (\Pi(k)-\bar{\Pi})\bar{H}_\theta\bar{f}\Big)^T\bar{P}\\
&\Big(\bar{A}_{\theta\theta}(k)\bar{x}(k) + \bar{A}_{d\theta}(k)\bar{x}(k-d(k)) + \bar{B}_{1\theta}\bar{w}(k) + \bar{\Pi}\bar{H}_\theta\bar{f} + (\Pi(k)-\bar{\Pi})\bar{H}_\theta\bar{f}\Big) - \bar{E}^T\bar{P}\bar{E}\Big\}\\
&= \mathbb{E}\Big\{\psi^T(k)\Big(\mathcal{A}_{\theta\theta}^T\bar{P}\mathcal{A}_{\theta\theta} - e_1^T\bar{E}^T\bar{P}\bar{E}e_1\Big)\psi(k)\Big\}\\
&\mathbb{E}\Big\{\Delta V_2(k)\Big\} \leq \mathbb{E}\Big\{\psi^T(k)\Big(e_1^T(\bar{Q}_1+\bar{Q}_2+\bar{Q}_3+(d_r+1)\bar{Q}_4)e_1 - e_2^T\bar{Q}_1e_2 - e_4^T\bar{Q}_2e_4 - e_5^T\bar{Q}_3e_5\\
&- e_3^T\bar{Q}_4e_3\Big)\psi(k)\Big\}\\
&\mathbb{E}\Big\{\Delta V_3(k)\Big\} = \mathbb{E}\Big\{\psi^T(k)\mathcal{A}_{E\theta\theta}^T\Big(\tau_m^2\bar{W}_1 + \tau_M^2\bar{W}_2\Big)\mathcal{A}_{E\theta\theta}\psi(k) - \tau_m\sum_{s=k-\delta}^{k-d_m-1}\eta^T(s)\bar{E}^T\bar{W}_1\bar{E}\eta(s)\\
&- \tau_M\sum_{s=k-d_M}^{k-\delta-1}\eta^T(s)\bar{E}^T\bar{W}_2\bar{E}\eta(s)\Big\}
\end{aligned} \tag{58}$$

Note that $\bar{E}^T\bar{R}_0 = 0$, for any matrix $\bar{S}$ with appropriate dimensions yields the following:

$$\mathbb{E}\Big\{2\bar{x}(k)\bar{S}\bar{R}_0^TE\bar{x}(k+1)\Big\} = \mathbb{E}\Big\{2\psi^T(k)e_1^T\bar{S}\bar{R}_0^T\mathcal{A}_{S\theta\theta}\Big\} = 0 \tag{59}$$

Furthermore, from assumption 2 the following inequality holds for any $\tau > 0$ as follows:

$$\begin{bmatrix}\bar{x}(k)\\ \bar{f}\end{bmatrix}^T\begin{bmatrix}-\tau\hat{\Psi} & -\tau\hat{\Omega}\\ * & -\tau I\end{bmatrix}\begin{bmatrix}\bar{x}(k)\\ \bar{f}\end{bmatrix} \geq 0 \tag{60}$$

Combining (58)–(60), it follows for $d_m \leq d(k) \leq \delta$ that we have the following:

$$\mathbb{E}\Big\{\Delta V(k)\Big\} \leq \mathbb{E}\Big\{\psi^T(k)\Big(\bar{\Phi}_0 - \Gamma_3^T\mathbb{W}_1\Gamma_3 + \bar{\Phi}(\bar{E},\bar{\mathcal{A}}_{\theta\theta},\bar{\mathcal{A}}_{E\theta\theta},\bar{\mathcal{A}}_{S\theta\theta})\Big)\psi(k)\Big\} \tag{61}$$

When $\delta \leq d(k) \leq d_M$, we derive the following:

$$\mathbb{E}\Big\{\Delta V(k)\Big\} \leq \mathbb{E}\Big\{\psi^T(k)\Big(\bar{\Phi}_0 - \Gamma_4^T\mathbb{W}_2\Gamma_4 + \bar{\Phi}(\bar{E},\bar{\mathcal{A}}_{\theta\theta},\bar{\mathcal{A}}_{E\theta\theta},\bar{\mathcal{A}}_{S\theta\theta})\Big)\psi(k)\Big\} \tag{62}$$

where

$$\begin{aligned}
&\bar{\Phi}(\bar{E},\bar{\mathcal{A}}_{\theta\theta},\bar{\mathcal{A}}_{E\theta\theta},\bar{\mathcal{A}}_{S\theta\theta}) = \mathcal{A}_{\theta\theta}^T\bar{P}\mathcal{A}_{\theta\theta} + \mathcal{A}_{E\theta\theta}^T\Big(\tau_m^2\bar{W}_1 + \tau_M^2\bar{W}_2\Big)\mathcal{A}_{E\theta\theta} + \mathrm{sym}\Big(e_1^T\bar{S}\bar{R}_0^T\mathcal{A}_{S\theta\theta}\Big)\\
&\psi(k) = \mathrm{col}\Big\{\bar{x}(k), \bar{x}(k-d_m), \bar{x}(k-d(k)), \bar{x}(k-d_M), \bar{x}(k-\delta), \bar{f}(x(k)), \bar{w}(k)\Big\}
\end{aligned} \tag{63}$$

Note that $\text{sym}\left(e_1^T\bar{S}\bar{R}_0^T\bar{\mathcal{A}}_{Sij}\right) = \text{sym}\left(e_1^T\bar{S}\bar{R}_0^T\bar{\mathcal{A}}_{Si}^0 + \bar{\mathcal{B}}_{Si}\bar{\mathcal{K}}_j\right)$. For an appropriate dimensional matrix $\bar{V} > 0$, the following inequality holds:

$$\left(\bar{\mathcal{B}}_{Si}^T - \bar{V}(\bar{\mathcal{K}}_j - \bar{\mathcal{K}}_j^0)\right)^T \bar{V}^{-1}\left(\bar{\mathcal{B}}_{Si}^T - \bar{V}(\bar{\mathcal{K}}_j - \bar{\mathcal{K}}_j^0)\right) \geq 0 \tag{64}$$

Equivalently, we obtain the following:

$$\text{sym}\left(\bar{\mathcal{B}}_{Si}\bar{\mathcal{K}}_j\right) \leq \left(\bar{\mathcal{B}}_{Si}\bar{V}^{-1}\bar{\mathcal{B}}_{Si}^T\right) + \left((\bar{\mathcal{K}}_j - \bar{\mathcal{K}}_j^0)^T\bar{V}(\bar{\mathcal{K}}_j - \bar{\mathcal{K}}_j^0)\right) + \text{sym}\left(\bar{\mathcal{B}}_{Si}\bar{\mathcal{K}}_j^0\right) \tag{65}$$

Then

$$\text{sym}\left(e_1^T S R_0^T \bar{\mathcal{A}}_{Sij}\right) \leq \text{sym}\left(e_1^T S R_0^T \bar{\mathcal{A}}_{Sij}^0\right) + \left(\bar{\mathcal{B}}_{Si}\bar{V}^{-1}\bar{\mathcal{B}}_{Si}^T\right) + \left((\bar{\mathcal{K}}_j - \bar{\mathcal{K}}_j^0)^T\bar{V}(\bar{\mathcal{K}}_j - \bar{\mathcal{K}}_j^0)\right) \tag{66}$$

Furthermore, by the Schur complement, we can deduce from (53)–(54) that the following holds:

$$\bar{\Phi}_0 - \Gamma_{2+l}^T\mathbb{W}_l\Gamma_{2+l} + \bar{\Phi}(E, \bar{\mathcal{A}}_{\theta\theta}, \bar{\mathcal{A}}_{E\theta\theta}, \bar{\mathcal{A}}_{S\theta\theta}) < 0, \quad l = 1, 2. \tag{67}$$

Thus, system (50) with $\varpi(k) = 0$ is admissible in the mean square.
To further investigate the mixed H_∞ /passive performance of the system in (11), we introduce the following performance index :

$$J_{zw} = \mathbb{E}\left\{\sum_{k=0}^{\infty}\left(\gamma^{-1}\phi z^T(k)z(k) - 2\gamma(1-\phi)z^T(k)\varpi(k) - \gamma\varpi^T(k)(k)\varpi(k)\right)\right\} \tag{68}$$

under zero initial condition, we have the following:

$$\begin{aligned} J_{zw} =& \mathbb{E}\left\{\sum_{k=0}^{\infty}\left(\Delta V(k) + \gamma^{-1}\phi z^T(k)z(k) - 2\gamma(1-\phi)z^T(k)\varpi(k) - \gamma\varpi^T(k)(k)\varpi(k)\right)\right\} \\ & - \mathbb{E}\left\{V(\infty)\right\} \\ \leq& \mathbb{E}\left\{\sum_{k=0}^{\infty}\left(\Delta V(k) + \gamma^{-1}\phi z^T(k)z(k) - 2\gamma(1-\phi)z^T(k)\varpi(k) - \gamma\varpi^T(k)(k)\varpi(k)\right)\right\} \end{aligned} \tag{69}$$

Let the following hold: $\bar{\Phi}_{c\theta} = -2\gamma(1-\phi)\,\text{sym}\left\{\mathcal{C}_\theta^T e_7\right\} - \gamma e_7^T e_7$. From (53) to (54), the following can be deduced:

$$\begin{aligned} & \mathbb{E}\left\{\left(\Delta V(k) + \gamma^{-1}\phi z^T(k)z(k) - 2\gamma(1-\phi)z^T(k)\varpi(k) - \gamma\varpi^T(k)\varpi(k)\right)\right\} = \\ & \mathbb{E}\left\{\psi^T(k)\left(\bar{\Phi}_0 - \Gamma_{l+2}^T\mathbb{W}_l\Gamma_{l+2} + \bar{\Phi}(E, \bar{\mathcal{A}}_{\theta\theta}, \bar{\mathcal{A}}_{E\theta\theta}, \bar{\mathcal{A}}_{S\theta\theta}) + \bar{\Phi}_{c\theta} + \gamma^{-1}\phi\mathcal{C}_\theta^T\mathcal{C}_\theta\right)\psi(k)\right\} < 0,\ l = 1, 2 \end{aligned} \tag{70}$$

Thus, system (50) is with a mixed weighted H_∞ /passive performance index γ in the sense of Definition 2. This completes the proof. □

Remark 4. *To reduce the conservativeness of the proposed conditions, matrix K_i^0 is introduced such that $A_i + B_{2i}K_i^0C_{2i}$ is Hurwitz.*

4.2. Reliable Controller Synthesis

Theorem 3. *Given positive integers d_m, d_M and a positive scalar $0 \leq \phi \leq 1$. The closed-loop system (Σ_c) is admissible in the mean square for $w(k) = 0$ and satisfies the mixed weighted H_∞ /passive performance index (9) under zero initial condition for any non-zero $w \in l_2[0, \infty)$, if* *there exist positive scalars τ, ε_i, κ_i and matrices $\bar{P} > 0$, $\hat{P} > 0$, $\bar{Q}_1 > 0$, $\bar{Q}_2 > 0$, $\bar{Q}_3 > 0$, $\bar{Q}_4 > 0$, $\bar{W}_1 > 0$, $\hat{W}_1 > 0$, $\bar{W}_2 > 0$, $\hat{W}_2 > 0$, $\bar{V} > 0$, $\hat{V} > 0$, $\bar{Z} > 0$, $\bar{S}$, $\bar{X}$ and $\bar{Y}$ satisfying the following:*

$$\begin{cases} \hat{\Phi}_{1ii} < 0, \quad i = 1, \cdots, r & (71) \\ \hat{\Phi}_{2ii} < 0, \quad i = 1, \cdots, r & (72) \\ \dfrac{2}{r-1}\hat{\Phi}_{1ii} + \hat{\Phi}_{1ij} + \hat{\Phi}_{1ji} < 0, \quad i \neq j = 1, \cdots, r & (73) \\ \dfrac{2}{r-1}\hat{\Phi}_{2ii} + \hat{\Phi}_{2ij} + \hat{\Phi}_{2ji} < 0, \quad i \neq j = 1, \cdots, r & (74) \\ \bar{P}\hat{P} = I, \quad \bar{W}_1\hat{W}_1 = I, \quad \bar{W}_2\hat{W}_2 = I, \quad \bar{V}\hat{V} = I & (75) \end{cases}$$

where

$$\hat{\Phi}_{lij} = \begin{bmatrix} \hat{\Phi}_{11lij} & \varepsilon_i \bar{\mathbf{N}}_i^T & \bar{\mathbf{M}}_i & \bar{\mathbf{B}}_i \bar{Z} & Q^T \bar{\mathbf{K}}_j^T \\ * & -\varepsilon_i I & 0 & 0 & 0 \\ * & * & -\varepsilon_i I & 0 & 0 \\ * & * & * & -\bar{Z} & 0 \\ * & * & * & * & -\bar{Z} \end{bmatrix}$$

$$\hat{\Phi}_{11lij} = \begin{bmatrix} \bar{\Phi}_{11lij} & \sqrt{\phi}\bar{\mathcal{C}}_i^T & \bar{\mathcal{A}}_{ij}^T & \tau_m \bar{\mathcal{A}}_{Eij}^T & \tau_M \bar{\mathcal{A}}_{Eij}^T & \bar{\mathcal{B}}_{Si} & \bar{\mathcal{K}}_j^T - \bar{\mathcal{K}}_j^{0T} \\ * & -\gamma I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\hat{P} & 0 & 0 & 0 & 0 \\ * & * & * & -\hat{W}_1 & 0 & 0 & 0 \\ * & * & * & * & -\hat{W}_2 & 0 & 0 \\ * & * & * & * & * & -\bar{V} & 0 \\ * & * & * & * & * & * & -\hat{V} \end{bmatrix} \quad (76)$$

$$\begin{aligned} \bar{\mathbf{M}}_i &= \begin{bmatrix} \bar{\mathcal{M}}_i^T & 0 & 0 & \tau_m \bar{M}_i^T & \tau_M \bar{M}_i^T & \bar{M}_i^T & 0 & 0 \end{bmatrix}^T, \\ \bar{\mathbf{N}}_i &= \begin{bmatrix} \bar{\mathcal{N}}_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \bar{\mathbf{B}}_i &= \begin{bmatrix} 0 & 0 & 0 & \bar{B}_i^T & \tau_m \bar{B}_i^T & \tau_M \bar{B}_i^T & 0 & I \end{bmatrix}^T, \\ \bar{\mathbf{K}}_i &= \begin{bmatrix} \bar{\mathcal{K}}_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \bar{\mathcal{N}}_i &= \bar{N}_i e_1 + \bar{N}_{di} e_3, \; \bar{\mathcal{M}}_i = e_1^T \bar{S} \bar{R}_0^T \bar{M}_i \end{aligned}$$

Proof. By substituting $\bar{A}_{ij}(k)$ and $\bar{A}_{di}(k)$ in (53) and (54), the following conditions holds:

$$\bar{\Phi}_{lij}(k) = \bar{\Phi}_{lij} + \mathrm{sym}\left(\bar{\mathbf{M}}_i F(k) \bar{\mathbf{N}}_i\right) + \mathrm{sym}\left(\bar{\mathbf{B}}_i \bar{\mathbf{K}}_j(k)\right) < 0 \quad (77)$$

Using Lemma 1, Equation (77) is equivalent to the following:

$$\begin{bmatrix} \bar{\Phi}_{lij} & \varepsilon_i \bar{\mathbf{N}}_i^T & \bar{\mathbf{M}}_i \\ * & -\varepsilon_i I & 0 \\ * & * & -\varepsilon_i I \end{bmatrix} + \mathrm{sym}\left(\bar{\mathbf{B}}_i^T \bar{\mathbf{K}}_j \mathbf{R}(k)\right) < 0 \quad (78)$$

where $\mathbf{R}(k) = \mathrm{diag}\left(\bar{R}(k), I, I, I, I, I, I, I\right)$ and $\bar{R}(k) = \mathrm{diag}\left(I, R(k)\right)$. For any appropriate matrix $\bar{Z} > 0$ yields the following:

$$\left(\bar{Z}\bar{\mathbf{B}}_i - \bar{\mathbf{K}}_j \mathbf{R}(k)\right)^T \bar{Z}^{-1} \left(\bar{Z}\bar{\mathbf{B}}_i - \bar{\mathbf{K}}_j \mathbf{R}(k)\right) \geq 0 \quad (79)$$

Using the fact that $G(k) \leq Q$, we obtain the following:

$$\mathrm{sym}\left(\bar{\mathbf{B}}_i^T \bar{\mathbf{K}}_j \mathbf{R}(k)\right) \leq \bar{\mathbf{B}}_i^T \bar{Z} \bar{Z}^{-1} \bar{Z} \bar{\mathbf{B}}_i + Q^T \bar{\mathbf{K}}_j^T \bar{Z}^{-1} \bar{\mathbf{K}}_j Q \quad (80)$$

then $\hat{\Phi}_{lij} < 0$ holds using Schur complement.

Based on the parametrized linear matrix inequality (PLMI) proposed in [39], relaxed conditions are obtained in (71)–(74). This completes the proof. □

Remark 5. *Due to equality constraints (75), conditions (71)–(74) are not in strict LMI form which cannot be solved directly using the standard LMI procedures. For applying the LMI technique, we can formulate this non-convex feasibility problem into a sequential optimization problem subject to LMIs constraints.*

Based on the cone complementarity linearization (CCL) technique [40], we propose the following minimization problem involving LMI conditions instead of the original non-convex condition (75).

$$min\ \mathbf{Tr}(\bar{P}\hat{P} + \bar{W}_1\hat{W}_1 + \bar{W}_2\hat{W}_2 + \bar{V}\hat{V}), \text{subject to} \tag{81}$$

$$\begin{cases} (71) - (74) \\ \begin{bmatrix} \bar{P} & I \\ I & \hat{P} \end{bmatrix} \geq 0 \begin{bmatrix} \bar{W}_1 & I \\ I & \hat{W}_1 \end{bmatrix} \geq 0 \begin{bmatrix} \bar{W}_2 & I \\ I & \hat{W}_2 \end{bmatrix} \geq 0 \begin{bmatrix} \bar{V} & I \\ I & \hat{V} \end{bmatrix} \geq 0 \end{cases} \tag{82}$$

If the solution of the above minimization problem is $3\bar{n} + m$, $(\bar{n} = n + p)$, that is,

$$min\ \mathbf{Tr}(\bar{P}\hat{P} + \bar{W}_1\hat{W}_1 + \bar{W}_2\hat{W}_2 + \bar{V}\hat{V}) = 3\bar{n} + m \tag{83}$$

then, the conditions in Theorem 2 are solvable. In order to find a feasible solution of the above minimization problem, we suggest the following Algorithm 1:

Algorithm 1: Find a feasible solution of the above minimisation problem

---

step 1 *Find a feasible set* $\bar{P}^{(0)}$, $\hat{P}^{(0)}$, $\bar{W}_1^{(0)}$, $\hat{W}_1^{(0)}$, $\bar{W}_2^{(0)}$, $\hat{W}_2^{(0)}$, $\bar{V}^{(0)}$, $\hat{V}^{(0)}$
satisfying (71)–(74). Set $k = 0$.

step 2 *Solve the following optimization problem:*

$$min\mathbf{Tr}(\hat{P}^{(k)}\bar{P} + \bar{P}^{(k)}\hat{P} + \hat{Z}_1^{(k)}\bar{W}_1 + \bar{W}_1^{(k)}\hat{W}_1 + \hat{W}_2^{(k)}\bar{W}_2 + \bar{W}_2^{(k)}\hat{W}_2 + \hat{V}^{(k)}\bar{V} + \bar{V}^{(k)}\hat{V})$$

subject to (82).

step 3 ***if*** $|\mathbf{Tr}(\bar{P}\hat{P} + \bar{W}_1\hat{W}_1 + \bar{W}_2\hat{W}_2 + \bar{V}\hat{V} - (3\bar{n} + m)| < \epsilon$,
for a sufficiently small scalar $\epsilon > 0$, *the solution* $K_i, i = 1, 2, \cdots, r$,
is the controller gains.
STOP.

else

Set $k = k + 1$, *set* $(\bar{P}^{(k)}, \hat{P}^{(k)}, \bar{W}_1^{(k)}, \hat{W}_1^{(k)}, \bar{W}_2^{(k)}, \hat{W}_2^{(k)}, \bar{V}^{(k)}, \hat{V}^{(k)})$
$= (\bar{P}, \hat{P}, \bar{W}_1, \hat{W}_1, \bar{W}_2, \hat{W}_2, \bar{V}, \hat{V})$, *and go to Step 2.*

step 4 *If* $k > N$, *where* N *is the maximum number of iterations allowed, EXIT.*
Our method fails to find feasible gains.

Remark 6. *The flowchart displayed in Figure 1 provides a clear description of the proposed design procedure. Moreover, this procedure can be also applied for a standard system with* $E = I$.

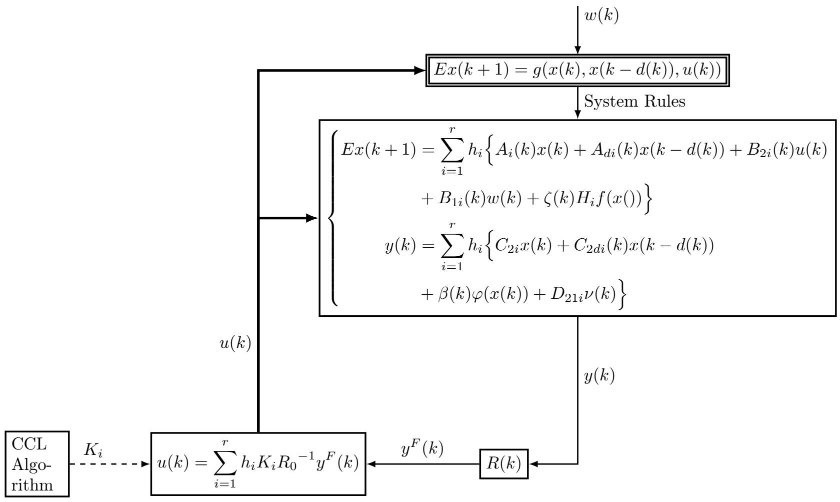

Figure 1. Flowchart of the control procedure.

Remark 7. *As mentioned in Remark 1, the non-linear functions are assumed that belong to sectors. For more general cases, if the non-linear perturbations satisfy the so-called one-sided Lipschitz constraint, the proposed approach cannot be applied. This interesting issue should be considered in future studies [27].*

5. Numerical Examples

In this section, three numerical examples are provided to validate the effectiveness and advantage of the developed results.

Example 1. *In this example, we demonstrate the advantage of the proposed admissibility method over some existing ones. Consider the following discrete-time singular system with delay.*

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{bmatrix}, \quad A = \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix}$$

For various values of d_m, Table 1 shows the admissible maximum values of d_M that guarantees the discrete singular time-delay system to be admissible by applying the result suggested in this study and the methods proposed in [5–11]. From the table, we observe that the admissibility criterion in Theorem 1 is less conservative than those proposed in [5–11].

Table 1. Calculated maximum d_M for various d_m.

Methods	$\boldsymbol{d_m = 3}$	$\boldsymbol{d_m = 6}$	$\boldsymbol{d_m = 9}$	$\boldsymbol{d_m = 12}$
[5]	8	10	13	15
[6]	15	16	17	19
[7]	16	19	22	25
[8]	19	21	24	27
[9]	28	30	33	36
[10]	37	39	42	44
[11]	45	45	48	51
Theorem 1	52	49	50	53

Example 2. *Referring to [41], we consider the following modified truck-trailer mapping system with stochastic disturbances:*

$$\begin{cases} x_1(k+1) = c(1-\frac{vt}{Lt_0})x_1(k) + (1-c)(1-\frac{vt}{Lt_0})x_1(k-d(k)) + \frac{vt}{lt_0}u(k) + 0.1w(k) \\ x_2(k+1) = c\frac{vt}{Lt_0}x_1(k) + (1-c)\frac{vt}{Lt_0}x_1(k-d(k)) + x_2(k) \\ x_3(k+1) = x_3(k) + \frac{vt}{t_0}\sin\left(x_2(k) + c\frac{vt}{2L}x_1(k) + (1-c)\frac{vt}{2L}x_1(k-d(k))\right) + 0.1f_1(x_k) \\ \quad x_4(k) = x_2(k) - c\frac{vt}{Lt_0}x_1(k) - (1-c)\frac{vt}{Lt_0}x_1(k-d(k)) + 0.2f_2(x_k) \\ \quad z(k) = 0.01x_3(k) + 0.01w(k) \\ \quad y(k) = \begin{bmatrix} c\frac{vt}{2L}x_1(k) + (1-c)\frac{vt}{2L}x_1(k-d(k)) + \varphi(x_k) \\ x_2(k) \\ x_3(k) + \nu(k) \end{bmatrix} \end{cases} \tag{84}$$

where $x_1(k)$ is the angle difference between truck and trailer, $x_2(k)$ is the angle of trailer, $x_3(k)$ is the vertical position of the rear end of the trailer, $x_4(k)$ is a new variable for the descriptor system, $u(k)$ is the steering angle, and $w(k)$ is the external disturbance. The model parameters are given as $t = 2s$, $t_0 = 0.5$, $l = 2.8$, $L = 5.5$, $v = -1$ and $c = 0.9$. Set $\beta = \frac{10t_0}{\pi}$. Non-linear system (84) can be exactly approximated by a TS fuzzy descriptor model as defined in (1). The system matrices are as follows:

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ A_1 = \begin{bmatrix} c(1-\frac{vt}{Lt_0}) & 0 & 0 & 0 \\ c\frac{vt}{Lt_0} & 1 & 0 & 0 \\ c\frac{v^2t^2}{2Lt_0} & \frac{vt}{t_0} & 1 & 0 \\ -c\frac{vt}{Lt_0} & 1 & 0 & -1 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} (1-c)(1-\frac{vt}{Lt_0}) & 0 & 0 & 0 \\ (1-c)\frac{vt}{Lt_0} & 0 & 0 & 0 \\ (1-c)\frac{v^2t^2}{2Lt_0} & 0 & 0 & 0 \\ -(1-c)\frac{vt}{Lt_0} & 0 & 0 & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} c(1-\frac{vt}{Lt_0}) & 0 & 0 & 0 \\ c\frac{vt}{Lt_0} & 1 & 0 & 0 \\ c\beta\frac{v^2t^2}{2Lt_0} & \beta\frac{vt}{t_0} & 1 & 0 \\ -c\frac{vt}{Lt_0} & 1 & 0 & -1 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} (1-c)(1-\frac{vt}{Lt_0}) & 0 & 0 & 0 \\ (1-c)\frac{vt}{Lt_0} & 0 & 0 & 0 \\ (1-c)\beta\frac{v^2t^2}{2Lt_0} & 0 & 0 & 0 \\ -(1-c)\frac{vt}{Lt_0} & 0 & 0 & 0 \end{bmatrix},\ B_{2i} = \begin{bmatrix} \frac{vt}{lt_0} \\ 0 \\ 0 \end{bmatrix},\ B_{1i} = \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix}$$

$$C_{1i} = \begin{bmatrix} 0.1 & 0 & 0 & 0 \end{bmatrix}, \qquad D_i = 0.1, \qquad C_{2i} = \begin{bmatrix} \frac{cvt}{2L} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{85}$$

$$C_{2d} = \begin{bmatrix} \frac{(1-c)vt}{2L} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad D_{2i} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T, \quad H_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.1 & 0 \\ 0 & 0.2 \end{bmatrix} \tag{86}$$

and the membership functions are defined as follows:

$$\begin{cases} h_1(\eta(k)) = \left(1 - \frac{1}{1+e^{-3(\eta(k)-\frac{\pi}{2})}}\right)\left(\frac{1}{1+e^{-3(\eta(k)+\frac{\pi}{2})}}\right), \quad h_2(\eta(k)) = 1 - h_1(\eta(k)) \\ \eta(k) = x_2(k) + c\frac{v\bar{t}}{2L}x_1(k) + (1-c)\frac{v\bar{t}}{2L}x_1(k-d(k)) \end{cases} \tag{87}$$

The uncertain matrices are the following:

$$M = \begin{bmatrix} 0.1 & 0 & 0 & 0 \end{bmatrix}^T, \quad N_i = \begin{bmatrix} \frac{0.1cvt}{2L} & 0 & 0 & 0 \end{bmatrix}, \quad N_{di} = \begin{bmatrix} \frac{0.1(1-c)vt}{2L} & 0 & 0 & 0 \end{bmatrix}$$

Our main aim is to design a reliable fuzzy SOF controller such that the closed-loop system is stochastically admissible with a mixed H_∞ /passive performance attenuation level. The nonlinear functions $f(x_k)$ and $\varphi(x_k)$ are, respectively, chosen as the following:

$$f(x_k) = \begin{bmatrix} -0.48x_1(k) + 0.2x_2(k) + \tanh(0.24x_1(k)) \\ 0.48x_2(k) - \tanh(0,16x_2(k)) \end{bmatrix} \tag{88}$$

$$\varphi(x_k) = 0.4x_1(k) + 0.1\sin(x_1(k)) \tag{89}$$

which can be bounded by the following:

$$\Phi_1 = \begin{bmatrix} 0.48 & 0.24 & 0 & 0 \\ 0 & 0.32 & 0 & 0 \end{bmatrix}, \qquad \Psi_1 = \begin{bmatrix} -0.24 & 0.24 & 0 & 0 \\ 0 & 0.48 & 0 & 0 \end{bmatrix}, \tag{90}$$

$$\Phi_2 = \begin{bmatrix} 0.5 & 0 & 0 & 0 \end{bmatrix}, \qquad \Psi_2 = \begin{bmatrix} 0.3 & 0 & 0 & 0 \end{bmatrix} \tag{91}$$

The disturbance inputs are given as $w(k) = 2e^{-2k}$ and $\nu(k) = \dfrac{sin(10k)}{k^2+1}$. The Bernoulli-distributed white noise sequences $\zeta(k)$ and $\beta(k)$ are assumed to satisfy condition (7) with $\bar{\zeta} = \bar{\beta} = 0.8$. The sensor fault matrix $R(k)$ is assumed to satisfy $\mathrm{diag}\{0.7, 0.6, 0.5\} \le R(k) \le \mathrm{diag}\{1, 1.2, 1.1\}$. Set $d_m = 2$, $d_M = 7$, $\gamma = 1$, and $\phi = 0.45$. By resorting to Yalmip software in MATLAB with the Sedumi solver, the CCL algorithm gives a feasible solution with fuzzy controller gains given as follows:

$$K_1 = \begin{bmatrix} -3.7268 & -0.17514 & 0.0012334 \end{bmatrix} \quad K_2 = \begin{bmatrix} -3.7278 & -0.17739 & 0.0012321 \end{bmatrix} \tag{92}$$

To study the effect of sensor failures, we consider a scenario defined by the following fault matrix for $2 \le k \le 100$ as follows:

$$R(k) = \mathrm{diag}\{\xi_1(k), \xi_2(k), \xi_3(k)\} \tag{93}$$

where

$$\xi_1(k) = 0.75 + 0.05sin(10k) \quad \xi_2(k) = 1 - 0.2e^{-0.03k}cos(20k) \quad \xi_3(k) = 0.65 - 0.15e^{-0.1k} \tag{94}$$

For the initial condition $x(0) = \begin{bmatrix} 1 & 1 & 0.5 & -0.499 \end{bmatrix}^T$, the numerical simulations are performed for the following two operational modes:

1. Normal mode, where reliable controller (92) is applied for a normal case without any failure;
2. Failure mode, where the proposed reliable controller (92) is implemented when the previous scenario affects the systems.

For both cases, the output and input responses of the closed-loop system are depicted in Figures 2–5. As expected, the truck–trailer system is stabilized, despite the sensor failures and the stochastic external disturbances. Additionally, the simulation result provides potent verification for the effectiveness of the proposed control scheme in accommodating the effect of sensor faults on the system, and shows its robustness in spite of the external disturbances and uncertainties.

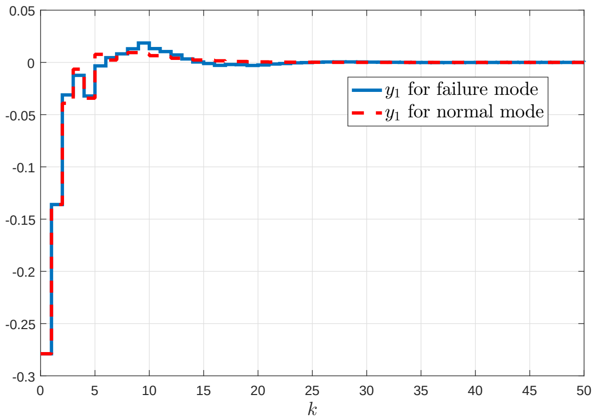

Figure 2. The response of system output variable y_1.

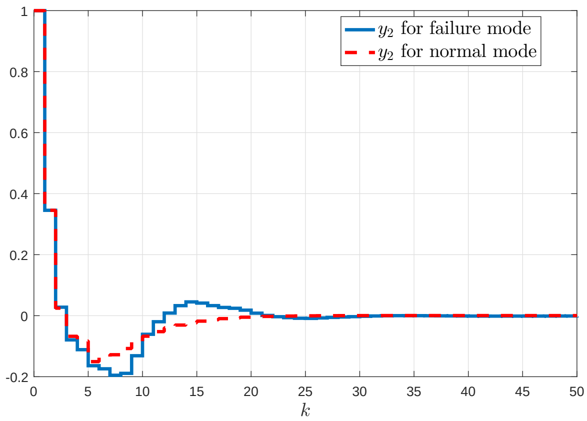

Figure 3. The response of system output variable y_2.

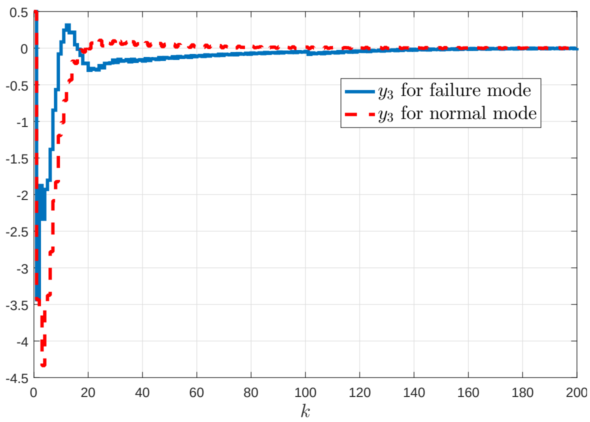

Figure 4. The response of system output variable y_3.

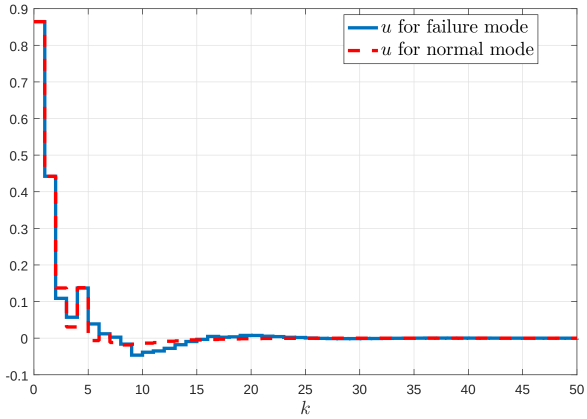

Figure 5. The response of system control input u.

Example 3. *As it is mentioned that the proposed control design is also suitable for standard systems. Based on the ball and beam system, we make a comparison with the design method suggested in [42]. The ball and beam system is described by the following dynamic equations:*

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = b\Big(x_1(t)x_4^2(t) - g\sin(x_3(t))\Big) \\ \dot{x}_3(t) = x_4(t) \\ \dot{x}_1(t) = u(t) + w(t) \end{cases} \tag{95}$$

Set $b = 0.7413$, $g = 9.81$ and $h = 0.1s$ as the sampling time. According to Euler's discretization method, we can obtain the following discrete-time dynamic system:

$$\begin{cases} x_1(k+1) = x_1(k) + hx_2(k) \\ x_2(k+1) = x_2(k) + hb\Big(x_1(k)x_4^2(k) - g\sin(x_3(k))\Big) \\ x_3(k+1) = x_3(k) + hx_4(k) \\ x_4(k+1) = x_2(k) + hu(k) + hw(k) \end{cases} \tag{96}$$

Assume that $x_3(k) \in [-\frac{\pi}{2} \quad \frac{\pi}{2}]$ and $\eta(k) = x_1(k)x_4(k) \in [-5 \quad 5]$. Using the sector nonlinearity approach, the TS fuzzy system can be established as follows:

$$x(k+1) = \sum_{i=1}^{2} h_i(\eta(k))\Big(A_i x(k) + A_{di}x(k-d_k) + B_{2i}u(k) + B_{1i}w(k) + Hf(x(k))\Big) \tag{97}$$

where $A_i = c\bar{A}_i$, $A_{di} = (1-c)\bar{A}_i$,

$$\bar{A}_1 = \begin{bmatrix} 1 & h & 0 & 0 \\ 0 & 1 & -\frac{2}{\pi}hbg & hbd \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \bar{A}_2 = \begin{bmatrix} 1 & h & 0 & 0 \\ 0 & 1 & -\frac{2}{\pi}hbg & -hbd \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B_{2i} = B_{1i} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ h \end{bmatrix}, \qquad H = \begin{bmatrix} 0 \\ -hbg \\ 0 \\ 0 \end{bmatrix}$$

The corresponding fuzzy membership functions are given as the following: $h_1(\eta(k)) = \dfrac{\eta(k)+5}{10}$ *and* $h_2(\eta(k)) = \dfrac{5-\eta(k)}{10}$, *while the non-linear function,* $f(x(k)) = \sin(x_3(k)) - \dfrac{2}{\pi}x_3(k)$, *is bounded by the following:*

$$\Phi_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}, \Psi_1 = \begin{bmatrix} 0 & 0 & 1-\dfrac{2}{\pi} & 0 \end{bmatrix} \tag{98}$$

Sensor faults are presented in Figure 6, and the uncertain parameters are assumed to be the following:

$$M = \begin{bmatrix} 0 & 0 & -hbg & 0 \end{bmatrix}^T, \quad N_i = \begin{bmatrix} 0 & 0 & 1-\dfrac{2}{\pi} & 0 \end{bmatrix}, \; i = 1,2$$

Set $c = 0.9$, $d_m = 1$, $d_M = 5$, $\gamma = 0.8$, $\phi = 0$, $K_1^0 = \begin{bmatrix} 1 & 2 & -10 & -4 \end{bmatrix}$, *and* $K_2^0 = \begin{bmatrix} 1 & 1 & -4 & -4 \end{bmatrix}$. *Assume that the sensor fault matrix* $R(k)$ *satisfies* $\mathrm{diag}\{0.7, 0.6, 0.5, 0.4\} \leq R(k) \leq \mathrm{diag}\{1, 1.2, 1.1, 0.8\}$. *Using the CCL algorithm, the controller gains can be computed as the following:*

$$K_1 = \begin{bmatrix} 0.69932 & 1.2392 & -16.079 & -8.0539 \end{bmatrix} \tag{99}$$
$$K_2 = \begin{bmatrix} 0.67935 & 1.7565 & -18.647 & -12.525 \end{bmatrix} \tag{100}$$

At this point, simulation studies are implemented with the initial condition $x(0) = \begin{bmatrix} 1 & -0.2 & 0.9 & -0.1 \end{bmatrix}^T$, *and external disturbance selected as follows:*

$$w(k) = \begin{cases} 2 & 1 \leq k \leq 3 \\ 0 & else. \end{cases} \tag{101}$$

The results of the simulation are depicted in Figure 7. It is clear from this figure that the suggested control law in (49) keeps the dynamic stability of the closed-loop system, even in the presence of the sensor faults, model uncertainties and external disturbances.
To further demonstrate the merit of the proposed control strategy, we perform, under the same conditions, a comparison with the method proposed in [42]. Figure 8 shows the state trajectories of the system, using the following control law

$$u(k) = \sum_{i=1}^{r} h_i\Big(R(k)K_{ai}x(k-d_k) + K_{bi}f(x(k))\Big) \tag{102}$$

with the following gains

$$\begin{aligned} K_{a1} &= \begin{bmatrix} -0.0167 & -0.0005 & -0.0350 & -0.0152 \end{bmatrix}, & K_{b1} &= 0.7476 \\ K_{a2} &= \begin{bmatrix} 0.0763 & 0.0335 & -0.0074 & 0.0298 \end{bmatrix}, & K_{b2} &= 0.2672 \end{aligned} \tag{103}$$

From this figure, it is clear that the control law developed in [42] may be incapable of dealing with a complex case, where the sensor faults and random non-linearity occur. Hence, the synthesized control law shows its superiority, compared to that proposed in [42].

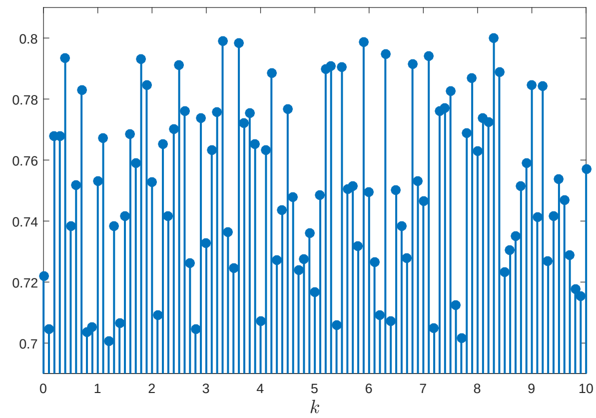

Figure 6. Sensor faults model.

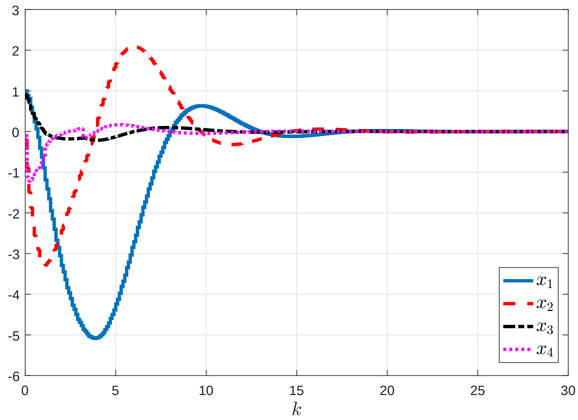

Figure 7. State trajectories in case 1.

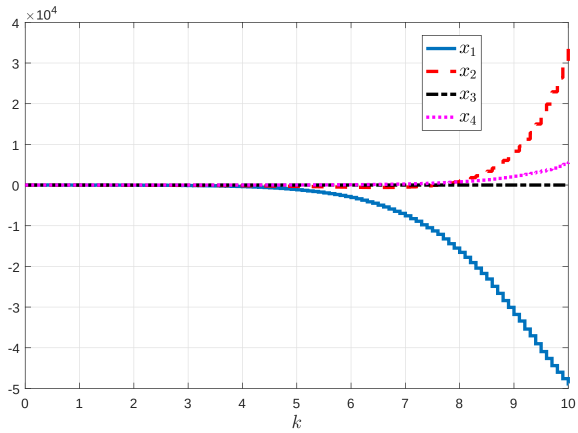

Figure 8. State trajectories in case 2.

6. Conclusions

In this paper, a new control scheme is developed for solving the main challenges faced while controlling discrete-time singular systems, such as the time-varying delay, randomly occurred non-linearities and sensor failures. Based on the TS fuzzy models and the Lyapunov–Krasovskii functional, the admissibility analysis is firstly studied by combining the delay decomposition and the reciprocally convex combination approaches. Then, a reliable static output feedback controller is designed such that the stability of the closed-loop system is ensured with accommodation of the negative effect caused by the sensors faults and noise. The feasibility of the addressed control problem is proved by establishing sufficient conditions such that the prescribed mixed H_∞ /passive performance level is achieved. The design of the controller gain is characterized by solving the CCL algorithm. Finally, the effectiveness of theoretical developments is verified by three examples. It should also be mentioned that many interesting types of research should be carried out in the future for Markovian jump singular systems described by interval type-2 fuzzy models with actuator/sensors faults, time-varying delay, and unknown transition probabilities.

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