physical sciences forum

Proceeding Paper

An Overview of Black Hole Chemistry †

Jamil Ahmed

School of Applied Sciences and Humanities, National University of Technology, Islamabad 44000, Pakistan; jamilahmed@nutech.edu.pk

† Presented at the 1st Electronic Conference on Universe, 22–28 February 2021; Available online: https://ecu2021.sciforum.net/.

Abstract: The understanding the thermodynamic behavior of black holes using the concepts of chemistry will be discussed here. To establish the complete correspondence between the thermodynamics of an ordinary system and the thermodynamics of black holes, recent proposals suggest the identification of the mass of a black hole as the chemical enthalpy of an ordinary thermodynamic system. Similarly, the negative cosmological constant, surface gravity, and horizon area of a black hole is identified as the pressure, temperature, and entropy of a thermodynamic system. Consequently, black holes behave analogously to a variety of everyday phenomena. This allows an understanding of black holes using concepts of chemistry such as Van der Waals fluids, phase transitions, etc.

Keywords: thermodynamic pressure; chemical enthalpy; Van der Waals fluids; phase transition

check for updates

Citation: Ahmed, J. An Overview of Black Hole Chemistry. *Phys. Sci. Forum* **2021**, *2*, 11. https://doi.org/10.3390/ECU2021-09308

Academic Editor: Gonzalo Olmo

Published: 22 February 2021

1. Introduction

Black holes are one of the most fascinating predictions of general relativity. According to Hawking [1], black holes can emit radiations similar to almost any black body using a semiclassical approach. Furthermore, Hawking and Page [2] studied the thermodynamical aspects of a black hole with an anti-de Sitter background that can undergo a phase transition between pure radiation and a stable black hole, known as a Hawking–Page phase transition. This prediction makes black holes of significant theoretical interest. Another interesting perspective [3] in this regard is the treatment of negative cosmological constant Λ of an anti-de Sitter background as a thermodynamic variable related to pressure. According to this perspective, black holes behave analogously to a variety of everyday phenomena. Here, an overview of this proposal will be discussed.

Thermodynamic quantities of a physical system, such as energy E, temperature T, and entropy S have well-known counterparts in black hole thermodynamics. The energy E of a physical system is related to the mass M of a black hole. Similarly, the temperature T is related to the surface gravity κ and the entropy S is related to the horizon area A of a black hole. Black hole thermodynamics is very important because it helps us understand the underlying structure of quantum gravity. Even though the above correspondence exists between the thermodynamics of a physical system and that of black holes, it is not evident if one compares the forms of the first law of thermodynamics for both. The form of the first law of thermodynamics for a physical system is as follows:

$$dE = TdS - PdV + \text{work terms}. \tag{1}$$

However, the form of the first law of thermodynamics for black holes is

$$dM = \frac{\kappa}{8\pi} dA + \omega dJ + \phi dQ, \tag{2}$$

where ωdJ and ϕdQ are the working terms. Please note that in the above relationship $G = c = \hbar = \kappa_\beta = 1$. It is now evident from the above two relationships that the pressure–volume term which is present in the first law of thermodynamics of a physical system has no counterpart in the form of the first law of black hole thermodynamics. Recently, to address this correspondence, new developments have taken place. These findings lead to a picture in which the mass M of a black hole is interpreted as the enthalpy of spacetime. This idea originates from the consideration of the Smarr relation [4,5]. It was originally presented in [6] for $d = 4$ dimensions.

2. Methodology

To elaborate the correspondence, consider the example of the following d-dimensional black holes

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{d-2}, \tag{3}$$

where $f(r) = 1 - \frac{1}{d-2}\frac{16\pi}{\lambda_{d-2}}\frac{M}{r^{d-3}} + \ldots$, and $d\Omega^2_d$ is the line element of the unit sphere S^d with a volume $\lambda_d = 2\pi^{\frac{d-1}{2}}/\Gamma(\frac{d+1}{2})$. Smarr's relationship for such black holes have the following form

$$(d-3)M = (d-2)TS. \tag{4}$$

If one includes the cosmological constant in the metric, then it will modify the Smarr relationship [5]. Euler's theorem for $M = M(A, \Lambda)$ implies

$$(d-3)M = (d-2)\frac{\partial M}{\partial A}A - 2\frac{\partial M}{\partial \Lambda}\Lambda, \tag{5}$$

Since $T = 4\frac{\partial M}{\partial A}$, the above equation suggests that one should regard $P = -\frac{\Lambda}{8\pi} = \frac{(d-1)(d-2)}{16\pi l^2}$ as a thermodynamic variable [7,8]. The conjugate of thermodynamic pressure is the thermodynamic volume given by $V = -8\pi\frac{\partial M}{\partial \Lambda}$. Due to this consideration, one can have the modified Smarr relationship given by

$$(d-3)M = (d-2)TS - 2PV. \tag{6}$$

Consequently, the first law of thermodynamics is also modified, and the modified first law of thermodynamics has the form

$$dM = TdS + VdP. \tag{7}$$

This consideration leads to complete thermodynamic correspondence. To include the case of charged and rotating black holes, one requires a more general form of the Smarr relationship, given by

$$\frac{d-3}{d-2}M = TS + \sum_i (K^i - K^i)J^i - \frac{2}{d-2}PV + \frac{d-3}{d-2}\Phi Q. \tag{8}$$

3. Charged Black Holes

As vacuum pressure is induced by a negative cosmological constant, from this perspective it is considered to be a thermodynamic pressure term. However, if one considers the gravitational version of enthalpy, this will correspond to the mass of a black hole, which means that it is the total energy required to create a black hole and place it in a cosmological environment. This change of perspective opens a new direction for understanding the behavior of black holes. This allows consideration of the black hole's behavior to be analogous to a variety of everyday phenomena. One such realization is that the charged black holes behave similar to Van der Waals fluids [3]. To approximate the behavior of real fluids, one needs to modify the equation of state for an ideal gas [9]

$$(P + \frac{a}{v^2})(v - b) = T, \tag{9}$$

it gives

$$P = \frac{T}{v-b} - \frac{a}{v^2}. \tag{10}$$

where a, b are constants and v represents the specific volume for the fluid under consideration. The locations of critical points can be found at $T = T_c$ where $P = P(v)$ will have a point of inflection at $P = P_c$ and $v = v_c$ and satisfy the universal relation $\frac{P_c v_c}{\kappa T_c} = \frac{3}{8}$ for the fluid under consideration. For the temperature $T < T_c$, a liquid–gas phase transition will occur.

To see such behavior for a black hole, one can consider the simplest example of a Reissner Nordström–AdS black hole, whose metrics have the form

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + sin^2\phi d\phi^2). \tag{11}$$

where $f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2}$. One can have the following relationships for its thermodynamics: $V = \frac{4}{3}\pi r_+^3$, where r_+ gives the event horizon of the black hole. The equation of state will have the form

$$P = \frac{T}{v} - \frac{1}{2\pi v^2} + \frac{2Q^2}{\pi v^4}, v = 2(\frac{3V}{4\pi})^{\frac{1}{3}}. \tag{12}$$

In the above relationship, the specific volume $v = 6V/N$ with $N = A/l_p^2, l_p = \sqrt{G\hbar/c^3}$. For any specific charge Q, the $P - V$ relationship and the $P - T$ relationship gives the behavior of a Van der Waals fluid, and a liquid–gas phase transition is replaced by a small/large black hole phase transition. Remarkably, the critical points satisfy the relationship $\frac{P_c v_c}{T_c} = \frac{3}{8}$, as is the case for Van der Waals fluids [10–13].

4. Discussion

An overview of black hole chemistry is presented in this article. Initially, black holes were only considered to be black, but after the work of Hawking it was realized that these are physical objects, and therefore their thermodynamic properties were explored. The perspective reviewed here is one step further in this direction. It allows a discussion of black hole behavior analogous to many everyday chemical phenomena. It is a new window through which to look at black holes, and one can expect variety of exciting new results about these ever-mysterious objects in our universe.

References

1. Hawking, S.W. Particle creation by black holes. *Commun. Math. Phys.* **1975**, *43*, 199–220. [CrossRef]
2. Hawking, S.W. The unpredictability of quantum gravity. *Commun. Math. Phys.* **1982**, *87*, 395–415. [CrossRef]
3. Kubiznak, D.; Mann, R.B. P-V criticality of charged AdS black holes. *JHEP* **2012**, *1207*, 033. [CrossRef]
4. Caldarelli, M.M.; Cognola, G.; Klemm, D. Thermodynamics of Kerr-Newman-AdS black holes and conformal field theories. *Class. Quantum Gravity* **2000**, *17*, 399. [CrossRef]
5. Kastor, D.; Ray, S.; Traschen, J. Enthalpy and the mechanics of AdS black holes. *Class. Quantum Gravity* **2009**, *26*, 195011. [CrossRef]
6. Smarr, L. Mass formula for Kerr black holes. *Phys. Rev. Lett.* **1973**, *30*, 71. [CrossRef]
7. Creighton, J.D.; Mann, R.B. Quasilocal thermodynamics of dilaton gravity coupled to gauge fields. *Phys. Rev. D* **1995**, *52*, 4569. [CrossRef] [PubMed]
8. Padmanabhan, T. Classical and quantum thermodynamics of horizons in spherically symmetric spacetimes. *Class. Quantum Gravity* **2002**, *19*, 5387. [CrossRef]
9. Lovelock, D. The Einstein tensor and its generalizations. *J. Math. Phys.* **1971**, *12*, 498–501. [CrossRef]
10. Gunasekaran, S.; Mann, R.B.; Kubiznak, D. Extended phase space thermodynamics for charged and rotating black holes and Born-Infeld vacuum polarization. *JHEP* **2012**, *1211*, 110. [CrossRef]
11. Dolan, B.P.; Kostouki, A.; Kubiznak, D.; Mann, R.B. Isolated critical point from Lovelock gravity. *Class. Quantum Gravity* **2014**, *31*, 242001. [CrossRef]

12. Altamirano, N.; Kubiznak, D.; Mann, R.B.; Sherkatghanad, Z. Kerr-AdS analogue of triple point and solid/liquid/gas phase transition. *Class. Quantum Gravity* **2014**, *31*, 042001. [CrossRef]
13. Kubiznak, D.; Mann, R.B.; Teo, M. Black hole chemistry: Thermodynamics with Lambda. *Class. Quantum Gravity* **2017**, *34*, 063001. [CrossRef]