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Recursive Convex Model for Optimal Power Flow Solution in Monopolar DC Networks

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Abstract: This paper presents a new optimal power flow (OPF) formulation for monopolar DC networks using a recursive convex representation. The hyperbolic relation between the voltages and power at each constant power terminal (generator or demand) is represented as a linear constraint for the demand nodes and generators. To reach the solution for the OPF problem a recursive evaluation of the model that determines the voltage variables at the iteration $t+1$ (v^{t+1}) by using the information of the voltages at the iteration t (v^t) is proposed. To finish the recursive solution process of the OPF problem via the convex relaxation, the difference between the voltage magnitudes in two consecutive iterations less than the predefined tolerance is considered as a stopping criterion. The numerical results in the 85-bus grid demonstrate that the proposed recursive convex model can solve the classical power flow problem in monopolar DC networks, and it also solves the OPF problem efficiently with a reduced convergence error when compared with semidefinite programming and combinatorial optimization methods. In addition, the proposed approach can deal with radial and meshed monopolar DC networks without modifications in its formulation. All the numerical implementations were in the MATLAB programming environment and the convex models were solved with the CVX and the Gurobi solver.

Keywords: recursive convex formulation; optimal power flow solution; monopolar DC networks; power losses minimization; convex optimization

MSC: 90C25; 90C26, 90C34

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1. Introduction

Monopolar DC networks represent an opportunity to distribute electrical energy from high- to low-voltage levels with a high efficiency regarding energy losses and excellent voltage profiles when compared to the classical AC electrical networks [1–3]. In general, a monopolar DC network is composed of two wires where one of them corresponds to the energized pole ($+V_{dc}$), and the other one corresponds to the return wire, which is typically solidly grounded, and each load terminal to ensure the adequate voltage regulation on these loads [4]. Even if monopolar DC networks are less complex than AC networks due to the non-existence of reactive power and frequency variables, these continue to offer important challenges in static and dynamic analyses [5,6]. This research focuses in the area of steady-state analyses (static studies) for monopolar DC networks by proposing a new recursive convex formulation to solve the optimal power flow (OPF) problem in this grid in the presence of multiple dispersed sources and constant power terminals [7].

In the current literature the OPF problem for monopolar DC networks has been widely studied via mathematical-based or combinatorial-based optimization [7,8]. Here, we present some important advances in this area. The authors of [9] proposed the application of the second-order cone programming to reformulate the power flow problem in radial monopolar DC networks. The proposed model is special for isolate microgrids with multiple constant power terminals and dispersed sources. The authors of [10] presented the application of the convex optimization to deal with the OPF problem in DC multiterminal systems with dispersed generators and controlled DC-DC converters. The OPF is formulated via the semidefinite programming (SDP) theory. The numerical results showed the convergence of the SDP reformulation to the global optimal solution with minimum (negligible) estimation errors. The authors of [11] present four different models to solve the OPF problem in monopolar DC networks. These OPF approaches are based on recursive approximations of the power balance equations into linear recursive expressions. To reduce the convergence error, an iterative procedure that starts with plane voltages is implemented and these are updated at each iteration until the desired convergence is reached. The numerical results confirm the efficiency of these approaches when compared with the exact nonlinear formulation of the OPF problem solved via interior point methods with logarithmic barriers.

Another important research area corresponds to the application of metaheuristic optimizers to deal with the OPF problem in monopolar DC networks. These combinatorial optimization methods are based on the well-known master–salve optimization concept [12]. In the OPF problem, the master stage is entrusted with determining the set of power injections in the dispersed generators, while the slave optimization stage evaluates an efficient power flow method to determine the feasibility of the proposed solution [13]. Ref. [14] has presented the application of the salp swarm optimization algorithm combined with the successive approximation method to deal with the optimal power dispatch in dispersed generation sources for monopolar DC network applications. Two test feeders composed of 21 and 69 nodes were used to validate the proposed master–slave optimizer. The numerical results demonstrated its efficiency when compared with metaheuristic techniques, such as particle swarm optimization, ant lion optimization, black hole optimization, the continuous genetic algorithm, and multiverse optimization. The authors of [15] have presented the application of the vortex search algorithm to solve the OPF problem in DC networks with multiple dispersed generation sources. The vortex search algorithm was used in the master stage to determine the optimal power injection in the distributed generators while the successive approximation method was used. The main advantage of the vortex search algorithm is that its formulation is based on Gaussian distributions which increases the probability to find the global optimum at each evaluation of the algorithm. A complete revision of the state of the art regarding combinatorial optimization methods applied to the OPF problem in monopolar DC networks was provided by the authors of [7]. In this research, the general formulation of the OPF model (the exact nonlinear programming model) and three solution techniques, such as the continuous genetic algorithm, the particle swarm optimizer, and the black hole optimizer, were provided. Test feeders composed of 21 and 69 nodes were employed for all the numerical validations with excellent numerical results for the different penetration levels of the dispersed generation from 20 to 60% of the total power consumption.

Table 1 summarizes the main approaches available in the current literature to solve the OPF problem in monopolar DC networks.

Table 1. Main literature reports applied to the OPF solution in monopolar DC networks.

Methodology	Classification	Year	Reference
Semidefinite programming	Convex optimization	2016	[10]
Second-order cone programming (SOCP)	Convex optimization	2018	[9]
Sequential quadratic programming	Convex optimization	2019	[11]
Black hole optimization	Combinatorial optimization	2019, 2020	[7]
Continuous genetic algorithm	Combinatorial optimization	2020	[7]
Particle swarm optimization	Combinatorial optimization	2020	[7]
Vortex search algorithm	Combinatorial optimization	2020	[15]
Sine–cosine algorithm	Combinatorial optimization	2019, 2022	[16]

Note that the optimization methodologies in Table 1 show two clear tendencies in the solution of the OPF problem for monopolar DC networks. The first tendency is oriented to the application of convex optimization methods [17]. These methods address the OPF problem from its mathematical structure in order to achieve equivalent models that ensure the convexity of the solution space and the objective function, which implies that the contribution of these methodologies is in the mathematical modeling and not in the solution technique itself [18]. The second tendency corresponds to the application of combinatorial optimization methods based on master–slave algorithms, where the master stage defines the power injections in the dispersed sources and the slave stage solves the resulting power flow problem [7]. The contribution of these approaches is the easy implementation of any metaheuristic optimizer to solve the exact nonlinear formulation of the OPF using an arbitrary programming language. However, when comparing the convex optimization methods and the combinatorial ones, it is worth mentioning that the first solution method ensures the global optimum finding [19], while the combinatorial optimization algorithms only ensure good solutions being in the general local optimums [20].

The main contribution of this research corresponds to the proposition of a new recursive convex model to deal with the OPF problem via convex approximations to find the voltage values v^{t+1} using the voltages v^t as the initial point, with t being the iterative counter. To reach a convex representation of the OPF model, the hyperbolic relation between the powers and voltages in the generation sources is relaxed, and the voltage variables in the demand nodes using Taylor's series expansion are linearized [21]. The proposed recursive convex model can be used to solve the classical power flow problem when the monopolar DC networks lack dispersed generators, and it also solves the OPF problem with multiple generators independent of the grid configurations, i.e., it is applicable to radial and meshed networks.

The remainder of this contribution is structured as follows: Section 2 presents the classical OPF formulation that exhibits the non-convexities of this problem in the power equilibrium constraint at each node. Section 3 presents the proposed recursive convex model by introducing current injections as auxiliary variables that allow transforming the hyperbolic constraint that relates currents and powers in second-order cones. Section 4 presents the main characteristics of the 85-bus grid as well as all the numerical validations. Finally, Section 5 lists the main concluding remarks derived from this research and some possible future works.

2. Optimal Power Flow Problem

The problem of the optimal power flow in monopolar DC networks corresponds to a nonlinear non-convex programming problem, where the main objective is to minimize the total grid power losses of the network for a particular load and demand conditions [22]. The nonlinear programming model that represents the OPF problem is presented below.

2.1. Objective Function

The objective function represents the minimization of the total grid power losses in all the branches of the network [5]. This function is presented in Equation (1) as a function of the conductance matrix and the grid voltage profiles [23].

$$\min z = \sum_{k \in \mathcal{N}} \sum_{m \in \mathcal{N}} G_{km} v_k v_m, \tag{1}$$

where z is the objective function value, v_k and v_m are the voltage values at nodes k and m, and the G_{km} is the value of the conductance matrix that connects nodes k and m, respectively.

Remark 1. *The objective function of the OPF problem in (1) is indeed a convex quadratic function of the voltage magnitudes due to the conductance matrix G is a positive semidefinite matrix that is defined as a function of the resistive connections of the network [24]. This matrix is positive semidefinite if only and only if all the nodes are connected, i.e., the system does not have isolated nodes [25].*

To illustrate that the conductance matrix is indeed a positive semidefinite matrix, let us consider that the monopolar DC network is represented by an oriented graph $G = \{\mathcal{N}, \mathcal{E}\}$, where $\mathcal{N}$ is the set of nodes and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ the set of branches. The cardinality of $\mathcal{N}$ is n and the cardinality of $\mathcal{E}$ is m. The nodal admittance matrix is $G \in \mathbb{R}^{n \times n}$. Note that the components of the conductance matrix are obtained as follows:

$$G = \begin{cases} G_{kk} = \sum_{m \in \mathcal{N}} \frac{1}{r_{km}} & k \neq m \\ G_{km} = -\frac{1}{r_{km}} & k \neq m \end{cases}$$

where r_{km} represents the resistive element associated with the branch that connects nodes k and m.

Note that the structure of the conductance matrix G is such that $\|G_{kk}| \geq \|G_{km}\|$, which implies that it is diagonal dominant and also positive semidefinite [25].

2.2. Set of Constraints

The OPF problem in monopolar DC networks is constrained by the power balance equations, voltage regulation, and capacities of generation in power sources among others. The set of constraints for the OPF problem are listed below.

$$p_k^g + p_k^{dg} - p_k^d = v_k \sum_{m \in \mathcal{N}} G_{km} v_m, \; \forall k \in \mathcal{N} \tag{2}$$

$$p_k^{g,\min} \leq p_k^g \leq p_k^{g,\max}, \; \forall k \in \mathcal{N} \tag{3}$$

$$p_k^{gd,\min} \leq p_k^{gd} \leq p_k^{gd,\max}, \; \forall k \in \mathcal{N} \tag{4}$$

$$v_k^{\min} \leq v_k \leq v_k^{\max}, \; \forall k \in \mathcal{N} \tag{5}$$

$$v_k = v_{\text{slack}}. \; k = \text{slack} \tag{6}$$

where p_k^g is the total power generation in the slack source, p_k^{dg} is the power generation in the dispersed source connected at node k, p_k^d represents the total power consumption at node k. Remember that G_{km} is the conductive effect that associates nodes k and m, which are obtained from the general admittance matrix that presents the physical connection of the system [25]. $p_k^{g,\min}$ and $p_k^{g,\max}$ are the minimum and maximum power inputs in the slack source; $p_k^{gd,\min}$ and $p_k^{gd,\max}$ are the minimum and maximum power limits in the dispersed source connected at node k; $v_k^{\min}$ and $v_k^{\max}$ correspond to the minimum and maximum voltage limits allowed for the voltage magnitudes as each node of the network; and v_{slack} is the voltage reference assigned to the slack source.

Note that (2) represents the power balance equilibrium at each node k of the network which is a nonlinear non-convex constraint due to the product among voltages in the right-hand side of the equation. Inequality constraints (3) and (4) define the minimum and maximum power generation capabilities of the slack and power sources connected in the monopolar DC network; inequality constraint (5) ensures the voltage regulation bounds in the monopolar DC network which is a constraint typically imposed by regulatory policies; and finally, Equation (6) defines the voltage output in the slack source, i.e., the voltage-controlled node.

Remark 2. *The unique nonlinear non-convex constraint of the OPF problem corresponds to Equation (2) because it has a product between voltage variables which makes it so that the OPF model takes a non-convex structure that complicates its solution using conventional optimization methods.*

3. Recursive Convex OPF Approach

To obtain a recursive convex model that permits to solve the OPF problem via convex optimization, the nonlinear set of equations regarding the power balance per node is approximated into an affine set of constraints [26]. Note that set of Equations (2) can be rewritten as presented in (7).

$$\frac{p_k^g}{v_k} + \frac{p_k^{dg}}{v_k} - \frac{p_k^d}{v_k} = \sum_{m \in \mathcal{N}} G_{km} v_m, \; \forall k \in \mathcal{N} \tag{7}$$

where in the left-hand side the hyperbolic relation between power and voltages is presented. In addition, we can observe the following facts:

i. The fractional relations between power injections in slack and dispersed generation sources involve each one of the two variables, i.e., the power injections p_k^g and p_k^{dg} in the numerator, and the voltage magnitude v_k at the denominator. However, we know that the voltage variable in generation sources presents small variations with respect to the ideal value. For this reason, we approximate these as presented in Equations (8) and (9).

$$\frac{p_k^g}{v_k} \approx \frac{p_k^g}{v_k^t}, \; \forall k \in \mathcal{N} \tag{8}$$

$$\frac{p_k^{dg}}{v_k} \approx \frac{p_k^{dg}}{v_k^t}, \; \forall k \in \mathcal{N} \tag{9}$$

where v_k^t represents the voltage value at the iteration t (current linearizing point), which is a predefined value that is being updated recursively as will be described ahead in this document.

ii. The hyperbolic relation between voltages and currents in the fraction term $\frac{p_k^d}{v_k}$ regarding constant power terminals only has a variable the voltage value at the denominator. This implies that different from generators where numerators and denominators are varying, in the demand nodes, only the denominator changes, which means that the value of the v_k variable defines the final value of this relation. For this reason, to approximate this component, we employ the first Taylor's series expansion of the variable v_k^{-1} in the linearizing point v_k^t [27]. This produces the linear equivalent relation in (10).

$$\frac{p_k^d}{v_k} \approx 2\frac{p_k^d}{v_k^t} - \frac{p_k^d}{(v_k^t)^2} v_k, \; \forall k \in \mathcal{N} \tag{10}$$

Remark 3. *Considering the approximations in Equations (8) to (10), the left-hand side component of the power balance constraint in (7) is now a linear affine expression that takes the structure in (11).*

$$\frac{p_k^g}{v_k^t} + \frac{p_k^{dg}}{v_k^t} - \left(2\frac{p_k^d}{v_k^t} - \frac{p_k^d}{(v_k^t)^2} v_k^{t+1}\right) = \sum_{m \in \mathcal{N}} G_{km} v_m^{t+1}, \ \forall k \in \mathcal{N} \tag{11}$$

where v_m^{t+1} will be the new updated value of the voltage variables.

In order to represent the convex equivalent model to solve the OPF problem in monopolar DC networks, the exact nonlinear programming models (1) to (6) are reformulated as presented below:

$$\textbf{Obj. Func.:} \quad \min z = \sum_{k \in \mathcal{N}} \sum_{m \in \mathcal{N}} G_{km} v_k^{t+1} v_m^{t+1}, \tag{12}$$

$$\textbf{Subject to:} \quad \frac{p_k^g}{v_k^t} + \frac{p_k^{dg}}{v_k^t} - \left(2\frac{p_k^d}{v_k^t} - \frac{p_k^d}{(v_k^t)^2} v_k^{t+1}\right) = \sum_{m \in \mathcal{N}} G_{km} v_m^{t+1}, \ \forall k \in \mathcal{N} \tag{13}$$

$$p_k^{g,\min} \leq p_k^g \leq p_k^{g,\max}, \ \forall k \in \mathcal{N} \tag{14}$$

$$p_k^{gd,\min} \leq p_k^{gd} \leq p_k^{gd,\max}, \ \forall k \in \mathcal{N} \tag{15}$$

$$v_k^{\min} \leq v_k^{t+1} \leq v_k^{\max}, \ \forall k \in \mathcal{N} \tag{16}$$

$$v_k^{t+1} = v_{\text{slack}}. \ k = \text{slack} \tag{17}$$

Note that in convex optimization models (12)–(17), only the variables regarding voltages are using superscripts t and $t+1$, which means that v_k^t is the predefined voltage value at node k where is the linearizing point to reach the next value v_k^{t+1}. For the remaining variables, i.e., powers in the slack and dispersed generation sources, this is not related to superscripts because the new value of them does not depend on the previous value assigned to them.

To illustrate the recursive implementation of the proposed convex model to solve the OPF problem in monopolar DC networks, Algorithm 1 is used.

Algorithm 1: Recursive OPF solution using a relaxed convex approximation.

Data: Select the monopolar DC network
Result: Present the final values for the OPF variables
Transform the DC system into its per-unit equivalent;
Select the maximum number of iteration $t_{\max}$;
Chose the convergence error ζ;
Make $t = 0$;
Set all the nodal voltages as $v_k^t = v_{\text{slack}}$;
for $t \leq t_{\max}$ **do**
  Solve the convex model (12)–(17);
  Calculate the convergence error $\varepsilon = \max_{k \in \mathcal{N}} \left\{ \left| \left| v_k^{t+1} \right| - \left| v_k^t \right| \right| \right\}$;
  if $\varepsilon \leq \zeta$ **then**
    for $k \leq n$ **do**
      Report voltages v_k^{t+1};
      Report power generations p_k^g and p_k^{dg};
    Report the final grid power losses z;
    break;
  else
    for $k \leq n$ **do**
      Make $v_k^{t+1} = v_k^t$;

4. Numerical Validations

To evaluate the efficiency of the proposed recursive formulation to deal with the OPF problem in monopolar DC networks, the 85-bus grid presented in [28] is used, by considering only the positive pole load information. The electrical configuration of this test feeder is presented in Figure 1, and all the parametric information can be consulted in [28].

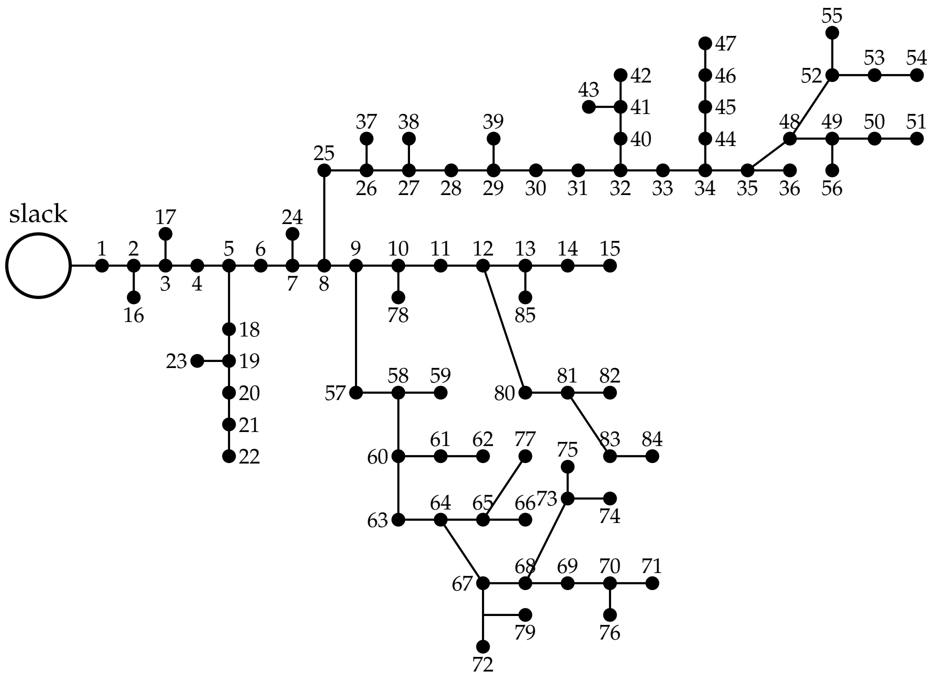

Figure 1. Grid topology of the IEEE 85-bus system for monopolar power flow applications.

For the 85-bus grid, two possible operative scenarios to validate the effectiveness of the proposed recursive convex formulation are considered .

i. The solution of the classical power flow problem, considering that in the grid there is no penetration of the dispersed generation.
ii. The location of four dispersed generators in the nodes 12, 19, 35, and 63, with nominal capacities of 750 kW each one.

4.1. Comparison with Specialized Power Flow Methods

To validate the first simulation scenario, we compare the power flow solution with efficient and well-known power flow methods for monopolar DC networks with a radial structure. The comparative power flow methods are: the Newton–Raphson (NR), triangular-based formulation (TBF), successive approximations method (SAM), matricial backward–forward method (MBFM), hyperbolic approximation method (HAM), and product approximation method (PAM). Each one of these power flow methodologies can be consulted in [29]. However, it is important to mention that all of these algorithms were programmed in this research in order to obtain a fair comparison among the literature reports and our proposed optimization model. Table 2 presents the comparative analysis between the aforementioned power flow methods and the proposed optimization models (12)–(17) solved in the convex tool CVX for MATLAB. For simplicity, the proposed approach is named the recursive convex formulation (RCF).

The numerical results in Table 2 show that the proposed RCF approach adequately solves the power flow problem in monopolar DC networks when compared with specialized numerical methods for a power flow analysis in this type of distribution network [29]. The final grid power losses reached by the RCF approach have an estimation error of 1.0159×10^{-7} when compared to the Newton–Raphson approach; however, this value implies that the first six decimals are completely equivalent, which is enough for any particular application in real DC systems. Note that due to the optimization nature of the RCF approach, it only takes two iterations to solve the power flow problem, while the derivative-based algorithms take four iterations, and the linear-based methods take eight iterations.

Table 2. Power flow solution for the 85-bus grid using different methodologies.

Method	Power Loss (kW)	Iterations	Error (%)
NR	58.6974817069044	4	—
TBF	58.6974816840913	8	3.8865×10^{-8}
SAM	58.6974817053999	8	2.5631×10^{-9}
MBFM	58.6974817051754	8	2.9456×10^{-9}
HAM	58.6974817059701	4	1.5917×10^{-9}
PAM	58.6974817055268	4	2.3469×10^{-9}
RCF	58.6974817665350	2	1.0159×10^{-7}

4.2. Comparison with Combinatorial Optimizers

This section presents a comparative analysis of the proposed RCF to solve the OPF problem in monopolar DC networks with respect to three specialized combinatorial optimization methods. The metaheuristic optimizers selected are: the black hole optimizer (BHO) [7], the sine–cosine algorithm (SCA) [16], and the vortex search algorithm (VSA) [15]. It is worth mentioning that these methodologies are selected based on the following factors: (i) the black hole optimization method is part of the metaheuristic optimizers based on the interaction between stars and black holes in the center of the galaxies. This method can be classified in the family of the particle swarm optimizers, and it includes the general inspiration of the metaheuristics with physical bases; (ii) the sine–cosine algorithm is selected as a comparative optimizer because this algorithm is inspired by mathematics. This algorithm explores and exploits the solution space using trigonometric functions (sine and cosine functions) which include the main characteristics of nonlinear functions in combinatorial optimization processes; and (iii) the vortex search algorithm is selected because it is the best metaheuristic optimizer reported in the current literature to deal with the OPF problem in monopolar DC networks. In addition, it is a metaheuristic optimization technique based on Gaussian distributions that explores and exploits the solution space using variable hyper-ellipses, maintaining the diversity of the solution during the iteration process, and introduces the advantages of the probability distribution functions for solving nonlinear optimization problems.

Table 3 presents the comparison between the proposed RCF and the selected metaheuristic optimizers. It is worth mentioning that the comparative algorithms were set with 10 individuals, 1000 iterations, and 100 consecutive evaluations to determine their average behavior.

Table 3. OPF solution for the 85-bus grid using different methodologies.

Method	Power (kW)	Min. (kW)	Mean (kW)	Max. (kW)	Std. (kW)	Time (s)
BHO	$\begin{bmatrix} 380.963290296850 \\ 177.993027750899 \\ 452.378953478316 \\ 438.980259314788 \end{bmatrix}$	3.30792201837848	3.88938176547841	4.85945934769858	3.7387×10^{-03}	5.6512
SCA	$\begin{bmatrix} 418.178542918465 \\ 201.118987781865 \\ 436.569616339663 \\ 439.104849610947 \end{bmatrix}$	3.26423557079929	344705722809309	3.44705722809309	3.7791×10^{-04}	2.5917
VSA	$\begin{bmatrix} 414.665838724249 \\ 199.172292151730 \\ 439.728488484532 \\ 439.040028810461 \end{bmatrix}$	3.26358341808515	3.26358341853136	3.26358341931154	2.2713×10^{-12}	2.8837
RCF	$\begin{bmatrix} 414.660935921302 \\ 199.172929051667 \\ 439.731195339216 \\ 439.036439456949 \end{bmatrix}$	3.26358363439168	3.26358363439168	3.26358363439168	0	1.8438

The numerical results in Table 3 show that:

i. As the literature mentions (see Ref. [15]), the VSA methodology is the most efficient algorithm regarding combinatorial optimization methods to deal with the OPF problem. Note that the difference between the minimum and maximum solutions is less than 1.2263×10^{-09}, with a standard deviation of 2.2713×10^{-12}.

ii. The BHO and the SCA approaches are stuck in locally optimal solutions with differences of about 1.3586 and 0.0200% with respect to the optimal solution found with the VSA method. These results show that, numerically speaking, the SCA approach can be considered accurate for solving the OPF problem in monopolar DC networks with the main advantage being that its implementation is very simple due to its basic evolution rules [30].

iii. The proposed RCF reaches the global optimal solution of the OPF problem, i.e., 3.2635 kW, considering between four to six decimals. This implies that the difference in the RCF, when compared with the VSA, is few in milliwatts. If we suppose that the global optimum corresponds to the VSA solution, then the RCF has an estimation error of about 6.6279×10^{-06}%, which implies that for any practical application the RCF method is effective to solve the OPF problem with the main advantage that, owing to the convexity of the solution space, statistical analyses are not required, which is not the case for metaheuristics.

Note that even if the statistical performance of the VSA method is enough for any practical application of the OPF studies in a monopolar DC network, its standard deviation is non-zero, which implies that each solution variates in some decimals at each ruining, which makes implementing statistical validations to confirm its effectiveness always necessary. On the other hand, regarding processing times, the proposed RCF only takes about 1.84 s to solve the OPF problem in the 85-bus grid, while the VSA takes an additional second, i.e., 2.88 s. Nevertheless, note that to confirm the effectiveness of the VSA method, 100 evaluations of the complete methodology were made and it took 288.37 s, while the proposed RCF does not require multiple evaluations because, via the convex theory, it is known that the optimal solution for the problem will be always the same (global optimization properties).

4.3. Comparison with a Semidefinite Programming Model

To demonstrate the effectiveness of the proposed RCF to solve the OPF problem in monopolar DC networks, a comparative analysis with the equivalent semidefinite programming (SDP) approach reported in [10] was conducted. The SDP approach is selected as the comparative algorithm because it can deal with radial and meshed distribution networks without any modification in its mathematical formulation. To obtain a meshed configuration for the 85-bus grid, four additional lines are included for the system in Figure 1. These lines connect nodes 15–32, 16–23, 59–80, and 74–83. For these lines, the equivalent resistance values are 0.520, 0.640, 0.495, and 0.635 Ω, respectively.

Table 4 presents the comparative analysis between the proposed RCF and the SDP method for the radial and meshed configurations of the 85-bus grid. Note that the solution of the SDP model was reached with the CVX tool in the MALTAB programming environment using the SDPT3 solver.

The numerical results in Table 4 reveal that:

i. Both convex optimization methods converge to the same global optimal solution with the main advantage that a statistical validation of the effectiveness of these algorithms is not required, because due to the convex nature of the solution space, the optimal solution reached is indeed the global optimum. Note that the difference between the objective functions is lower than some milliwatts, confirming their efficiency to solve the OPF problem in monopolar DC networks.
ii. The numerical results confirm that the SDP and the RCF methods allow solving the OPF problem in radial and meshed monopolar DC distribution networks. In addition, the meshed configuration presents 0.2676 kW of additional energy losses when compared with the meshed configuration. This is an expected behavior in electrical networks with meshes because the voltage profile is improved and the power flows present a better redistribution.

With respect to the processing times, it is clear that the RCF method is more efficient to solve the OPF problem in monopolar DC networks, because in both configurations, it takes less than 2 s, while the processing times of the SDP in both cases are higher than 35 s. However, this behavior is easily explained due to the number of variables regarding the voltages in the SDP model, which is n^2 (note that the SDP problems are defined in the space of square matrices), while the recursive model does not increase the number of variables, and these remain as n in the case of the voltages for each recursive evaluation.

Table 4. OPF solution for the 85 bus with radial and meshed configurations using the proposed RCF and the SDP method.

Method	Power (kW)	Min. (kW)	Time (s)
Radial configuration			
SDP	$\begin{bmatrix} 414.667268948386 \\ 199.170800020030 \\ 439.727950779580 \\ 439.041025339755 \end{bmatrix}$	3.26360276596915	38.5165
RCF	$\begin{bmatrix} 414.660935921302 \\ 199.172929051667 \\ 439.731195339216 \\ 439.036439456949 \end{bmatrix}$	3.26358363439168	1.8438
Meshed configuration			
SDP	$\begin{bmatrix} 499.999555544086 \\ 218.553723985719 \\ 381.320450037291 \\ 414.949506471181 \end{bmatrix}$	2.99597169762933	35.1618
RCF	$\begin{bmatrix} 500.034832543591 \\ 218.550039816306 \\ 381.327927092341 \\ 414.97099629762 \end{bmatrix}$	2.99596545080895	1.8125

4.4. MATLAB Implementation

To illustrate the general implementation of the proposed RCF, here we present a small numerical example composed of a monopolar DC network with six nodes. This is a test feeder operated in the substation bus with a voltage of 220 V. The parametric information for this test feeder is presented in Table 5.

Table 5. Branch and load parameters for the six-bus grid in monopolar DC applications.

Node j	Node k	R_{jk} (Ω)	P_{dk} (W)
1	2	0.25	1500
2	3	0.50	1750
3	4	0.45	1250
2	5	0.35	1350
3	6	0.40	1500

Figure 2 depicts the MATLAB implementation of the proposed RCF to solve the OPF problem in monopolar DC networks. Note that for this numerical example, the presence of two disperse generators with maximum power generations about 2750 W was considered.

```
%% RECURSIVE CONVEX FORMULATION
clc; clear; tic
% SYSTEM DATA
Vb = 220; Pb = 1000;% V and W
Rb = (Vb^2)/Pb; % (Ohm)
%       [i j Rij(Ohm)]
LINES = [1 2 0.25;2 3 0.50;3 4 0.45;2 5 0.35;3 6 0.40];
%%         [i Pkd Pkg]
NODES = [1 0 0;2 1500 0;3 1750 0;4 1250 0;5 1350 0;6 1500 0];
NODES([4 6],3) = 2750; % Dispersed generation
% Per-Unit transformation
LINES(:,3) = LINES(:,3)/Rb; NODES(:,2:3) = NODES(:,2:3)/Pb;
%% INCIDENCE MATRIX
N = size(NODES,1); L = size(LINES,1); A = zeros(N,L);
for k = 1:L
    n = LINES(k,1); m = LINES(k,2); A(n,k) = 1; A(m,k) = -1;
end
%% PRIMITIVE ADMITANCE MATRIX
G = diag(1./LINES(:,3)); Gbus = A*G*A.';
Vmin = 0.90; Vo = ones(N,1)*Vmin;
zeta = 1e-10; tmax = 20; Keep = [];
for t = 1:tmax
    %% RCF modeling
    cvx_begin
    cvx_solver gurobi
    variable Pgs(N,1);
    variable Id(N,1);
    variable Pgd(N,1);
    variable V(N,1);
    minimize transpose(V)*(Gbus*V);
    subject to
    V(1,1) == 1;
    for k = 1:N
        %% Balances
        Pgs(k,1)/Vo(k,1) - NODES(k,2)*(2/Vo(k,1) - (1/Vo(k,1))^2*V(k,1)) + 1*Pgd(k,1)/Vo(k,1)
            == Gbus(k,:)*V(:,1);
        if k > 1
            Pgs(k,1) == 0;
        end
            0 <= Pgd(k,1) <= NODES(k,3);
    end
    cvx_end
    Keep(t,:) = [t max(abs(abs(Vo) - abs(V))) cvx_optval*Pb];
    if max(abs(abs(Vo) - abs(V))) <= zeta
        Vo = V;
        break
    else
        Vo = V;
    end
end
Time = toc;
```

Figure 2. MATLAB implementation of the proposed RCF to the six-bus system.

Once the MATLAB code in Figure 2 is solved for the six-bus grid, voltage profiles when dispersed generators are or are not connected are presented in Figure 3.

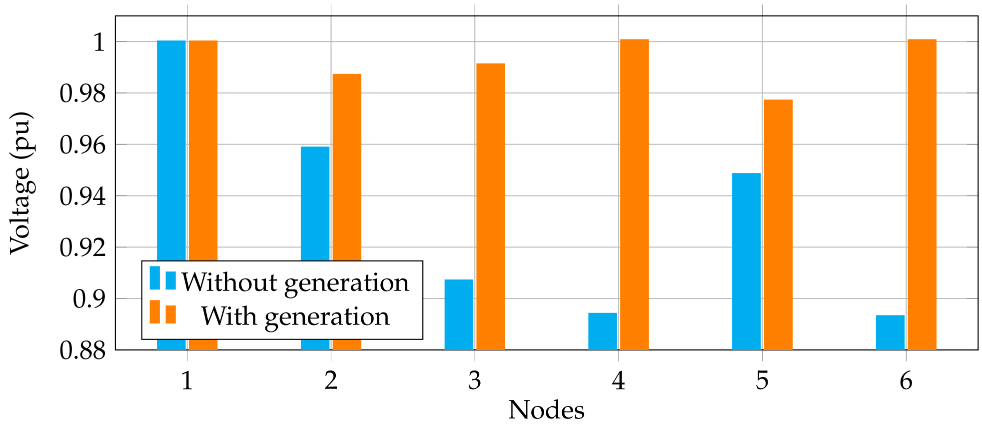

Figure 3. Voltage profiles for the six-bus system when dispersed generation is or is not considered.

The voltage profiles in Figure 3, as expected, clearly improve when the dispersed generators are connected to the network. Note that the voltage regulation without dispersed generators is higher than 10%; however, with the inclusion of dispersed sources, it improves until 2.30%. In addition, the initial power losses for this test feeder are 645.3576 W, which are reduced to 68.2905 W when the dispersed sources in nodes 4 and 6 generate 2266.1062 and 2643.2839 W, respectively.

5. Conclusions

The problem regarding the OPF solution in monopolar DC networks was addressed in this research through the implementation of a recursive convex approximation approach. The hyperbolic relation between the powers and voltages in demand nodes was reformulated as an affine equation through the linear approximation based on Taylor's series expansion for the function $\frac{1}{v_k}$. In addition, the nonlinear relation between powers and voltage in the slack and dispersed generations was relaxed using the value of the voltages at the iteration t in order to determine their new values at the iteration $t+1$. To eliminate the estimation error between the linear approximation of the nonlinear function $\frac{1}{v_k}$, a recursive solution of the equivalent convex model is implemented until the error between two consecutive iterations, i.e., $\varepsilon = \max_{k \in \mathcal{N}}\left\{\left|\left|v_k^{t+1}\right| - \left|v_k^t\right|\right|\right\}$, reached the desired convergence.

The numerical simulations demonstrated that: (i) the proposed RCF can efficiently solve the classical power flow problem by fixing all the power generation in the dispersed sources as zero, and the error between the RCF model and the Newton–Raphson method was less than 1×10^{-06}, which confirms the effectiveness of the RCF as a power flow solver; (ii) the comparative analysis between the RCF approach and the metaheuristic optimizers shows that the proposed approach effectively reaches the global optimal solution of the problem without recurring in a statistical analysis, while the BHO and the SCA are stuck in locally optimal solutions and the VSA is the only metaheuristic approach that can deal with the OPF solution in monopolar DC networks with low standard deviations; and (iii) the comparative analysis of the RCF with respect to the SDP approach confirmed that both convex methods reached the global optimal solution with the main advantage that these are directly applicable to radial and meshed distribution networks without any modification in their mathematical structures; however, the RCF corresponded to the most efficient method to solve the OPF problem when compared with the SDC approach and the combinatorial optimizers regarding processing times.

In future works, it will be possible to develop the following research: (i) to extend the proposed convex recursive model to operate batteries and renewable generators in monopolar DC networks; (ii) to propose a recursive convex formulation to solve the OPF problem in bipolar DC networks, including unbalanced loads and non-grounded neutral wire; and (iii) to obtain a mixed-integer convex model to locate and size renewable generators in monopolar DC networks.

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