



Proceeding Paper

Accuracy of an EGB Exponential Inflationary Scenario †

Ekaterina Pozdeeva

Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Leninskie Gory, GSP-1, 119991 Moscow, Russia; ekatpozdeeva@gmail.com

† Presented at the 1st Electronic Conference on Universe, 22–28 February 2021; Available online: <https://ecu2021.sciforum.net/>.

Abstract: Earlier we constructed a model with exponential form potential and of Gauss–Bonnet interaction. This model can be considered as an appropriate inflationary scenario. In this model, the attractor inflationary parameters correspond to ones from the cosmological attractor model in leading order approximation in an inverse e-folding number. We study how many orders of inverse e-folding numbers are included in the spectral index in exponential inflationary scenario in the Einstein–Gauss–Bonnet gravity.

Keywords: Gauss-Bonnet term; inflation; cosmological attractors

1. Introduction

Earlier we constructed inflationary scenarios of exponential type [1] in Einstein–Gauss–Bonnet gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} - \frac{\partial^\nu \phi \partial_\nu \phi}{2} - V(\phi) - \frac{\xi(\phi)}{2} \mathcal{G} \right], \quad (1)$$

where $\mathcal{G} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2$.

We reformulated equations of motion in spatially flat FLRW space–time in slow-roll regime

$$H^2 \simeq \frac{V}{3}, \quad (\phi')^2 \simeq \frac{V'}{V} + \frac{4}{3} \xi' V = \frac{(H^2)'}{H^2} + 4H^2 \xi'. \quad (2)$$

and inflationary parameters

$$n_s = 1 - 2\epsilon_1 + \frac{r'}{r}, \quad r = 8(\phi')^2, \quad \epsilon_1 = \frac{1}{2} \frac{(H^2)'}{H^2}. \quad (3)$$

in terms of e-folding numbers. According to the inflationary parameters of cosmological-attractor models [2] without the Gauss–Bonnet term, the spectral index includes only the logarithmic derivative of the tensor-to-scalar ratio

$$r \approx \frac{12C_\alpha}{(N + N_0)^2}, \quad \frac{r'}{r} = -\frac{2}{N + N_0}, \quad \text{and} \quad n_s \approx 1 + \frac{r'}{r} \approx 1 - \frac{2}{N + N_0}. \quad (4)$$

in the leading order of $1/N$ approximation.

2. Exponential Model

In [1] the exponential inflationary scenario with the Gauss–Bonnet which allows reproduction of cosmological attractor prediction in leading order approximation was obtained. The model obtained within the framework of slow-roll approximation has the following form:



Citation: Pozdeeva, E. Accuracy of an EGB Exponential Inflationary Scenario. *Phys. Sci. Forum* **2021**, *2*, 20. <https://doi.org/10.3390/ECU2021-09294>

Academic Editor: David L. Wiltshire

Published: 22 February 2021

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

$$\xi = \xi_0 \exp\left(\frac{3C_\beta}{2(N + N_0)}\right), \quad H^2 = H_0^2 \exp\left(-\frac{3C_\beta}{2(N + N_0)}\right), \quad V = 3H^2, \quad (5)$$

where C_β is a constant

$$C_\beta = \frac{C_\alpha}{1 - 4\xi_0 H_0^2}, \quad H_0^2 \neq \frac{1}{4\xi_0}. \quad (6)$$

According to (3) and (4), the derivative of field is related to e-folding number:

$$(\phi')^2 = \frac{3C_\alpha}{2(N + N_0)^2}, \quad \phi' = \frac{\omega_\phi \sqrt{\frac{3C_\alpha}{2}}}{N + N_0}, \quad \omega_\phi = \pm 1 \quad (7)$$

from here

$$\phi = \omega_\phi \sqrt{\frac{3C_\alpha}{2}} \ln\left(\frac{N + N_0}{N_\phi}\right), \quad N + N_0 = N_\phi \exp\left(\omega_\phi \sqrt{\frac{2}{3C_\alpha}} \phi\right). \quad (8)$$

Using (2), (5) and (8), we constructed the family of the models with the Gauss–Bonnet interaction and potential with variable parameter C_α :

$$V = 3H_0^2 \exp\left(-\frac{3}{2} \frac{C_\beta}{N_\phi} \exp\left(-\omega_\phi \sqrt{\frac{2}{3C_\alpha}} \phi\right)\right), \quad \xi = \xi_0 \exp\left(\frac{3}{2} \frac{C_\beta}{N_\phi} \exp\left(-\omega_\phi \sqrt{\frac{2}{3C_\alpha}} \phi\right)\right) \quad (9)$$

leading to appropriate inflationary scenarios.

Now, we would like to compare the order of inflationary parameters of the obtained model (9) using field formulation of inflationary parameters.

3. Inflationary Parameters

In this subsection, we obtain expressions for inflationary parameters in terms of fields and, after that, we reformulate results in terms of the e-folding number. We use the tensor-to-scalar ratio and spectral index of scalar perturbations in the following form [1,3]:

$$r = 8Q^2, \quad n_s = 1 - Q \frac{V_\phi}{V} + 2Q_{,\phi}, \quad \text{where } Q = V_{,\phi}/V + 4\xi_{,\phi} V/3. \quad (10)$$

For the exponential models we obtain

$$r = \frac{64(1 - 4H_0^2 \xi_0)^2}{3C_\alpha} \left(\exp\left(-\omega_\phi \sqrt{\frac{2}{3C_\alpha}} \phi\right)\right)^2 \quad (11)$$

$$n_s = 1 - \frac{8 \exp\left(-\omega_\phi \sqrt{\frac{2}{3C_\alpha}} \phi\right) \left(1 + \exp\left(-\omega_\phi \sqrt{\frac{2}{3C_\alpha}} \phi\right)\right) (1 - 4H_0^2 \xi_0)}{3C_\alpha} \quad (12)$$

Now we apply the relation between the field and e-folding number (8) and obtain:

$$r = \frac{64(1 - 4H_0^2 \xi_0)^2}{3C_\alpha} \left(\frac{N_\phi}{N + N_0}\right)^2 \quad (13)$$

$$n_s = 1 - \frac{8(1 - 4H_0^2 \xi_0)}{3C_\alpha} \left(\frac{N_\phi}{N + N_0} + \left(\frac{N_\phi}{N + N_0}\right)^2\right) \quad (14)$$

To present these expressions in a more regular form, we apply (6)

$$r = \frac{64N_\phi^2(1 - 4H_0^2\xi_0)}{3C_\beta(N + N_0)^2} = \frac{64}{3} \frac{C_\alpha N_\phi^2}{(N + N_0)^2} \quad (15)$$

$$n_s = 1 - \frac{8}{3C_\beta} \left(\frac{N_\phi}{N + N_0} + \left(\frac{N_\phi}{N + N_0} \right)^2 \right) \quad (16)$$

substituting $N_\phi = \frac{3C_\beta}{4}$, $C_\beta = 1$ we obtain

$$r = \frac{64N_\phi^2(1 - 4H_0^2\xi_0)}{3C_\beta(N + N_0)^2} = \frac{12C_\alpha}{(N + N_0)^2} \quad (17)$$

$$n_s = 1 - \left(\frac{2}{N + N_0} + \frac{9}{16(N + N_0)^2} \right) \quad (18)$$

It is evident that, at the beginning of inflation, $N + N_0 \approx 60$, so we can roughly suppose

$$r \approx \frac{12C_\alpha}{(N + N_0)^2}, \quad n_s \approx 1 - \frac{2}{N + N_0} \quad (19)$$

The obtained approximations coincide with inflationary parameters of the attractor approximation [2] and, in the case of $C_\alpha \approx 1$, with parameters of R^2 inflation [4]. The case $C_\alpha = 1$ leads to switching of Gauss–Bonnet interaction and coinciding with the exponential scenario earlier obtained in Einstein Gravity [5].

4. Conclusions

We considered the exponential inflationary scenario in Einstein–Gauss–Bonnet gravity and found that the direct calculations of spectral index for this model include a second order inverse e-folding number term. However, such as in the beginning of inflation $N + N_0 \approx 60$, this term is negligible in relation to the first order inverse e-folding number term. Roughly we can suppose that the cosmological attractor approximation for inflationary parameters is satisfied for the considered model. Moreover the switch of Gauss–Bonnet interaction can lead to R^2 gravity prediction for inflationary parameters in leading order approximation in the inverse e-folding number.

Funding: This work was partially supported by the Russian Foundation for Basic Research grant No. 20-02-00411.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

References

1. Pozdeeva, E.O. Generalization of cosmological attractor approach to Einstein–Gauss–Bonnet gravity. *Eur. Phys. J. C* **2020**, *80*, 612. [[CrossRef](#)]
2. Galante, M.; Kallosh, R.; Linde, A.; Roest, D. Unity of Cosmological Inflation Attractors. *Phys. Rev. Lett.* **2015**, *114*, 141302. [[CrossRef](#)] [[PubMed](#)]
3. Guo, Z.; Schwarz, D.J. Slow-roll inflation with a Gauss-Bonnet correction. *Phys. Rev. D* **2010**, *81*, 123520. [[CrossRef](#)]
4. Starobinsky, A.A. Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations. *Phys. Lett. B* **1982**, *117*, 175. [[CrossRef](#)]
5. Mukhanov, V. Quantum Cosmological Perturbations: Predictions and Observations. *Eur. Phys. J. C* **2013**, *72*, 2486. [[CrossRef](#)]