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Single-Valued Neutrosophic Hybrid Arithmetic and Geometric Aggregation Operators and Their Decision-Making Method

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Abstract: Single-valued neutrosophic numbers (SVNNs) can express incomplete, indeterminate, and inconsistent information in the real world. Then, the common weighted aggregation operators of SVNNs may result in unreasonably aggregated results in some situations. Based on the hybrid weighted arithmetic and geometric aggregation and hybrid ordered weighted arithmetic and geometric aggregation ideas, this paper proposes SVNN hybrid weighted arithmetic and geometric aggregation (SVNNHWAGA) and SVNN hybrid ordered weighted arithmetic and geometric aggregation (SVNNHOWAGA) operators and investigates their rationality and effectiveness by numerical examples. Then, we establish a multiple-attribute decision-making method based on the SVNNHWAGA or SVNNHOWAGA operator under a SVNN environment. Finally, the multiple-attribute decision-making problem about the design schemes of punching machine is presented as a case to show the application and rationality of the proposed decision-making method.

Keywords: multiple-attribute decision-making; single-valued neutrosophic number; single-valued neutrosophic number hybrid weighted arithmetic and geometric aggregation (SVNNHWAGA) operator; single-valued neutrosophic number hybrid ordered weighted arithmetic and geometric aggregation (SVNNHOWAGA) operator

1. Introduction

Fuzzy decision-making theory has been an important research topic. Many fuzzy theories, especially like fuzzy (linguistic) sets [1,2], intuitionistic fuzzy sets [3], interval-valued intuitionistic fuzzy sets [4] etc., have been proposed and applied to decision-making problems [5–8]. However, there exists incomplete, indeterminate, and inconsistent information in the real life, which is not expressed by the aforementioned fuzzy sets. To represent incomplete, indeterminate, and inconsistent information, Smarandache [9] firstly proposed neutrosophic sets as the generalization of fuzzy sets, intuitionistic fuzzy sets, and interval-valued intuitionistic fuzzy sets. In the neutrosophic set, its truth, indeterminacy, and falsity membership degrees lie in the real standard/nonstandard interval $]^{-}0, 1^{+}[$. To constrain them in the real standard interval [0, 1] for convenient engineering applications, single-valued neutrosophic sets [10], interval neutrosophic sets [11], and simplified neutrosophic sets (including single-valued neutrosophic sets and interval neutrosophic sets) [12] were introduced as the subclasses of the neutrosophic sets. Recently, many researchers have developed various aggregation operators of simplified neutrosophic numbers (including single-valued/interval neutrosophic numbers) to be used for multiple-attribute decision-making problems with simplified (single-valued/interval) neutrosophic information. For example, they proposed simplified (single-valued/interval) weighted aggregation operators [12], interval neutrosophic number-weighted arithmetic average and interval neutrosophic number-weighted geometric average operators [13], single-valued neutrosophic number normalized weighted Bonferroni mean operators [14], generalized neutrosophic number Hamacher aggregation operators [15], interval neutrosophic number generalized weighted aggregation operator [16], interval neutrosophic number Choquet integral operator [17], interval neutrosophic number ordered weighted arithmetic average and interval neutrosophic number ordered weighted geometric average operators [18], interval neutrosophic number prioritized ordered weighted average operator [19], simplified neutrosophic number prioritized aggregation operator [20], single-valued neutrosophic number and interval neutrosophic number exponential weighted aggregation operators [21,22] etc. They have been wildly used for various decision-making problems in engineering, economics, and management areas. Furthermore, some scholars also proposed various multiple-attribute decision-making methods with single-valued neutrosophic numbers and interval neutrosophic numbers [23–27].

In the aforementioned aggregation operators, however, the interval neutrosophic number weighted arithmetic average and interval neutrosophic number weighted geometric average operators and the interval neutrosophic number ordered weighted arithmetic average and interval neutrosophic number ordered weighted geometric average operators are common aggregation operations in information fusion and decision-making areas. Especially when interval membership values in interval neutrosophic numbers are degenerated to any real numbers between 0 and 1, the interval neutrosophic number weighted arithmetic average (INNWAA) and interval neutrosophic number weighted geometric average (INNWGA) operators [13] can be reduced to the single-valued neutrosophic number weighted arithmetic average (SVNNWAA) and single-valued neutrosophic number weighted geometric average (SVNNWGA) operators, and the interval neutrosophic number ordered weighted arithmetic average (INNOWAA) and interval neutrosophic number ordered weighted geometric average (INNOWGA) operators [18] can be reduced to the single-valued neutrosophic number ordered weighted arithmetic average (SVNNOWAA) and single-valued neutrosophic number ordered weighted geometric average (SVNNOWGA) operators, respectively, as special cases of the existing interval neutrosophic number aggregation operators [13,18]. However, they imply the drawbacks of their unreasonably aggregated results in some cases (see Section 2 in detail). For example, in the information aggregations of the SVNNWAA and SVNNOWAA operators, their aggregated results may result in tendency to the maximum value in some cases, while the aggregated results of the SVNNWGA and SVNNOWGA operators may result in tendency to the maximum weight value in some cases. Also, the SVNNWGA and SVNNOWGA operators emphasize personal major points [13,18] and the SVNNWAA and SVNNOWAA operators emphasize group's major points [13,18]. Motivated by the hybrid arithmetic and geometric aggregation operators of intuitionistic fuzzy numbers [28], this paper proposes the single-valued neutrosophic number hybrid weighted arithmetic and geometric aggregation (SVNNHWAGA) and single-valued neutrosophic number hybrid ordered weighted arithmetic and geometric aggregation (SVNNHOWAGA) operators to realize more reasonable results in information aggregations of single-valued neutrosophic numbers, and then indicates some properties of the SVNNHWAGA and SVNNHOWAGA operators. Furthermore, a single-valued neutrosophic multiple-attribute decision-making method is established by using the SVNNHWAGA or SVNNHOWAGA operator, and then used for the decision-making problem of design schemes of punching machine under a single-valued neutrosophic environment. The main advantage of this study is that the proposed SVNNHWAGA and SVNNHOWAGA operators can overcome the drawbacks of the existing arithmetic/geometric average aggregation operators of single-valued neutrosophic numbers in some situations and reach the moderate aggregation values.

The remainder of this paper is structured as the following. In Section 2, we introduce some basic concepts and operations of single-valued neutrosophic numbers and investigate some drawbacks of the SVNNWAA, SVNNOWAA, SVNNWGA, and SVNNOWGA operators in some cases. In Section 3, we propose the SVNNHWAGA and SVNNHOWAGA operators and investigate their effectiveness and rationality based on numerical examples. Section 4 develops a single-valued neutrosophic multiple-attribute decision-making method based on the SVNNHWAGA or

SVNNHOWAGA operator. Section 5 presents a multiple-attribute decision-making problem about the design schemes of punching machine as a case to illustrate the application and effectiveness of the presented decision-making method. Section 6 gives some conclusions and further research.

2. Some Concepts and Operations of Single-Valued Neutrosophic Numbers

Definition 1 [1]. *Let X be a universal of discourse. A single-valued neutrosophic set N in X is characterized by truth, indeterminacy, and falsity membership functions $T_N(x)$, $U_N(x)$, and $V_N(x)$, respectively, where the values of the three functions $T_N(x)$, $U_N(x)$, and $V_N(x)$ are real numbers between 0 and 1, satisfying $T_N(x)$, $U_N(x)$, $V_N(x) \in [0, 1]$ and $0 \le T_N(x) + U_N(x) + V_N(x) \le 3$ for $x \in X$. Thus, the single-valued neutrosophic set N is denoted as the following form*:

$$N = \{ \langle x, T_N(x), U_N(x), V_N(x) \rangle \mid x \in X \}$$

For convenience, a basic element $<x, T_N(x), U_N(x), V_N(x)>$ in the single-valued neutrosophic set N is denoted by $z = <T, U, V>$ for short, which is called a single-valued neutrosophic number.

Since a single-valued neutrosophic number is a special case of an interval neutrosophic number, the concepts and operations of interval neutrosophic numbers can be introduced to single-valued neutrosophic numbers.

Let two single-valued neutrosophic numbers be $z_1 = <T_1, U_1, V_1>$ and $z_2 = <T_2, U_2, V_2>$. Then, there are the following relations [9–13]:

(1) $(z_1)^c = <V_1, 1 - U_1, T_1>$ (complement of z_1);
(2) $z_1 \subseteq z_2$ if and only if $T_1 \le T_2$, $U_1 \ge U_2$ and $V_1 \ge V_2$;
(3) $z_1 = z_2$ if and only if $z_1 \subseteq z_2$ and $z_2 \subseteq z_1$.
(4) $z_1 \oplus z_2 = \langle T_1 + T_2 - T_1T_2, U_1U_2, V_1V_2 \rangle$;
(5) $z_1 \otimes z_2 = \langle T_1T_2, U_1 + U_2 - U_1U_2, V_1 + V_2 - V_1V_2 \rangle$;
(6) $\alpha z_1 = \langle 1-(1-T_1)^\alpha, U_1^\alpha, V_1^\alpha \rangle$ for $\alpha > 0$;
(7) $z_1^\alpha = \langle z_1^\alpha, 1-(1-U_1)^\alpha, 1-(1-V_1)^\alpha \rangle$ for $\alpha > 0$.

For any single-valued neutrosophic number $z = <T, U, V>$, its score and accuracy functions [13] are defined, respectively, as follows:

$$E(z) = (2 + T - U - V)/3, \quad E(z) \in [0,1] \tag{1}$$

$$H(z) = T - V, \quad H(z) \in [-1,1] \tag{2}$$

Definition 2 [13]. *Let two single-valued neutrosophic numbers be $z_1 = <T_1, U_1, V_1>$ and $z_2 = <T_2, U_2, V_2>$, then the ranking method based on both the score values of $E(z_1)$ and $E(z_2)$ and the accuracy degrees of $H(z_1)$ and $H(z_2)$ has the following relations*:

(1) If $E(z_1) > E(z_2)$, then $z_1 \succ z_2$;
(2) If $E(z_1) = E(z_2)$ and $H(z_1) > H(z_2)$, then $z_1 \succ z_2$;
(3) If $E(z_1) = E(z_2)$ and $H(z_1) = H(z_2)$, then $z_1 = z_2$.

Let $z_j = <T_j, U_j, V_j>$ ($j = 1, 2, \ldots, n$) be a collection of single-valued neutrosophic numbers. Then, the SVNNWAA and SVNNWGA operators [13] are introduced, respectively, as follows:

$$SVNNWAA(z_1, z_2, \ldots, z_n) = \sum_{j=1}^{n} w_j z_j = \left\langle 1 - \prod_{j=1}^{n} (1-T_j)^{w_j}, \prod_{j=1}^{n} (U_j)^{w_j}, \prod_{j=1}^{n} (V_j)^{w_j} \right\rangle \tag{3}$$

$$SVNNWGA(z_1, z_2, \ldots, z_n) = \prod_{j=1}^{n} z_j^{w_j} = \left\langle \prod_{j=1}^{n} (T_j)^{w_j}, 1 - \prod_{j=1}^{n} (1 - U_j)^{w_j}, 1 - \prod_{j=1}^{n} (1 - V_j)^{w_j} \right\rangle \quad (4)$$

where w_j (j = 1, 2, …, n) is the weight of z_j (j = 1, 2, …, n), satisfying $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

When the orders of all the arguments are considered by important positions in the aggregation process of single-valued neutrosophic numbers, the SVNNOWAA and SVNNOWGA operators [18] are introduced, respectively, as follows:

$$SVNNOWAA(z_1, z_2, \ldots, z_n) = \sum_{j=1}^{n} \zeta_j z_{p(j)} = \left\langle 1 - \prod_{j=1}^{n} (1 - T_{p(j)})^{\zeta_j}, \prod_{j=1}^{n} (U_{p(j)})^{\zeta_j}, \prod_{j=1}^{n} (V_{p(j)})^{\zeta_j} \right\rangle \quad (5)$$

$$SVNNOWGA(z_1, z_2, \ldots, z_n) = \prod_{j=1}^{n} z_{p(j)}^{\zeta_j} = \left\langle \prod_{j=1}^{n} (T_{p(j)})^{\zeta_j}, 1 - \prod_{j=1}^{n} (1 - U_{p(j)})^{\zeta_j}, 1 - \prod_{j=1}^{n} (1 - V_{p(j)})^{\zeta_j} \right\rangle \quad (6)$$

where (p(1), p(2), …, p(n)) is a permutation of (1, 2, …, n) based on $p(j-1) \geq p(j)$ for j = 2, …, n; (ζ_1, ζ_2, …, ζ_n) is an associated weight vector, satisfying $\zeta_j \in [0, 1]$ and $\sum_{j=1}^{n} \zeta_j = 1$. Then, the SVNNOWAA and SVNNOWGA operators can reflect the important degrees of the ordered positions of arguments.

Although the above four aggregation operators are common aggregation operations in information fusion and decision-making areas, they imply some drawbacks, which result in tendency to the maximum arguments or weight values of their aggregated values. For example, some drawbacks are shown by the following two numerical examples.

Example 1. *Let two single-valued neutrosophic numbers be* z_1 = <0.001, 0, 0> *and* z_2 = <1, 0, 0> *with their weights and associated weights* $w_1 = \zeta_1 = 0.9$ *and* $w_2 = \zeta_2 = 0.1$, *respectively.*

Then, by using Equations (3)–(6) we can yield SVNNWAA(z_1, z_2) = <1, 0, 0>, SVNNWGA(z_1, z_2) = <0.002, 0, 0>, SVNNOWAA(z_1, z_2) = <1, 0, 0>, and SVNNOWGA(z_1, z_2) = <0.5012, 0, 0>.

Example 2. *Also take two single-valued neutrosophic numbers* z_1 = <0.001, 0, 0> *and* z_2 = <1, 0, 0> *with their weights* $w_1 = \zeta_1 = 0.1$ *and* $w_2 = \zeta_2 = 0.9$, *respectively.*

Then, we can obtain SVNNWAA(z_1, z_2) = <1, 0, 0>, SVNNWGA(z_1, z_2) = <0.5012, 0, 0>, SVNNOWAA(z_1, z_2) = <1, 0, 0>, and SVNNOWGA(z_1, z_2) = <0.002, 0, 0>.

From the aggregated results of the two examples, it is obvious that the aggregated values of the SVNNWAA and SVNNOWAA operators indicate tendency to the maximum value, and then the aggregated values of the SVNNWGA and SVNNOWGA operators indicate tendency to the maximum weight value. Therefore, the SVNNWAA, SVNNOWAA, SVNNWGA and SVNNOWGA operators may result in unreasonably aggregated results of single-valued neutrosophic numbers in some cases. To overcome these drawbacks, we need to improve these aggregation operators and to propose hybrid arithmetic and geometric aggregation operators of single-valued neutrosophic numbers as the extension of hybrid arithmetic and geometric aggregation operators of intuitionistic fuzzy numbers in [28].

3. Hybrid Arithmetic and Geometric Aggregation Operators of Single-Valued Neutrosophic Numbers

3.1. SVNNHWAGA Operator

Definition 3. *Let* z_j = <T_j, U_j, V_j> (j = 1, 2, …, n) *be a collection of single-valued neutrosophic numbers. Then, the SVNNHWAGA operator is defined by*

$$SVNNHWAGA(z_1, z_2, ..., z_n) = \left(\sum_{j=1}^{n} w_j z_j \right)^{\alpha} \left(\prod_{j=1}^{n} z_j^{w_j} \right)^{(1-\alpha)} \tag{7}$$

where w_j (j = 1, 2, …, n) is the weight of z_j (j = 1, 2, …, n), satisfying $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, and $\alpha \in [0, 1]$.

Theorem 1. *Let z_j = <T_j, U_j, V_j> (j = 1, 2, …, n) be a collection of single-valued neutrosophic numbers and α be any real number in [0,1]. Thus, the aggregated value of the SVNNHWAGA operator is also a single-valued neutrosophic number, which is calculated by*

$$SVNNHWAGA(z_1, z_2, ..., z_n) = \left(\sum_{j=1}^{n} w_j z_j \right)^{\alpha} \left(\prod_{j=1}^{n} z_j^{w_j} \right)^{(1-\alpha)} = \left\langle \begin{array}{c} \left(1 - \prod_{j=1}^{n} (1-T_j)^{w_j}\right)^{\alpha} \left(\prod_{j=1}^{n} T_j^{w_j} \right)^{(1-\alpha)}, \\ 1 - \left(1 - \prod_{j=1}^{n} U_j^{w_j}\right)^{\alpha} \left(\prod_{j=1}^{n} (1-U_j)^{w_j} \right)^{(1-\alpha)}, \\ 1 - \left(1 - \prod_{j=1}^{n} V_j^{w_j}\right)^{\alpha} \left(\prod_{j=1}^{n} (1-V_j)^{w_j} \right)^{(1-\alpha)} \end{array} \right\rangle \tag{1}$$

where w_j is the weight of z_j (j = 1, 2, …, n), satisfying $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Proof:

Corresponding to the operational laws of single-valued neutrosophic numbers in Section 2 and the SVNNWAA and SVNNWGA operators, we have the following result:

$$SVNNHWAGA(z_1, z_2, \ldots, z_n) = \left(\sum_{j=1}^{n} w_j z_j\right)^{\alpha} \left(\prod_{j=1}^{n} z_j^{w_j}\right)^{(1-\alpha)}$$

$$= \left\langle 1-\prod_{j=1}^{n}(1-T_j)^{w_j}, \prod_{j=1}^{n}U_j^{w_j}, \prod_{j=1}^{n}V_j^{w_j} \right\rangle^{\alpha} \left\langle \prod_{j=1}^{n}T_j^{w_j}, 1-\prod_{j=1}^{n}(1-U_j)^{w_j}, 1-\prod_{j=1}^{n}(1-V_j)^{w_j} \right\rangle^{(1-\alpha)}$$

$$= \left\langle \left(1-\prod_{j=1}^{n}(1-T_j)^{w_j}\right)^{\alpha}, 1-\left(1-\prod_{j=1}^{n}U_j^{w_j}\right)^{\alpha}, 1-\left(1-\prod_{j=1}^{n}V_j^{w_j}\right)^{\alpha} \right\rangle$$

$$\times \left\langle \left(\prod_{j=1}^{n}T_j^{w_j}\right)^{(1-\alpha)}, 1-\left(\prod_{j=1}^{n}(1-U_j)^{w_j}\right)^{(1-\alpha)}, 1-\left(\prod_{j=1}^{n}(1-V_j)^{w_j}\right)^{(1-\alpha)} \right\rangle$$

$$= \left\langle \begin{array}{l} \left(1-\prod_{j=1}^{n}(1-T_j)^{w_j}\right)^{\alpha} \left(\prod_{j=1}^{n}T_j^{w_j}\right)^{(1-\alpha)}, \left[1-\left(1-\prod_{j=1}^{n}U_j^{w_j}\right)^{\alpha} + 1-\left(\prod_{j=1}^{n}(1-U_j)^{w_j}\right)^{(1-\alpha)}\right] \\ -\left[1-\left(1-\prod_{j=1}^{n}U_j^{w_j}\right)^{\alpha}\right]\left[1-\left(\prod_{j=1}^{n}(1-U_j)^{w_j}\right)^{(1-\alpha)}\right], \\ \left[1-\left(1-\prod_{j=1}^{n}V_j^{w_j}\right)^{\alpha} + 1-\left(\prod_{j=1}^{n}(1-V_j)^{w_j}\right)^{(1-\alpha)}\right] - \left[1-\left(1-\prod_{j=1}^{n}V_j^{w_j}\right)^{\alpha}\right]\left[1-\left(\prod_{j=1}^{n}(1-V_j)^{w_j}\right)^{(1-\alpha)}\right] \end{array} \right\rangle$$

$$= \left\langle \begin{array}{l} \left(1-\prod_{j=1}^{n}(1-T_j)^{w_j}\right)^{\alpha} \left(\prod_{j=1}^{n}T_j^{w_j}\right)^{(1-\alpha)}, \left[1-\left(1-\prod_{j=1}^{n}U_j^{w_j}\right)^{\alpha} + 1-\left(\prod_{j=1}^{n}(1-U_j)^{w_j}\right)^{(1-\alpha)}\right] \\ -\left[1-\left(\prod_{j=1}^{n}(1-U_j)^{w_j}\right)^{(1-\alpha)} - \left(1-\prod_{j=1}^{n}U_j^{w_j}\right)^{\alpha} + \left(1-\prod_{j=1}^{n}U_j^{w_j}\right)^{\alpha}\left(\prod_{j=1}^{n}(1-U_j)^{w_j}\right)^{(1-\alpha)}\right], \\ \left[1-\left(1-\prod_{j=1}^{n}V_j^{w_j}\right)^{\alpha} + 1-\left(\prod_{j=1}^{n}(1-V_j)^{w_j}\right)^{(1-\alpha)}\right] \\ -\left[1-\left(\prod_{j=1}^{n}(1-V_j)^{w_j}\right)^{(1-\alpha)} - \left(1-\prod_{j=1}^{n}V_j^{w_j}\right)^{\alpha} + \left(1-\prod_{j=1}^{n}V_j^{w_j}\right)^{\alpha}\left(\prod_{j=1}^{n}(1-V_j)^{w_j}\right)^{(1-\alpha)}\right] \end{array} \right\rangle$$

$$= \left\langle \begin{array}{l} \left(1-\prod_{j=1}^{n}(1-T_j)^{w_j}\right)^{\alpha} \left(\prod_{j=1}^{n}T_j^{w_j}\right)^{(1-\alpha)}, \\ 1-\left(1-\prod_{j=1}^{n}U_j^{w_j}\right)^{\alpha}\left(\prod_{j=1}^{n}(1-U_j)^{w_j}\right)^{(1-\alpha)}, \\ 1-\left(1-\prod_{j=1}^{n}V_j^{w_j}\right)^{\alpha}\left(\prod_{j=1}^{n}(1-V_j)^{w_j}\right)^{(1-\alpha)} \end{array} \right\rangle.$$

Therefore, this completes the proof of Equation (8). □

Based on the properties of the INNWAA and INNWGA operators [13], the SVNNHWAGA operator also contains these properties:

(1) Idempotency: If $z_j = z$ for $j = 1, 2, \ldots, n$, then there is $SVNNHWAGA(z_1, z_2, \cdots, z_n) = z$.

(2) Boundedness: If $z_{\min} = \min(z_1, z_2, \ldots, z_n)$ and $z_{\max} = \max(z_1, z_2, \ldots, z_n)$ for $j = 1, 2, \ldots, n$, then there exists $z_{\min} \le SVNNHWAGA(z_1, z_2, \cdots, z_n) \le z_{\max}$.

(3) Monotonicity: If $z_j \le z_j^*$ for j = 1, 2, …, n, then $SVNNHWAGA(z_1, z_2, \cdots, z_n) \le SVNNHWAGA(z_1^*, z_2^*, \cdots, z_n^*)$ holds.

When the some values of $\alpha \in [0,1]$ are specified as the special cases, we can investigate the families of the SVNNHWAGA operator below:

(1) The SVNNHWAGA operator reduces to the SVNNWAA operator if $\alpha = 1$;
(2) The SVNNHWAGA operator reduces to the SVNNWGA operator if $\alpha = 0$;
(3) The SVNNHWAGA operator is the mean of the SVNNWAA and SVNNWGA operators if $\alpha = 0.5$.

3.2. SVNNHOWAGA Operator

Definition 4. *Let z_j = <T_j, U_j, V_j> (j = 1, 2, …, n) be a collection of single-valued neutrosophic numbers. Then, the SVNNHOWAGA operator is defined by*

$$SVNNHOWAGA(z_1, z_2, ..., z_n) = \left(\sum_{j=1}^{n} \zeta_j z_{p(j)}\right)^{\alpha} \left(\prod_{j=1}^{n} z_{p(j)}^{\zeta_j}\right)^{(1-\alpha)} \tag{9}$$

where ($p(1)$, $p(2)$, …, $p(n)$) is a permutation of (1, 2, …, n) based on $p(j-1) \ge p(j)$ for j = 2, …, n; (ζ_1, ζ_2, …, ζ_n) is an associated weight vector, satisfying $\zeta_j \in [0, 1]$ and $\sum_{j=1}^{n} \zeta_j = 1$; α is any real number between 0 and 1. Then, the SVNNHOWAGA operator can reflect the important degrees of the ordered positions of arguments in the aggregated process of single-valued neutrosophic numbers.

Theorem 2. *Let z_j = <T_j, U_j, V_j> (j = 1, 2, …, n) be a collection of single-valued neutrosophic numbers and $\alpha \in [0, 1]$. Thus, the aggregated value of the SVNNHOWAGA operator is also a single-valued neutrosophic number, which is calculated by*

$$SVNNHOWAGA(z_1, z_2, ..., z_n) = \left(\sum_{j=1}^{n} \zeta_j z_{p(j)}\right)^{\alpha} \left(\prod_{j=1}^{n} z_{p(j)}^{\zeta_j}\right)^{(1-\alpha)} = \left\langle \begin{array}{c} \left(1-\prod_{j=1}^{n}(1-T_{p(j)})^{\zeta_j}\right)^{\alpha} \left(\prod_{j=1}^{n} T_{p(j)}^{\zeta_j}\right)^{(1-\alpha)}, \\ 1-\left(1-\prod_{j=1}^{n} U_{p(j)}^{\zeta_j}\right)^{\alpha} \left(\prod_{j=1}^{n}(1-U_{p(j)})^{\zeta_j}\right)^{(1-\alpha)}, \\ 1-\left(1-\prod_{j=1}^{n} V_{p(j)}^{\zeta_j}\right)^{\alpha} \left(\prod_{j=1}^{n}(1-V_{p(j)})^{\zeta_j}\right)^{(1-\alpha)} \end{array} \right\rangle \tag{10}$$

where ($p(1)$, $p(2)$, …, $p(n)$) is a permutation of (1, 2, …, n) based on $p(j - 1) \ge p(j)$ for j = 2, …, n; (ζ_1, ζ_2, …, ζ_n) is an associated weight vector with $\sum_{j=1}^{n} \zeta_j = 1$ for $\zeta_j \in [0,1]$.

Based on the similar proof manner of Theorem 1, it is obvious that Equation (10) can hold. Here, we omit the proof.

Corresponding to the properties of the INNOWAA and INNOWGA operators in [18], the SVNNHOWAGA operator also contains the following properties:

(1) Idempotency: If $z_j = z$ for j = 1, 2, …, n, then there exists $SVNNHOWAGA(z_1, z_2, ..., z_n) = z$.
(2) Boundedness: If $z_{\min} = \min(z_1, z_2, ..., z_n)$ and $z_{\max} = \max(z_1, z_2, ..., z_n)$ for j = 1, 2, …, n, then there exists $z_{\min} \le SVNNHOWAGA(z_1, z_2, \cdots, z_n) \le z_{\max}$.
(3) Monotonicity: If $z_j \le z_j^*$ for j = 1, 2, …, n, then $SVNNHOWAGA(z_1, z_2, \cdots, z_n) \le SVNNHOWAGA(z_1^*, z_2^*, \cdots, z_n^*)$ holds.
(4) Commutativity: If $(z_1^{'}, z_2^{'}, \cdots, z_n^{'})$ is any permutation of $(z_1, z_2, \cdots, z_n)$, then $SVNNHOWAGA(z_1, z_2, \cdots, z_n) = SVNNHOWAGA(z_1^{'}, z_2^{'}, \cdots, z_n^{'})$ holds.

When some values of $\alpha \in [0, 1]$ are specified as the special cases, we can investigate the families of the SVNNHOWAGA operator below:

(1) The SVNNHOWAGA operator reduces to the SVNNOWAA operator if $\alpha = 1$;
(2) The SVNNHOWAGA operator reduces to the SVNNOWGA operator if $\alpha = 0$;
(3) The SVNNHOWAGA operator is the mean of the SVNNOWAA and SVNNOWGA operators if $\alpha = 0.5$.

3.3. Numerical Examples

We still consider the aforementioned two numerical examples to illustrate the effectiveness and rationality of the aggregated values of the SVNNHWAGA and SVNNHOWAGA operators. Generally taking $\alpha = 0.5$, we apply the SVNNHWAGA and SVNNHOWAGA operators to calculate the two numerical examples in Section 2.

For Example 1, by using Equations (8) and (10) we can obtain SVNNHWAGA(z_1, z_2) = <0.0447, 0, 0>, which is the moderate value between SVNNWAA(z_1, z_2) = <1, 0, 0> and SVNNWGA(z_1, z_2) = <0.002, 0, 0>, and SVNNHOWAGA(z_1, z_2) = <0.7079, 0, 0>, which is the moderate value between SVNNOWAA(z_1, z_2) = <1, 0, 0> and SVNNOWGA(z_1, z_2) = <0.5012, 0, 0>.

For Example 2, by using Equations (8) and (10) we get SVNNHWAGA(z_1, z_2) = <0.7079, 0, 0>, which is the moderate value between SVNNWAA(z_1, z_2) = <1, 0, 0> and SVNNWGA(z_1, z_2) = <0.5012, 0, 0>, and SVNNHOWAGA(z_1, z_2) = <0.0447, 0, 0>, which is the moderate value between SVNNOWAA(z_1, z_2) = <1, 0, 0> and SVNNOWGA(z_1, z_2) = <0.002, 0, 0>.

From the above aggregated results of the two numerical examples, the SVNNHWAGA and SVNNHOWAGA operators indicate their moderate values. Obviously, the SVNNHWAGA and SVNNHOWAGA operators demonstrate their effectiveness and rationality in the information aggregations.

4. Decision-Making Method Using the SVNNHWAGA or SVNNHOWAGA Operator

This section develops a multiple-attribute decision-making method by using the SVNNHWAGA or SVNNHOWAGA operator.

In a multiple-attribute decision-making problem, suppose that $Z = \{Z_1, Z_2, \ldots, Z_m\}$ is a set of alternatives and $A = \{A_1, A_2, \ldots, A_n\}$ is a set of attributes. By decision-makers' suitability evaluation for each attribute A_j over each alternative Z_i, all the evaluation values are expressed by single-valued neutrosophic numbers $z_{ij} = \langle T_{ij}, U_{ij}, V_{ij}\rangle$, where $T_{ij}, U_{ij}, V_{ij} \in [0,1]$ and $0 \le T_{ij} + U_{ij} + V_{ij} \le 3$ ($j = 1, 2, \ldots, n$; $i = 1, 2, \ldots, m$). In the single-valued neutrosophic number z_{ij}, T_{ij} indicates the degree that the alternative Z_i is suitable for the attribute A_j, U_{ij} indicates the degree that the alternative Z_i is unsure/indeterminate for the attribute A_j, and V_{ij} indicates the degree that the alternative Z_i is unsuitable for the attribute A_j. Thus, all the evaluation values can be constructed as a single-valued neutrosophic decision matrix $D = (z_{ij})_{m\times n}$.

Hence, we can apply the proposed decision-making method based on the SVNNHWAGA or SVNNHOWAGA operator to the multiple-attribute decision-making problem and give the following decision procedures:

Step 1. Suppose that the weight vector of attributes is $\boldsymbol{w} = (w_1, w_2, \ldots, w_n)$ and satisfies $\sum_{j=1}^{n} w_j = 1$ for $w_j \in [0, 1]$. Then, the aggregated value of z_i ($i = 1, 2, \ldots, m$) for each alternative Z_i ($i = 1, 2, \ldots, m$) is calculated by the following SVNNHWAGA operator:

$$z_i = SVNNHWAGA(z_{i1}, z_{i2}, \ldots, z_{in}) = \left(\sum_{j=1}^{n} w_j z_{ij}\right)^{\alpha} \left(\prod_{j=1}^{n} z_{ij}^{w_j}\right)^{(1-\alpha)} = \left\langle \begin{array}{c} \left(1-\prod_{j=1}^{n}(1-T_{ij})^{w_j}\right)^{\alpha}\left(\prod_{j=1}^{n} T_{ij}^{w_j}\right)^{(1-\alpha)}, \\ 1-\left(1-\prod_{j=1}^{n} U_{ij}^{w_j}\right)^{\alpha}\left(\prod_{j=1}^{n}(1-U_{ij})^{w_j}\right)^{(1-\alpha)}, \\ 1-\left(1-\prod_{j=1}^{n} V_{ij}^{w_j}\right)^{\alpha}\left(\prod_{j=1}^{n}(1-V_{ij})^{w_j}\right)^{(1-\alpha)} \end{array} \right\rangle \quad (11)$$

On the other hand, suppose that the ordered important positions of all the arguments are given by the associated weight vector $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)$, satisfying $\sum_{j=1}^{n} \zeta_j = 1$ for $\zeta_j \in [0,1]$. Thus, the aggregated value of z_i (i = 1, 2, …, m) for each alternative Z_i (i = 1, 2, …, m) is calculated by the following SVNNHOWAGA operator:

$$z_i = SVNNHOWAGA(z_{i1}, z_{i2}, \ldots, z_{in}) = \left(\sum_{j=1}^{n} \zeta_j z_{ip(j)}\right)^{\alpha} \left(\prod_{j=1}^{n} z_{ip(j)}^{\zeta_j}\right)^{(1-\alpha)} = \left\langle \begin{array}{l} \left(1-\prod_{j=1}^{n}(1-T_{ip(j)})^{\zeta_j}\right)^{\alpha} \left(\prod_{j=1}^{n} T_{ip(j)}^{\zeta_j}\right)^{(1-\alpha)}, \\ 1-\left(1-\prod_{j=1}^{n} U_{ip(j)}^{\zeta_j}\right)^{\alpha} \left(\prod_{j=1}^{n}(1-U_{ip(j)})^{\zeta_j}\right)^{(1-\alpha)}, \\ 1-\left(1-\prod_{j=1}^{n} V_{ip(j)}^{\zeta_j}\right)^{\alpha} \left(\prod_{j=1}^{n}(1-V_{ip(j)})^{\zeta_j}\right)^{(1-\alpha)} \end{array} \right\rangle \quad (12)$$

Step 2. By Equation (1) (Equation (2) if necessary), we calculate the score values of $E(z_i)$ (accuracy degrees of $H(z_i)$ if necessary) (i = 1, 2, …, m).

Step 3. Corresponding to the score values (accuracy degrees), we rank all the alternatives in a descending order and determine the best choice based on the alternative with the largest value.

Step 4. End.

5. MADM Problem of the Design Schemes of Punching Machine

In this section, an applied example about the multiple-attribute decision-making (MADM) problem of the design schemes (alternatives) of punching machine is introduced from [29] to illustrate the application and rationality of the proposed decision-making method by the actual case.

The conceptual design stage mainly contains two tasks, in which designers firstly provide different design schemes (potential alternatives) according to their knowledge and experience, and then decision makers (designers) give their suitability evaluation (decision making) of all the potential design schemes.

Some manufacturing company wants to design the punching machine to develop a new mechanical product. Generally speaking, the punching machine consists of the reducing mechanism, punching mechanism, and feed intermittent mechanism to construct its motion scheme. Based on the motion scheme, a group of designers provides a set of four potential design schemes (alternatives) $Z = \{Z_1, Z_2, Z_3, Z_4\}$ by their knowledge and experiences, which are shown in Table 1. The designers (decision makers) must make a decision depending on the requirements of the four attributes: (a) A_1 is the manufacturing cost; (b) A_2 is the structure complexity; (c) A_3 is the transmission effectiveness; (d) A_4 is the reliability; (e) A_5 is the maintainability. By the decision makers' suitability evaluation for the alternatives of Z_i (i = 1, 2, 3, 4) with respect to the attributes of A_j (j = 1, 2, 3, 4, 5), their evaluation values are expressed by the form of single-valued neutrosophic numbers. Thus, the single-valued neutrosophic number decision matrix $D = (z_{ij})_{4\times5}$ can be constructed as follows:

$$D = \begin{bmatrix} (0.75, 0.1, 0.4) & (0.8, 0.1, 0.3) & (0.85, 0.1, 0.2) & (0.85, 0.1, 0.3) & (0.9, 0.1, 0.2) \\ (0.7, 0.1, 0.5) & (0.75, 0.1, 0.1) & (0.75, 0.2, 0.1) & (0.8, 0.1, 0.1) & (0.8, 0.2, 0.3) \\ (0.8, 0.2, 0.3) & (0.78, 0.1, 0.2) & (0.8, 0.1, 0.2) & (0.8, 0.2, 0.2) & (0.75, 0.1, 0.3) \\ (0.9, 0.1, 0.2) & (0.85, 0.1, 0.1) & (0.9, 0.1, 0.2) & (0.85, 0.1, 0.3) & (0.85, 0.2, 0.3) \end{bmatrix}$$

Table 1. Four design schemes (potential alternatives) of punching machine [29].

Potential Alternative	Z_1	Z_2	Z_3	Z_4
Reducing mechanism	Gear reducer	Gear head motor	Gear reducer	Gear head motor
Punching mechanism	Crank-slider mechanism	Six-bar punching mechanism	Six-bar punching mechanism	Crank-slider mechanism

Dial feed intermittent mechanism	Sheave mechanism	Ratchet feed mechanism

Then, we can apply the developed decision-making method based on the SVNNHWAGA or SVNNHOWAGA operator to the multiple-attribute decision-making problem of the design schemes of punching machine.

If the weight vector of the five attributes is considered as $\boldsymbol{w}$ = (0.25, 0.2, 0.25, 0.15, 0.15) in the multiple-attribute decision-making problem, then the decision steps are presented as follows:

Step 1. By Equation (11) (generally take α = 0.5), we calculate the aggregated values of z_i (i = 1, 2, 3, 4) for each alternative Z_i (i = 1, 2, 3, 4) as the following results:
$z_1 = <0.8255, 0.1000, 0.2818>$, $z_2 = <0.7534, 0.1367, 0.2149>$, $z_3 = <0.7888, 0.1367, 0.2384>$, and $z_4 = <0.8761, 0.1134, 0.2049>$.

Step 2. By Equation (1), we calculate the score values of $E(z_i)$ for each alternative Z_i (i = 1, 2, 3, 4) as the following values:
$E(z_1) = 0.8145$, $E(z_2) = 0.8006$, $E(z_3) = 0.8045$, and $E(z_4) = 0.8526$.

Step 3. According to $E(z_4) > E(z_1) > E(z_3) > E(z_2)$, the ranking of the four design schemes is $Z_4 \succ Z_1 \succ Z_3 \succ Z_2$. So, the best design scheme is Z_4. These results are the same as in [29].

If the associated weight vector ζ = (0.4, 0.3, 0.1, 0.1, 0.1) is considered as the ordered important positions of all the given arguments in the multiple-attribute decision-making problem, the decision steps are described as the follows:

Step 1′. By Equation (12) (in general take α = 0.5), we calculate the aggregated values of z_i (i = 1, 2, 3, 4) for each alternative Z_i (i = 1, 2, 3, 4) as the following results:
$z_1 = <0.8577, 0.1000, 0.2378>$, $z_2 = <0.7708, 0.1179, 0.1520>$, $z_3 = <0.7892, 0.1179, 0.2190>$, and $z_4 = <0.8711, 0.1089, 0.1740>$.

Step 2′. By Equation (1), we calculate the score values of $E(z_i)$ for each scheme Z_i (i = 1, 2, 3, 4) as the following values:
$E(z_1) = 0.8400$, $E(z_2) = 0.8336$, $E(z_3) = 0.8174$, and $E(z_4) = 0.8627$.

Step 3′. According to $E(z_4) > E(z_1) > E(z_2) > E(z_3)$, the ranking of the four design schemes is $Z_4 \succ Z_1 \succ Z_2 \succ Z_3$. Thus, the best design scheme is also Z_4.

Although the above two ranking orders show little difference, the best scheme Z_4 is identical.

For comparative convenience, all the results of the proposed decision-making approach and the related decision-making methods based on the SVNNWAA, SVNNWGA, SVNNOWAA, and SVNNOWGA operators are summarized in Table 2.

Table 2. Decision-making results of different aggregation operators.

Aggregation Operator	Aggregated Result	Score Value	Ranking
SVNNWAA ($\alpha = 1$)	z_1 = <0.8301, 0.1000, 0.2741>, z_2 = <0.7553, 0.1320, 0.1763>, z_3 = <0.7892, 0.1320, 0.2352>, z_4 = <0.8775, 0.1110, 0.1966>	$E(z_1) = 0.8187$, $E(z_2) = 0.8157$, $E(z_3) = 0.8073$, $E(z_4) = 0.8566$	$Z_4 \succ Z_1 \succ Z_2 \succ Z_3$
SVNNWGA ($\alpha = 0$)	z_1 = <0.8209, 0.1000, 0.2895>, z_2 = <0.7516, 0.1414, 0.2517>, z_3 = <0.7883, 0.1414, 0.2416>, z_4 = <0.8746, 0.1158, 0.2131>	$E(z_1) = 0.8105$, $E(z_2) = 0.7861$, $E(z_3) = 0.8018$, $E(z_4) = 0.8486$	$Z_4 \succ Z_1 \succ Z_3 \succ Z_2$
SVNNHWAGA ($\alpha = 0.5$)	z_1 = <0.8255, 0.1000, 0.2818>, z_2 = <0.7534, 0.1367, 0.2149>, z_3 = <0.7888, 0.1367, 0.2384>, z_4 = <0.8761, 0.1134, 0.2049>	$E(z_1) = 0.8145$, $E(z_2) = 0.8006$, $E(z_3) = 0.8045$, $E(z_4) = 0.8526$	$Z_4 \succ Z_1 \succ Z_3 \succ Z_2$
SVNNOWAA ($\alpha = 1$)	z_1 = <0.8619, 0.1000, 0.2325>, z_2 = <0.7723, 0.1149, 0.1311>, z_3 = <0.7896, 0.1149, 0.2169>, z_4 = <0.8725, 0.1072, 0.1644>	$E(z_1) = 0.8431$, $E(z_2) = 0.8421$, $E(z_3) = 0.8193$, $E(z_4) = 0.8670$	$Z_4 \succ Z_1 \succ Z_2 \succ Z_3$
SVNNOWGA ($\alpha = 0$)	z_1 = <0.8536, 0.1000, 0.2432>, z_2 = <0.7693, 0.1210, 0.1724>, z_3 = <0.7888, 0.1210, 0.2211>, z_4 = <0.8697, 0.1105, 0.1835>	$E(z_1) = 0.8368$, $E(z_2) = 0.8253$, $E(z_3) = 0.8156$, $E(z_4) = 0.8585$	$Z_4 \succ Z_1 \succ Z_2 \succ Z_3$
SVNNHOWAGA ($\alpha = 0.5$)	z_1 = <0.8577, 0.1000, 0.2378>, z_2 = <0.7708, 0.1179, 0.1520>, z_3 = <0.7892, 0.1179, 0.2190>, z_4 = <0.8711, 0.1089, 0.1740>	$E(z_1) = 0.8400$, $E(z_2) = 0.8336$, $E(z_3) = 0.8174$, $E(z_4) = 0.8627$	$Z_4 \succ Z_1 \succ Z_2 \succ Z_3$

The results of Table 2 show that all the aggregated results of the SVNNHWAGA and SVNNHOWAGA operators tend toward moderate values between the aggregated values of the SVNNWAA and SVNNWGA operators or the SVNNOWAA and SVNNOWGA operators. Hence, the SVNNHWAGA and SVNNHOWAGA operators are suitable and effective and can overcome the drawbacks of the SVNNWAA, SVNNOWAA, SVNNWGA and SVNNOWGA operators. Furthermore, the different aggregation operators may show different ranking orders. Then, the different values of α may result in different ranking orders.

Compared with the decision-making method based on the hybrid arithmetic and geometric aggregation operators of intuitionistic fuzzy numbers introduced by Ye [28], Ye's method [28] uses the hybrid arithmetic and geometric aggregation operators of intuitionistic fuzzy numbers to aggregate intuitionistic fuzzy numbers and applied them to multiple-attribute decision-making problems under an intuitionistic fuzzy number environment; while the multiple-attribute decision-making method proposed in this paper uses the hybrid arithmetic and geometric aggregation operators of single-valued neutrosophic numbers to aggregate single-valued neutrosophic information as the extension of the hybrid arithmetic and geometric aggregation operators of intuitionistic fuzzy numbers and extends the decision-making method based on the hybrid arithmetic and geometric aggregation operators of intuitionistic fuzzy numbers [28], because an intuitionistic fuzzy number is a special case of a single-valued neutrosophic number and cannot express and handle indeterminate and inconsistent information. Thus, it is obvious that the SVNNHWAGA and SVNNHOWAGA operators and their decision-making method proposed in this paper are superior to the previous hybrid arithmetic and geometric aggregation operators and decision-making method with intuitionistic fuzzy numbers [28], because the latter cannot express and deal with the decision-making problems with indeterminate and inconsistent information. Hence, the proposed SVNNHWAGA and SVNNHOWAGA operators and their decision-making method in this study are more general and more suitable under indeterminate and inconsistent environments than the previous Ye's method [28].

However, the presented decision-making method based on the SVNNHWAGA or SVNNHOWAGA operator demonstrates its suitability and effectiveness in some decision-making situations since it can overcome the drawbacks of the unreasonably aggregated results in some cases (as mentioned in Section 2). Since some value of α is specified by the preference and actual requirements of decision makers, the decision-making method proposed in this study appears to be more flexible and more reasonable than the decision-making method based on one of the SVNNWAA, SVNNOWAA, SVNNWGA, and SVNNOWGA operators.

6. Conclusions

This paper presented the SVNNHWAGA and SVNNHOWAGA operators to overcome some drawbacks of the SVNNWAA, SVNNOWAA, SVNNWGA and SVNNOWGA operators to aggregate single-valued neutrosophic numbers in some cases, and then investigated their some properties and rationality. Furthermore, we established a multiple-attribute decision-making method based on the SVNNHWAGA or SVNNHOWAGA operator. Finally, a multiple-attribute decision-making problem about design schemes of punching machine is presented as a case to show the application and rationality of the proposed decision-making method. However, the proposed decision-making method based on the SVNNHWAGA or SVNNHOWAGA operator provides an effective and reasonable way for multiple-attribute decision-making problems since the SVNNWAA, SVNNOWAA, SVNNWGA and SVNNOWGA operators are special cases of the SVNNHWAGA and SVNNHOWAGA operators under a single-valued neutrosophic environment. Similarly, this study will be also further extended to interval neutrosophic sets and applications, like group decision making, pattern recognition, fault diagnosis, and medical diagnosis, and so on.

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