

Article

Entropy Production of Main-Sequence Stars

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Abstract: The entropy production (inside the volume bounded by a photosphere) of main-sequence stars is calculated based on $B-V$ photometry data. The entropy-production distribution function and the dependences of entropy production on temperature and luminosity are obtained for these stars for the first time. A very small range of variation of specific (per volume) entropy production discovered for main-sequence stars (only 0.5 to 1.8 solar magnitudes) is an interesting result that can be crucial for understanding thermodynamic processes of stars.

Keywords: non-equilibrium thermodynamics; open star clusters; main-sequence stars; entropy production

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1. Introduction

For more than a century, entropy and its production have been key quantities not only for non-equilibrium statistical physics and thermodynamics, but for natural science in general. They are of paramount importance when discussing variational principles of non-equilibrium physics, issues of order and disorder in nature, origin and transfer of information, problems of irreversibility and direction of time, *etc.* (see, e.g., [1–5]). Starting from the studies by Clausius and his concept of heat death of the Universe and until now, the papers related to entropy have always been in the centre of attention among astrophysicists and cosmologists. Currently, this quantity and its derivatives are used

to discuss of black holes, questions connected with the accelerated expansion of the Universe, to build and generalize gravitation theories, *etc.* (see, e.g., [6–10]). The majority of the papers, being strictly theoretical, place principal emphasis on functional relations between variables established through the analysis of entropy and its production. There are almost no quantitative calculations of entropy and entropy production for astrophysical objects; rough estimates are applied in rare cases even for relatively well-studied objects, such as, for instance, stars [11–15]. However, it is obvious that such calculations are extremely important for our understanding of the physics of the world around us. So, stars are the most common objects in the Universe, they contain more than 97% of the mass of all visible matter. But what is their entropy production, how does it depend on the type of a star, on the time of its life? Despite the fact that the quantities needed for calculating the entropy production are known (effective temperature and luminosity), modern literature has no answers to these questions. Presently, there is no other information, except for entropy production estimates of the Sun (the entropy productions of the Sun are $9 \times 10^{22} \text{ J} \cdot \text{K}^{-1} \cdot \text{s}^{-1}$ [14] and $8 \times 10^{22} \text{ J K}^{-1} \text{ s}^{-1}$ [15]). Thus, entropy production, a crucial quantity from the standpoint of non-equilibrium physics, was neither calculated nor quantitatively analyzed for the most important and widespread objects in the Universe: stars [16]. This paper is the first step to solve this issue. The study considers only main-sequence stars as the most common type of stars. The satisfactory accuracy of this investigation based on observational data has been achieved thanks to the recently appeared techniques that enabled to determine, with sufficiently high accuracy, effective temperature T_{eff} and bolometric correction BC of stars.

2. Object of Study

Stars belonging to open clusters were chosen as the object of study hereof. This is very convenient because, according to the modern concepts, stars of the same cluster are formed from the same molecular cloud and therefore have the same age (which is relatively easy to determine) and the same composition (e.g., metallicity $[\text{Fe}/\text{H}]$). Furthermore, cluster stars are located at the same distance from the Earth, which makes it possible to use the same color excess $E(B-V)$ and distance modulus $(V-M_V)$ for conversion of their photometry. Due to a relatively low density of stars in open clusters, photometry of individual stars can be performed with higher accuracy as compared with stars of globular clusters. All the above reasons represent a significant advantage of open-cluster stars and allow reducing calculation errors and obtaining uniform samples of stars with different masses and known ages.

In accordance with the modern concepts, the distribution of stars in clusters by their masses is similar to the distribution of stars in a galaxy. Therefore, by studying stars in the nearest clusters, we gain understanding of the stars of considerably larger (galactic) spatial scales.

Classification of stars is based on the so-called HR diagram. Usually, the HR diagram is plotted either in the coordinates of color index $(B-V)$ versus absolute stellar magnitude M_V or in the coordinates of effective temperature T_{eff} versus luminosity L . Stars tend to fall only into certain regions of the HR diagram. About 90% of the observed stars belong to the main sequence that represents a narrow strip running from hot stars with high luminosity to brown dwarfs with low luminosity. There are also star groups of giants, supergiants, and several others. Location of a star in the HR diagram is determined by the star's mass, age, and chemical composition. The bigger the mass of a star belonging

to the main sequence (MS), the higher its luminosity and temperature. Further, the more massive a star is, the faster it evolves and leaves the main sequence because nuclear fusion reactions in its core are more intensive.

3. Photometry-Based Method of Entropy-Production Calculation

This study uses photometric data of open star clusters from the WEBDA database (<http://webda.physics.muni.cz>) [17]. WEBDA contains photometric data for every cluster star obtained from direct observations: apparent stellar magnitude V and color index $(B-V)$, as well as metallicity $[\text{Fe}/\text{H}]$, color excess $E(B-V)$, and distance modulus $(m-M)_V$. For calculations, initial photometric data are converted into absolute stellar magnitude M_V and normal color index $(B-V)_0$ using the following formulae [18,19]:

$$M_V = V - (m - M)_V - A_V, \quad (1)$$

$$(B-V)_0 = (B-V) - E(B-V), \quad (2)$$

where $A_V = R_V \times E(B-V)$ is the total extinction at the V filter band, R_V is the constant magnitude for the given photometric band $(B-V)$ [20].

The effective temperature T_{eff} was determined using the values of $(B-V)_0$ through semi-empirical methods proposed in the papers [21–24]. Depending on the metallicity and class of a star, which was established based on [25,26], we selected the most up-to-date semi-empirical methods of temperature calculation. Specifically, it was found that: (1) for approximately 60% of all the stars that we have studied, the calibration [21] (that takes into consideration $[\text{Fe}/\text{H}]$ and is applicable to main-sequence stars and subgiants with considerable limitations to the range $(B-V)_0$ and $[\text{Fe}/\text{H}]$) is suitable; (2) for approximately 10% of the stars, the calibration [22] (that also takes in consideration $[\text{Fe}/\text{H}]$ and is applicable to main-sequence stars and giants) is suitable; and (3) for approximately 30%, the calibration [23,24] (that fails to take into consideration $[\text{Fe}/\text{H}]$ but, on the other hand, has no considerable limitations to the range $(B-V)_0$ and applies to almost all the classes of stars (main sequence, subgiants, giants, and supergiants)) is suitable.

To calculate the luminosity L , Pogson's ratio was used [18]:

$$\log_{10} L/L_{\odot} = 0.4 \times (M_{\odot} - M_{\text{bol}}), \quad (3)$$

where $L_{\odot}$, $M_{\odot}$ are the luminosity and the absolute bolometric stellar magnitude for the Sun [27]; $M_{\text{bol}} = M_V + BC$; BC is the bolometric correction that was calculated on the basis of T_{eff} using a semi-empirical method proposed in [23,24].

The entropy production Σ in a star (in the volume inside its photosphere) is determined by nuclear fusions, convection, interaction between radiation and matter, and the like [15]. Let us adopt the most common assumption that a star's photosphere is, on the first approximation, a black body [18,19]. In this case, the radiation thermodynamics is well studied and the simplest. As is known, a star can be considered, for the most part of its life, as a system in a non-equilibrium steady state [15]. Consequently, the total entropy production Σ inside the photosphere of a star is equal to an entropy flux from the photosphere's surface [15], *i.e.*,

$$\Sigma = 4L / (3T_{\text{eff}}). \quad (4)$$

In addition to the total entropy production of a star, which is its integral characteristic, let us introduce a local characteristic [28]—specific (per unit volume) entropy production Σ_V for a star with the radius R —as:

$$\Sigma_V = L / (\pi R^3 T_{\text{eff}}), \quad (5)$$

or, using the Stefan–Boltzmann law, Equation (5) can be rearranged in the form:

$$\Sigma_V = \chi \times T_{\text{eff}}^5 / \sqrt{L}, \quad (6)$$

where $\chi = 8 \times \sigma^{3/2} \times \pi^{1/2}$ and σ is the Stefan–Boltzmann constant.

As is seen from the above, the quantities Σ and Σ_V can be found from direct photometric measurements. For this reason, only their values are calculated and analyzed below. Mass-specific entropy production Σ_M , another known quantity of non-equilibrium thermodynamics, is not considered herein. This is connected with the fact that in order to determine a star's mass, some theoretical stellar model is required, and therefore, additional assumptions are necessary. As a result, the calculation of Σ_M turns out to be combined rather than based on experimental data only. This complicates the analysis of Σ_M calculation error.

4. Data for Calculations and Their Accuracy

To select clusters from the WEBDA database for investigation, the following criteria were applied: (1) availability of all the photometric information necessary to perform calculations using Equations (1)–(6); (2) availability of information on the probability that stars belong to a cluster and on binary stars; (3) clusters must contain a sufficiently large number of stars and be at approximately the same distance from the Sun; (4) the HR diagram of clusters must be well approximated by theoretical isochrones plotted according to the model (the Padova database of evolutionary tracks and isochrones web-site: <http://pleiadi.oapd.inaf.it/>) [29,30] using the values of age and metallicity from the WEBDA database.

As a result, a sample of 11 clusters was obtained (NGC 884, NGC 869, IC 4725, NGC 2516, NGC 1039, NGC 3532, NGC 2099, NGC 2281, NGC 2506, NGC 2682, and NGC 188) containing only single stars (from 56 to 900 in each cluster) with the membership probability above 0.5. The age of the clusters varied from 12.6 Myr to 6.3 Gyr.

Based on the accuracy of the given photometric data and the calibration errors [21–24], we have estimated the error of entropy production calculation. The relative error due to T_{eff} calibration does not exceed 3% for Σ and 12% for Σ_V . The relative error of entropy production due to the accuracy of photometric data considerably depends on the temperature and luminosity of stars. So, for the MS stars with the temperature of up to 5000 K, the error for Σ and Σ_V does not exceed 19% and 15%, respectively; for the MS stars with the temperature of 5000 K to 10,000 K, the error does not exceed 17% and 32%, respectively; and for the MS stars with the temperature above 10,000 K, the error value is found to be higher than 23% and 40%, respectively.

To facilitate the calculation of the temperature, luminosity, and entropy production (and the statistic characteristics thereof) of cluster stars based on the above algorithm, we have developed and used a special software complex, Star Clusters (SC; <http://www.fisica.hol.es>).

5. Results and Discussion

For ease of analysis, the data below was normalized to solar magnitudes that we had calculated on the basis of Equations (1)–(6) and the reference information contained in [31,32]. Their values are as follows: $T_{\text{eff } \odot} = (57 \pm 2) \times 10^2 \text{ K}$, $L_{\odot} = (3.8 \pm 0.6) \times 10^{26} \text{ W}$, $\Sigma_{\odot} = (9 \pm 1) \times 10^{22} \text{ W} \cdot \text{K}^{-1}$, and $\Sigma_{V \odot} = (6 \pm 2) \times 10^{-5} \text{ W} \cdot \text{K}^{-1} \cdot \text{m}^{-3}$. These results conform well with the data given in [14,15,27].

Calculation results of Σ and Σ_V for the MS stars are shown in Figures 1–3. Using these results, the following conclusions can be made:

- (1) For the studied stars, the range of entropy-production variation is considerably wider than the corresponding range for specific entropy production (Figure 1). Thus, for the first case, 90% of all the stars at hand fall within the range of 0.3 to 400 solar units (median is 6.3); and for the second case, the same percentage of stars is within the range of 0.5 to 1.8 (median is 0.9).
- (2) As is seen (Figure 2), the total entropy production of the stars grows strongly with the increase of luminosity, while the specific entropy production of the stars depends on luminosity to a very small extent. Indeed, with the increase of L in more than 10^4 times (0.1 to $10^3 L_{\odot}$), the growth of Σ_V is only 0.4 to 1.
- (3) Both Σ and Σ_V grow with the increase of temperature roughly in accordance with the exponential law but they demonstrate distinctively different growth rates. $\Sigma(T_{\text{eff}})$ grows approximately 10^3 times faster than $\Sigma_V(T_{\text{eff}})$ (Figure 3).

The above distinctions in the behavior of specific and total entropy productions can be explained by a considerable difference in the volumes of MS stars having various luminosities and temperatures.

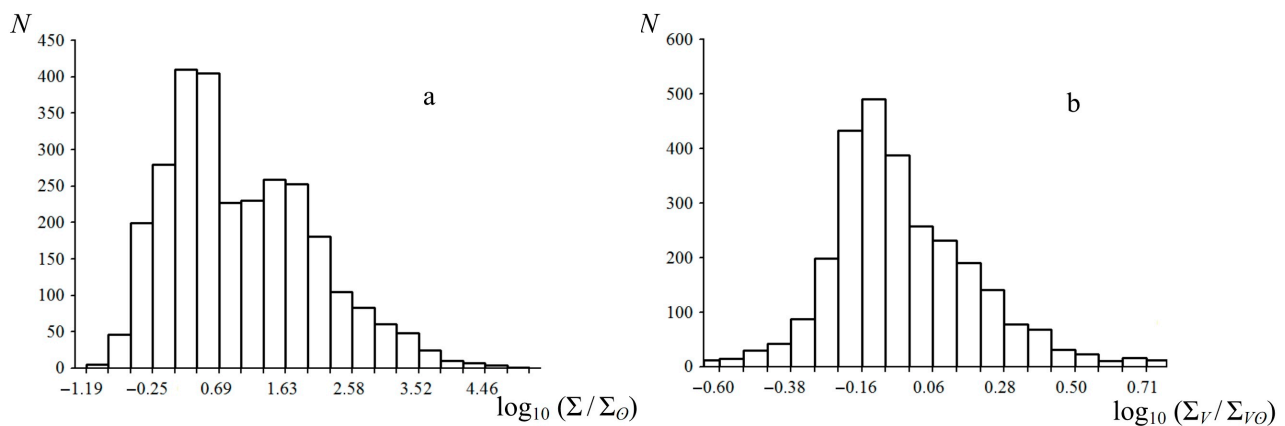


Figure 1. Bar charts for distribution of total Σ (a) and specific Σ_V (b) entropy production of the main-sequence stars (all the studied stars of 11 clusters are shown). N is the number of stars falling into the range. For clarity, Σ and Σ_V are normalized to solar magnitudes and presented in logarithmic form.

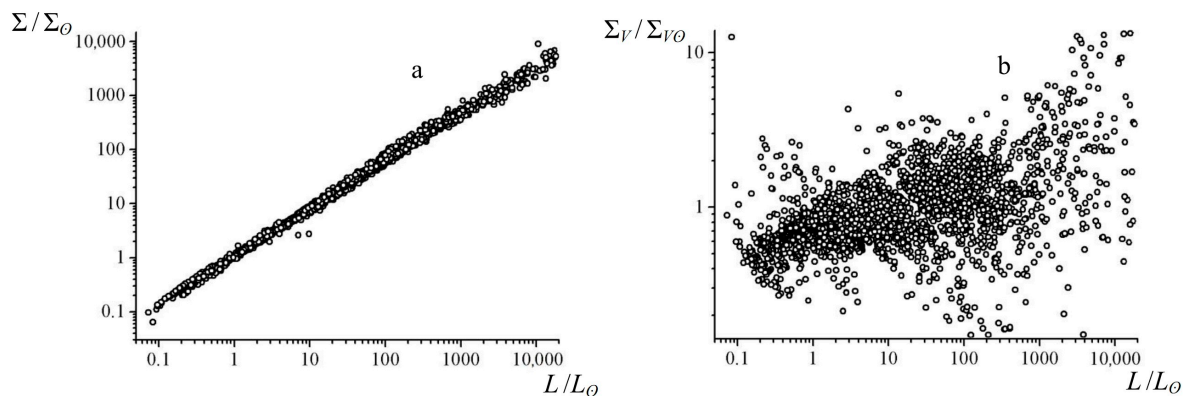


Figure 2. Dependence of total Σ (a) and specific Σ_V (b) entropy production of the main-sequence stars on their luminosity (L). All the quantities are given in solar units. The graphs show the data for all the studied stars of 11 clusters.

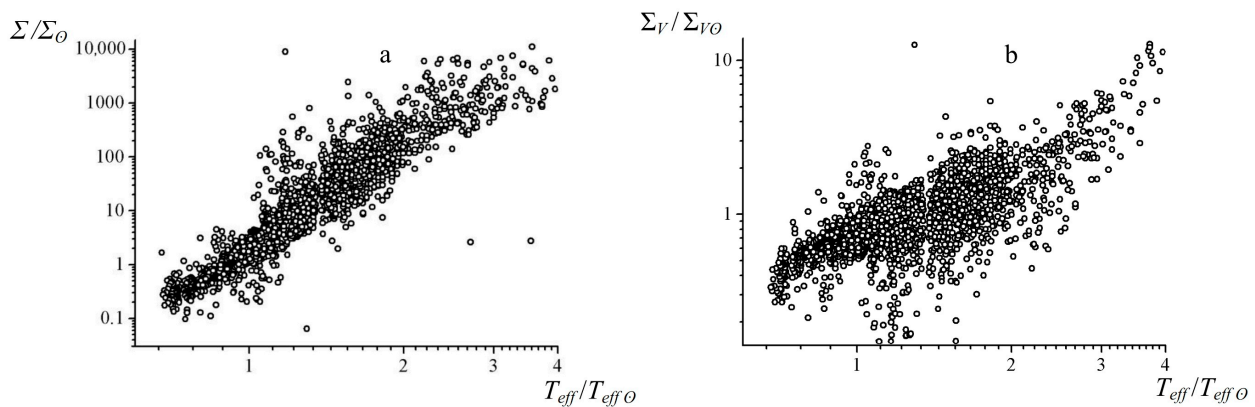


Figure 3. Dependence of total Σ (a) and specific Σ_V (b) entropy production of the main-sequence stars on their effective temperature (T_{eff}). All the quantities are given in solar units. The graphs show the data for all the studied stars of 11 clusters.

Data related to the entropy production of stars with different masses is given above. As is seen, Σ_V changes insignificantly for the stars belonging to MS. To study this quantity in more detail, let us consider distribution of Σ_V for stars belonging to the same cluster. Examples of frequency bar charts of Σ_V as well as the dependence of Σ_V on age are shown in Figures 4 and 5. It is seen that, independent of the age of an open star cluster, the distribution of specific entropy production remains almost the same in terms of both position and type. The majority of stars belonging to the cluster have specific entropy productions very similar to the solar one; and this value is almost independent of the ages which differ by three orders of magnitude. These results are no surprise. Indeed, as is well known, the overwhelming majority of stars in a cluster are MS stars. The fraction of high-temperature stars in a cluster decreases with age (by evolving, these stars move to the right of the main sequence. So, the older the cluster is, the less high-luminosity stars belong to the main sequence). However, the number of these stars relative to the total number of MS stars is rather small; and additionally, the values of Σ_V of the stars leaving the main-sequence show little difference from Σ_V of those remaining (see Figures 1–3). All this makes the result illustrated in Figures 4 and 5 predictable. Furthermore,

there is one interesting result here: according to the calculations made, the range of Σ_V , where the majority (90%) of cluster stars is distributed, shrinks with age (approximately in 35 times) (Figure 4). Such a shrinkage occurs for both high and low specific entropy productions. Whereas the former can be explained by the decrease of the fraction of high-temperature stars, the reasons of the latter are not completely known and require further investigation.

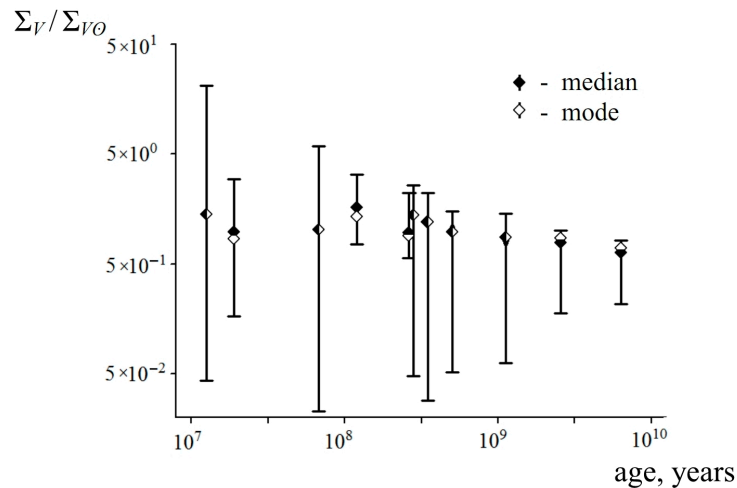


Figure 4. Specific entropy production Σ_V of the eleven star clusters under consideration: NGC 884 (7.10), NGC 869 (7.28), IC 4725 (7.83), NGC 2516 (8.08), NGC 1039 (8.42), NGC 3532(8.45), NGC 2099 (8.54), NGC 2281 (8.70), NGC 2506 (9.05), NGC 2682 (9.41), NGC 188 (9.80). The cluster age is given in parentheses and presented in logarithmic form: $\log_{10}(t, \text{year})$. The lines form the areas including Σ_V for 90% of all the studied cluster stars (they contain Σ_V for 45% of the cluster stars bigger and smaller than the median).

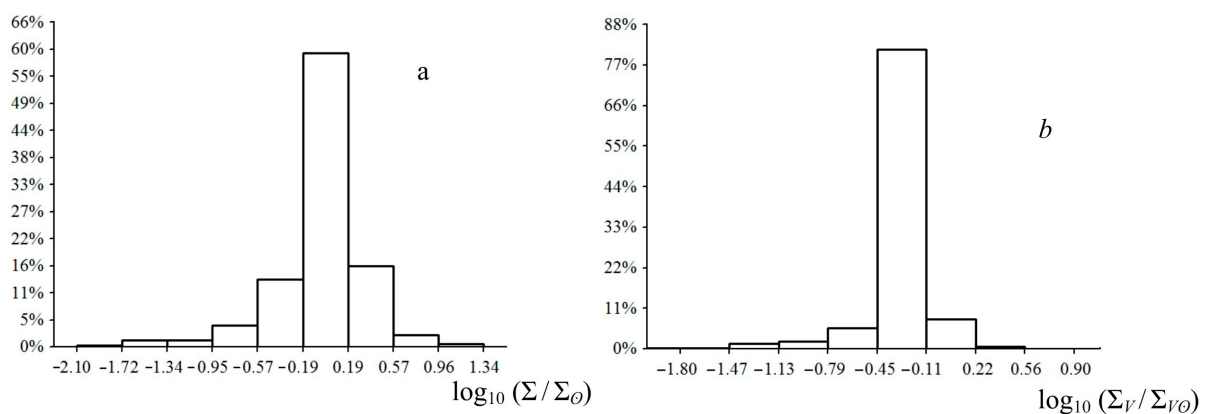


Figure 5. Bar charts of Σ_V distribution for the cluster stars: NGC 869 (age = 7.28, $N = 331$) (a); NGC 188 (age = 9.8, $N = 817$) (b). The age is presented in logarithmic form: $\log_{10}(t, \text{year})$. N is the total number of stars calculated in the cluster.

The empirically-found weak dependence of Σ_V on temperature (luminosity, mass) of MS stars does not qualitatively contradict the known stellar models. Thus, according to Refs. [18,33]:

$$\Sigma_V \propto \rho_c^2 \times T_c^\nu, \quad (7)$$

where ρ_c and T_c are the density and the temperature in the centre of a star, respectively; and ν is a number from 4 to 8, in the case of a nuclear fusion according to the PP chain, and from 15 to 16, in the case of a nuclear fusion according to the CNO cycle. It is known that the PP chain prevails for the MS stars whose masses do not greatly exceed that of the Sun, whereas CNO-cycle fusions are typical for massive MS stars, giants, and supergiants [18,33]. The central density of stars decreases and their central temperature grows with the increase of mass. Moreover, as is well known [34], the growth of temperature is considerably slower than the reduction of density. Due to such an opposite behavior, specific entropy production insignificantly changes with the change of mass. Our more detailed numerical calculations following the theoretical models of evolutionary tracks and isochrones [29,30] confirm the above estimation based on Equation (7).

6. Conclusions

The found relatively narrow range of specific (per volume) entropy productions of 0.5 to 1.8 solar magnitudes (median equals 0.9) for stars belonging to the main sequence is the most important result of this study. It is surprising that this quantity insignificantly differs for MS stars when their luminosities change in $\approx 10^5$ times, their temperatures change in ≈ 4 times, and their volumes change in $\approx 2 \times 10^3$ times. From the perspective of stellar astrophysics, the found invariance can be useful for formulating stellar models (particularly, as an additional constraint on the behavior of temperatures and densities in the reaction zone) as well as for testing numerical calculations of stars. From the perspective of non-equilibrium thermodynamics, the obtained result is important as it confirms, using a specific example, a hypothesis advanced in a number of recent papers (see, for example, [3,5]). So, these papers propose, on the basis of the maximum entropy production principle, a hypothesis that co-existing dissipative non-equilibrium systems have the same local (specific) entropy productions. This hypothesis was previously verified by the results of experiments related to non-equilibrium growth of crystals and hydrodynamic instabilities. The present manuscript confirms this hypothesis on a space scale using the data of stars. Indeed, it proves to be that the MS stars co-existing in a cluster have almost constant specific entropy productions.

We see three main areas to continue this study: (1) Determination of a functional relation between specific entropy production and temperature (luminosity) based on a rigorous statistical sample of stellar parameters; (2) Calculation of specific entropy production for the stars different from main-sequence stars (primarily, supergiants, giants, and white dwarfs); (3) Analysis of the result obtained herein from the standpoint of non-equilibrium thermodynamics.

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study described herein. This research used the WEBDA database established at the Department of Theoretical Physics and Astrophysics of Masaryk University.

Author Contributions

Leonid M. Martyushev proposed the idea of research and the method of entropy-production calculation demonstrated in the paper. Sergey N. Zubarev calculated the data. Both authors analyzed the data, prepared the manuscript, and read and approved its final version.

Conflicts of Interest

The authors declare no conflict of interest.

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