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Multiperson Decision-Making Using Consistent Interval-Valued Fuzzy Information with Application in Supplier Selection

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Abstract: This study describes a consistency-based approach for multiperson decision-making (MPDM) in which decision-makers' suggestions are expressed as incomplete interval-valued fuzzy preference relations. The presented approach utilizes Lukasiewicz's t-norm in conjunction with additive reciprocity to obtain comprehensive interval valued fuzzy preference relations from each expert, and the transitive closure formula also produces L-consistency. We would evaluate the consistency weights of the experts using consistency analysis. Experts are allocated final priority weights by combining the consistency weights and preset weights. A collective consistency matrix is then constructed from the weighted sum of preference matrices. After computing the possibility degrees, the normalization procedure is utilized to generate complimentary matrices, and the final ranking values of alternatives are derived as well. Finally, a numerical example demonstrates the efficacy of the suggested approach following a comparison analysis.

Keywords: multiperson decision-making; incomplete interval valued fuzzy preference relation; L-consistency; priority weights

MSC: 3B52; 15B15; 94D05

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1. Introduction

Decision-making (DM) problems are associated with all aspects of modern life, including the assessment of human resource quality, location of facilities and selection of infrastructure projects. MPDM is a situation in which a number of experts collaborate on some possible set of available alternatives for choosing the best option, although each expert could have specific inspirations or goals and a separate decision-making process. Several techniques to handle MPDM situations have been proposed recently [1,2]. Preference relation seems to be the most famous expression included in MPDM since it is a powerful resource to model decision procedures when we need to incorporate expert preferences into group preferences [3]. Primarily, two kinds of pairwise comparisons have been used to incorporate decision models; multiplicative preference relations (MPRs) [4] and fuzzy preference relations (FPRs) [5], the latter being more common for expressing the expert's preferences over alternatives.

In the case of FPR, the expert allocates a numerical value to each pair of alternatives within [0, 1] and indicates the preferable strength of one alternative over the other. By doing so, the first and logical question immediately arises: what criteria must be met in order to obtain consistent results in the final ranking? There are three important and graded stages of rationality in connection to preference relations:

- Indifference,

- Reciprocity,
- Transitivity.

The mathematical modeling of all such rationale assumptions depends greatly on the scales used to set the choice values [6].

In the literature, it is frequently described that the third level guarantees consistent information, and the property is known as a consistency property. The lack of consistency leads to unreliable judgments, and it is, therefore, important to analyze the conditions that remain a part of consistency. On the other hand, it is difficult to achieve fully consistent information in practice, particularly while evaluating choices for a large range of possibilities. Many structures for the transitivity of FPRs have been proposed in the literature, and precision can be seen explicitly. The notion of transitivity that has been found to be most appropriate for fuzzy ordering is L-transitivity, i.e., $r_{ik} \geq \max(r_{ij} + r_{jk} - 1, 0)$, and finds the weakest one [7]. In this paper, L-transitivity is used for preference ordering and ranking of alternatives by constructing fully consistent preference relationships. In addition, it is a really radical concept to model such types of transitivity, and there is very little published research on this context.

It is frequently difficult to evaluate an expert's preference using a precise, crisp scale in real-world scenarios since they may have a vague understanding of the preferred values of one alternative over another. An interval valued fuzzy preference relation (IVFPR) is one such strategy where decisions are made in the form of intervals. IVFPRs may flexibly indicate ambiguous preferences over alternatives because intervals can rationally cover and convey the ambiguity and uncertainty of human judgment. Recent studies on IVFPRs focus mainly on the nature of:

(i) The consistency analysis;
(ii) The evaluation of priority weights.

The consistency of the IVFPRs does not necessitate any contradiction in the judgments of experts. For certain cases, however, it might be difficult for experts to come forward with absolutely clear results on alternatives. However, priority weights resulting from inconsistent IVFPRs contribute to an untrustworthy judgment. The consistency of IVFPRs is, therefore, an important issue. In 2008, Xu and Chen [8] presented additive and multiplicative consistent IVFPRs as an extension of the additive and multiplicative consistent FPRs. Chen and Zhou [9] suggested an approach for MPDM with IVFPRs in 2011 and provided a quality test for IVFPRs. Liu et al. [10] developed a definition for consistent IVFPR in 2012 by assessing whether or not two FPRs generated by the IVFPR were consistent. After introducing the interval [0.5, 0.5] in 2014, Xu et al. [11] established the additive consistency of the IVFPR and looked into the relationship between the multiplicative consistency and the additive consistency of IVFPR. In 2015, Wang and Li [12] introduced some properties for multiplicative consistent IVFPR after investigating the multiplicative transitivity defined in [13]. In 2018, Wang et al. [14] investigated a MPDM method with IVFPRs based on geometric consistency. The max-consistency index and min-consistency index of an IVFPR are derived based on their geometric consistency. In 2021, Cheng et al. [15] introduced a new consistency definition of interval multiplicative preference relation with desirable properties, and the sufficient and necessary conditions of the new consistency definition were also provided. In 2023, Shu et al. [16] proposed a method for group decision-making with interval multiplicative preference relations based on geometric consistency.

Almost all of the preceding work is focused on IVFPRs for comprehensive data. However, in the case of MPDM, certain circumstances are inescapable in which an expert lacks appropriate information on the issue owing to time limits, a lack of experience, and a lack of skill within the problem area [17–22]. An incomplete preference structure will be constructed as a result of the expert's potential inability to provide his or her perspective on the particular aspects of the situation. There are a few MPDM studies that deal with incomplete interval valued fuzzy preference relations (IIVFPRs), although there has been research based on incomplete FPRs published in the literature [11,12].

This study introduces an innovative MPDM method for scenarios where group recommendations are given as IIVFPRs. The algorithm focuses on obtaining complete L-consistent interval valued fuzzy relations in the form of matrices. First, the proposed method estimates the missing preference values and constructs the consistent matrix for each IVFPR for each expert. Secondly, in order to calculate the average degree of preference of each alternative over all remaining alternatives, an interval normalizing approach is used to integrate all consistent IVFPRs into a collective consistent matrix. Thirdly, it calculates the possibility degree and constructs the complementary matrix. Finally, it determines the ranking values of each alternative.

2. Preliminaries

As is generally known, a fuzzy subset A of a set U is a mapping $A : U \to [0,1]$, and the value $A(u)$ for a certain u is typically connected with some expert's level of confidence. An accepted viewpoint holds that attributing an exact number to an expert's judgment is overly limiting and that assigning a range of values is more realistic. This means replacing the interval $[0,1]$ of fuzzy values by the set $L = \left\{[l^-, l^+] \in [0,1]^2 \text{ with } l^- \leq l^+\right\}$.

Definition 1 ([23,24])**.** *A fuzzy set A on universe of discourse X is known as an interval valued fuzzy set if it is described by a mapping $A : X \to L$, where $L = \left\{[l^-, l^+] \in [0,1]^2 \text{ with } l^- \leq l^+\right\}$.*

T-norms are employed as conjuctors in fuzzy logic. Therefore, we will define one of them and explain how its interval valued counterpart works.

Definition 2 ([25])**.** *A mapping $T : [0,1] \times [0,1] \to [0,1]$ having commutative, associative, and increasing nature with $T(1,a) = a$ for all $a \in [0,1]$ is known as triangular norm (t-norm). The concept of a t-norm on $[0,1]$ can be extended to subintervals of $[0,1]$.*

Definition 3 ([23])**.** *An extended t-norm, T_e, is an increasing, commutative, associative, and $L \times L \to L$ mapping that satisfies:*

$$T_e([1,1],[l^-,l^+]) = [l^-,l^+] \text{ for all } [l^-,l^+] \in L.$$

Let T be a triangular norm. The mapping T_e is defined as:

$$T_e([l^-,l^+],[m^-,m^+]) = [T(l^-,m^-),T(l^+,m^+)]$$

for $[l^-,l^+]$, $[m^-,m^+] \in L$, is an extended t-norm on $(L, \subseteq)$, where $\subseteq$ represents the crisp set inclusion. The extended interval t-norm corresponding to Lukasiewicz's t-norm can be computed by:

$$T_L([l^-,l^+],[m^-,m^+]) = [\max(l^- + m^- - 1, 0), \max(l^+ + m^+ - 1, 0)]. \quad (1)$$

Definition 4 ([3])**.** *A fuzzy preference relation R over a finite set X of alternatives, $X = \{x_1, x_2, x_3, \ldots, x_n\}$, is a fuzzy set on the product set $X \times X$, i.e., it is characterized by a membership function $\mu_R : X \times X \to [0,1]$.*

On s similar pattern, an interval valued preference relation (IVPR) $\overline{R}$ over a finite set X of alternatives, $X = \{x_1, x_2, \ldots, x_n\}$, is an interval valued fuzzy set on the product set $X \times X$, i.e., it is characterized by a membership function $\mu_{\overline{R}} : X \times X \to L$. Hence $\overline{R} = \left(\overline{r}_{ij}\right)_{n \times n}$ where $\overline{r}_{ij} = [r_{ij}^-, r_{ij}^+]$, $0 \leq r_{ij}^- \leq r_{ij}^+ \leq 1$, $\overline{r}_{ij} = [1,1] - \overline{r}_{ji}$ and $\overline{r}_{ii} = [0.5, 0.5]$ for all $i, j \in N$.

As is common in the literature on FPRs, an IVFPR on X can also be conveniently expressed by an $n \times n$ matrix $R = (\overline{r}_{ij} = [r_{ij}^-, r_{ij}^+])_{n \times n}$, where $\overline{r}_{ij}$ denotes the degree of preference of alternative x_i over the alternative x_j with $\overline{r}_{ij} \in L$, $\overline{r}_{ij} + \overline{r}_{ji} = \overline{1}$ (additive reciprocity) for $1 \leq i \leq n$ and $1 \leq j \leq n$. If $\overline{r}_{ij} = [0.5, 0.5]$, then there is no difference between the alternatives x_i and x_j.

Definition 5. *An IVFPR $\overline{R}$ is said to be L-consistent, if for $i \neq j \neq k \in \{1, 2, 3, \ldots, n\}$ it holds*:

$$\overline{r}_{ik} \geq T_L(\overline{r}_{ij}, \overline{r}_{jk}) \qquad (L\text{-}transitivity) \tag{2}$$

which implies that $r^-_{ik} \geq \max(r^-_{ij} + r^-_{jk} - 1, 0)$ and $r^+_{ik} \geq \max(r^+_{ij} + r^+_{jk} - 1, 0)$ hold at the same time.

Definition 6. *An IVFPR relation $\overline{R} = (\overline{r}_{ij})_{n \times n}$ is said to be incomplete if it contains at least one unknown preference value $\overline{r}_{ij}$ for which the expert has no idea about the degree of preference of alternative x_i over the alternative x_j.*

Next, we present the procedure to estimate the missing preference values in an IIVFPR and to construct the L-consistent complete fuzzy preference matrix.

3. Procedure to Receive Complete IVFPRs

For the determination of missing preferences in the IIVFPR $\overline{R} = (\overline{r}ij)n \times n$, the measures for the pairs of alternatives for known and unknown preference values are shown in the following sets:

$$Av = \{(i, j) \mid i \neq j \wedge i, j \in \{1, 2, 3, \ldots, n\}\} \tag{3}$$

$$Ev = \{(i, j) \in Av \mid \overline{r}_{ij} \text{ is known}\} \tag{4}$$

$$Mv = \{(i, j) \in Av \mid \overline{r}_{ij} \text{ is unknown}\} \tag{5}$$

where Av is associated with the overall set of pairs of alternatives with known and unknown preference values of one alternative over another; Ev and Mv are related to the sets of pairs of alternatives for which the corresponding preference values of the one alternative over the other are known and unknown, respectively. It is to be noted that the preference value of alternative x_i over x_j belongs to the family of closed subintervals of $[0, 1]$ (i.e., $\overline{r}_{ij} \in L([0, 1])$). Since $\overline{r}_{ij} = [1, 1] - \overline{r}_{ji}$, $\overline{r}_{ii} = [0.5, 0.5]$ for $1 \leq i \leq n$ and $1 \leq j \leq n$, therefore, based on L-transitivity $\overline{r}_{ik} \geq T_L(\overline{r}_{ij}, \overline{r}_{jk})$, the following set can be defined to determine the unknown preference value $\overline{r}_{ik}$ of alternative x_i over alternative x_k:

$$H^1_{ik} = \{(i, k), j \neq i, \mid (i, j) \in Ev, (j, k) \in Ev \text{ and } (i, k) \in Mv\}, \tag{6}$$

for $i = \{1, 2, 3, \ldots, n\}$, $j = \{1, 2, 3, \ldots, n\}$ and $k = \{1, 2, 3, \ldots, n\}$. Based on (6), we can determine the unknown preference value $\overline{r}_{ik}$ for x_i over x_k as follows:

$$\overline{r}_{ik} = \begin{cases} \max\limits_{j \in H^1_{ik}}(T_L(\overline{r}_{ij}, \overline{r}_{jk})), & \text{if } H^1_{ik} \neq \varnothing \\ [0.5, 0.5], & \text{otherwise} \end{cases} \tag{7}$$

By using the condition $\overline{r}_{ij} = [1, 1] - \overline{r}_{ji}$, the value of $\overline{r}_{ki}$ can be calculated as follows:

$$\overline{r}_{ki} = \left[1 - r^+_{ik}, 1 - r^-_{ik}\right] \tag{8}$$

Therefore, the new sets of pairs of alternatives for which preference values of one alternative over the other are known and unknown will be:

$$Ev' = Ev \cup \{(i, k), (k, i)\}, \tag{9}$$

$$Mv' = Mv / \{(i, k), (k, i)\}. \tag{10}$$

To get the L-consistency, compute the transitive closure of the IVFPR after having the complete form by using:

$$\bar{r}^*_{ik} = \max_{j \neq i,k}\left(\bar{r}_{ik}, T_L\left(\bar{r}_{ij}, \bar{r}_{jk}\right)\right) \text{ such that } \bar{r}^*_{ik} = [1,1] - \bar{r}^*_{ki}. \tag{11}$$

Example 1. *Let $\overline{R} = (\bar{r}_{ij})_{4\times 4}$ be an IIVFPR for the alternatives x_1, x_2, x_3, and x_4, given as follows:*

$$\overline{R} = \begin{bmatrix} [0.5,0.5] & \bar{r}_{12} & [0.6,0.9] & [0.6,0.8] \\ \bar{r}_{21} & [0.5,0.5] & [0.4,0.9] & \bar{r}_{24} \\ [0.1,0.4] & [0.1,0.6] & [0.5,0.5] & [0.6,0.9] \\ [0.2,0.4] & \bar{r}_{42} & [0.1,0.4] & [0.5,0.5] \end{bmatrix}$$

where $\bar{r}_{12}, \bar{r}_{21}, \bar{r}_{24}$ and $\bar{r}_{42}$ are unknown preference values. Now, applying (6)–(10) to estimate the unknown preference values for the alternative x_i over x_k, $1 \le i \le 4$ and $1 \le k \le 4$, we obtain:

$$\begin{aligned} Ev &= \{(1,3),(1,4),(2,3),(3,1),(3,2),(3,4),(4,1),(4,3)\}, \\ Mv &= \{(1,2),(2,1),(2,4),(4,2)\}. \\ H_{12} &= \{3\}, \\ \bar{r}_{12} &= T_L(\bar{r}_{13}, \bar{r}_{32}) = [\max(r^-_{13} + r^-_{32} - 1, 0), \max(r^+_{13} + r^+_{32} - 1, 0)], \\ &= [0, 0.5]. \\ \bar{r}_{21} &= [1,1] - [0,0.5] = [0.5,1]. \end{aligned}$$

$$\begin{aligned} Ev' &= \{(1,2),(1,3),(1,4),(2,1),(2,3),(3,1),(3,2),(3,4),(4,1), \\ &\quad (4,3)\}, \\ Mv' &= \{(2,4),(4,2)\}. \\ H_{24} &= \{1,3\}, \\ \bar{r}_{24} &= \max(T_L(\bar{r}_{21}, \bar{r}_{14}), T_L(\bar{r}_{23}, \bar{r}_{34})) = \max([0.1,0.8],[0,0.8]) \\ &= [0.1,0.8]. \\ \bar{r}_{21} &= [1,1] - [0.1,0.8] = [0.2,0.9]. \end{aligned}$$

Hence, the complete IVFPR is obtained under the above process as:

$$\overline{R} = \begin{bmatrix} [0.5,0.5] & [0,0.5] & [0.6,0.9] & [0.6,0.8] \\ [0.5,1] & [0.5,0.5] & [0.4,0.9] & [0.1,0.8] \\ [0.1,0.4] & [0.1,0.6] & [0.5,0.5] & [0.6,0.9] \\ [0.2,0.4] & [0.2,0.9] & [0.1,0.4] & [0.5,0.5] \end{bmatrix} \tag{12}$$

By applying (11) on (12), $\overline{R}$ becomes an L-consistent IVFPR $\widetilde{\overline{R}}$ as follows:

$$\begin{aligned} \bar{r}^*_{12} &= \max_{j=3,4}\left([0,0.5], T_L\left(\bar{r}_{1j}, \bar{r}_{j2}\right)\right) = \max([0,0.5],[0,0.5],[0,0.7]) = [0,0.7] \\ \bar{r}^*_{21} &= [1,1] - [0,0.7] = [0.3,1.0] \\ \bar{r}^*_{13} &= \max_{j=2,4}\left([0.6,0.9], T_L\left(\bar{r}_{1j}, \bar{r}_{j3}\right)\right) = \max([0.6,0.9],[0,0.5],[0,0.2]) = [0.6,0.9] \\ \bar{r}^*_{31} &= [1,1] - [0.6,0.9] = [0.1,0.4] \\ \bar{r}^*_{14} &= \max_{j=2,3}\left([0.6,0.8], T_L\left(\bar{r}_{1j}, \bar{r}_{j3}\right)\right) = \max([0.6,0.8],[0,0.5],[0.2,0.8]) = [0.6,0.8] \\ \bar{r}^*_{41} &= [1,1] - [0.6,0.8] = [0.2,0.4] \\ \bar{r}^*_{23} &= \max_{j=1,4}\left([0.4,0.9], T_L\left(\bar{r}_{2j}, \bar{r}_{j3}\right)\right) = \max([0.4,0.9],[0,0.9],[0,0.2]) = [0.4,0.9] \\ \bar{r}^*_{32} &= [1,1] - [0.4,0.9] = [0.1,0.6] \end{aligned}$$

$$\bar{r}^*_{24} = \max_{j=1,3}\left([0.1, 0.8], T_L(\bar{r}_{1j}, \bar{r}_{j2})\right) = \max([0.1, 0.8], [0, 0.8], [0, 0.8]) = [0.1, 0.8]$$
$$\bar{r}^*_{42} = [1, 1] - [0.1, 0.8] = [0.2, 0.9]$$
$$\bar{r}^*_{34} = \max_{j=1,2}\left([0.6, 0.9], T_L(\bar{r}_{3j}, \bar{r}_{j4})\right) = \max([0.6, 0.9], [0, 0.2], [0, 0.4]) = [0.6, 0.9]$$
$$\bar{r}^*_{43} = [1, 1] - [0.6, 0.9] = [0.1, 0.4]$$

Hence

$$\overline{R}^* = \begin{bmatrix} [0.5, 0.5] & [0, 0.7] & [0.6, 0.9] & [0.6, 0.8] \\ [0.3, 1] & [0.5, 0.5] & [0.4, 0.9] & [0.1, 0.8] \\ [0.1, 0.4] & [0.1, 0.6] & [0.5, 0.5] & [0.6, 0.9] \\ [0.2, 0.4] & [0.2, 0.9] & [0.1, 0.4] & [0.5, 0.5] \end{bmatrix}. \tag{13}$$

However, $\overline{R}^*$ is not L-consistent so far because the following inequalities do not hold:

$$\begin{aligned} r^{*+}_{31} \geq \max(r^{*+}_{32} + r^{*+}_{21} - 1, 0) &\implies 0.4 \geq 0.6; \\ r^{*+}_{32} \geq \max(r^{*+}_{34} + r^{*+}_{42} - 1, 0) &\implies 0.6 \geq 0.8; \\ r^{*+}_{41} \geq \max(r^{*+}_{42} + r^{*+}_{21} - 1, 0) &\implies 0.4 \geq 0.9; \\ r^{*+}_{43} \geq \max(r^{*+}_{42} + r^{*+}_{23} - 1, 0) &\implies 0.4 \geq 0.8. \end{aligned}$$

Therefore, we have to apply (11) again on (13), and the repeated application of (11) results in an L-consistent IVFPR $\widetilde{\overline{R}}$ given as:

$$\widetilde{\overline{R}} = \begin{bmatrix} [0.5, 0.5] & [0, 0.7] & [0.2, 0.9] & [0.1, 0.8] \\ [0.3, 1] & [0.5, 0.5] & [0.2, 0.9] & [0.1, 0.8] \\ [0.1, 0.8] & [0.1, 0.8] & [0.5, 0.5] & [0.2, 0.9] \\ [0.2, 0.9] & [0.2, 0.9] & [0.1, 0.8] & [0.5, 0.5] \end{bmatrix} \tag{14}$$

one can easily check that (14) is fully L-consistent because $\widetilde{\bar{r}}_{ik} \geq \max(\widetilde{\bar{r}}_{ij} + \widetilde{\bar{r}}_{jk} - 1, 0)$ i.e., $\widetilde{r}^-_{ik} \geq \max(\widetilde{r}^-_{ij} + \widetilde{r}^-_{jk} - 1, 0)$ and $\widetilde{r}^+_{ik} \geq \max(\widetilde{r}^+_{ij} + \widetilde{r}^+_{jk} - 1, 0)$, hold for $i, j, k \in \{1, 2, 3, 4\}$ $(i \neq j \neq k)$.

4. Selection of Best Alternative(s) in MPDM with IIVFPRs

This section addresses a technique for MPDM based on incomplete interval valued fuzzy preference relations (IIVFPRs), and obtaining L-consistency is proposed. The procedure works in the following steps:

(i) Estimation of the missing preference values;
(ii) Conversion of the completed preference matrix into an L-consistent matrix;
(iii) Consistency analysis of IVFPRs and measure of the consistency weight vector of the experts;
(iv) Derivation of priority weights of the experts;
(v) Aggregation of all consistent preference relations to receive the weighted collective matrix;
(vi) Construction of complementary preference relation by calculating the possibility degree;
(vii) Finally, the algorithm determines the score of each alternative for ranking the preference order of the alternatives.

Hence, in this section, the procedure given in Section 2 is extended for MPDM in IIVFPRs environment and toward the end, an explanatory example to validate the anticipated technique is also given. Suppose that there are n alternatives $x_1, x_2, \ldots, x_n$, and m experts

$E_1, E_2, \ldots, E_m$ with weight vector $\lambda_i (i = 1, 2, 3 \ldots, m)$. Let $\overline{R}^q$ be the IVFPR for expert E_q shown as follows:

$$\overline{R}^q = \left(\overline{r}_{ij}^q\right)_{n\times n} = \begin{bmatrix} [0.5, 0.5] & \overline{r}_{12}^q & . & . & \overline{r}_{1n}^q \\ \overline{r}_{21}^q & [0.5, 0.5] & . & . & \overline{r}_{2n}^q \\ . & . & . & & . \\ . & . & & . & . \\ \overline{r}_{n1}^q & \overline{r}_{n2}^q & . & . & [0.5, 0.5] \end{bmatrix},$$

where $\overline{r}_{ij}^q \in L([0,1])$ is the preference value given by expert E_q for alternative x_i over x_j, $\overline{r}_{ij}^q = [1,1] - \overline{r}_{ji}^q$, $\overline{r}_{ii}^q = [0.5, 0.5]$, $1 \le i \le n, 1 \le j \le n$ and $1 \le q \le m$. The proposed MPDM technique is given as follows:

Step-i: Determine the sets Av^q, Ev^q and Mv^q of pairs of alternatives for known and unknown preference values, respectively, shown as follows:

$$Av^q = \{(i,j) \mid i \ne j \wedge i, j \in \{1, 2, 3, \ldots, n\}\}, \tag{15}$$

$$Ev^q = \{(i,j) \in Av \mid \overline{r}_{ij}^q \text{ is known}\} \tag{16}$$

$$Mv^q = \{(i,j) \in Av \mid \overline{r}_{ij}^q \text{ is unknown}\}, \tag{17}$$

where $1 \le i \le n, 1 \le j \le n$, and $1 \le q \le m$. Now, construct the set H_{ik}^q based on the sets Ev^q and Mv^q, and is used to estimate the unknown preference values $\overline{r}_{ik}^q$ for the alternative x_i over x_k by expert E_q as follows:

$$H_{ik}^q = \{j \ne i, k \mid (i,j) \in Ev^q, (j,k) \in Ev^q \text{ and } (i,k) \in Mv^q\},$$

$$\overline{r}_{ik}^q = \begin{cases} \max\limits_{j \in H_{ik}^q} (T_L(\overline{r}_{ij}^q, \overline{r}_{jk}^q)), & \text{if } H_{ik}^q \ne \emptyset \\ [0.5, 0.5], & \text{otherwise.} \end{cases} \tag{18}$$

By using the condition, $\overline{r}_{ji}^q = [1,1] - \overline{r}_{ij}^q$, the value of $\overline{r}_{ki}$ can be calculated as follows:

$$\overline{r}_{ki}^q = \left[1 - r_{ik}^{q+}, 1 - r_{ik}^{q-}\right] \tag{19}$$

Hence

$$Ev'^q = Ev^q \cup \{(i,k), (k,i)\}, \tag{20}$$

$$Mv'^q = Mv^q - \{(i,k), (k,i)\}. \tag{21}$$

Step-ii: An L-consistent matrix $\widetilde{\overline{R}}^q = \left(\widetilde{\overline{r}}_{ij}^q\right)_{n\times n}$ is obtained by using the following formula:

$$\widetilde{\overline{r}}_{ik}^q = \max_{j \ne i,k}\left(\overline{r}_{ik}^q, T_L\left(\overline{r}_{ij}^q, \overline{r}_{jk}^q\right)\right) \text{ such that } \widetilde{\overline{r}}_{ik}^q = [1,1] - \widetilde{\overline{r}}_{ki}^q. \tag{22}$$

Step-iii: After evaluating the complete IVFPRs, we can then approximate the degrees of consistency of IVFPRs $\overline{R}^q$ based on its similarity with the corresponding L-consistent $\widetilde{\overline{R}}^q$ by computing their distances. To estimate the consistency level of IVFPR given by the expert E_q, construct the FPR $A^q = \left(a_{ij}^q\right)_{n\times n}$ and the consistency matrix $\widetilde{A}^q = \left(\widetilde{a}_{ij}^q\right)_{n\times n}$ for expert E_q using average values as:

$$A^q = \left(a_{ij}^q\right)_{n\times n} = \left(\frac{1}{2}\left(a_{ij}^{q-} + a_{ij}^{q+}\right)\right)_{n\times n} \tag{23}$$

$$\widetilde{A}^q = \left(\widetilde{a}_{ij}^q\right)_{n\times n} = \left(\frac{1}{2}\left(\widetilde{a}_{ij}^{q-} + \widetilde{a}_{ij}^{q+}\right)\right)_{n\times n} \tag{24}$$

where preference values a_{ij}^q and $\widetilde{a}_{ij}^q$ fall in $[0,1]$, $1 \le i \le n, 1 \le j \le n$, and $1 \le q \le m$. Now, to estimate the consistency degree of an FPR A^q based on its similarity with the corresponding L-transitive FPR $\widetilde{A}^q$, we use three levels:

1. The L-consistent index (LCI) of a pair of alternatives is determined by using:

$$LCI(\overline{r}_{ij}^q) = 1 - d(a_{ij}^q, \widetilde{a}_{ij}^q) \tag{25}$$

   where $d(a_{ij}^q, \widetilde{a}_{ij}^q)$ is the error (distance) measured by $\varepsilon\overline{r}_{ij}^q = d(a_{ij}^q, \widetilde{a}_{ij}^q) = \left|a_{ij}^q - \widetilde{a}_{ij}^q\right|$. Seemingly, the higher the value of $LCI(\overline{r}_{ij}^q)$, the more consistent $\overline{r}_{ij}^q$ is with respect to the rest of the preference values involving alternatives x_i and x_j.
2. The LCI of alternatives x_i, $1 \le i \le n$, is evaluated by:

$$LCI(x_i) = \frac{1}{2(n-1)}\sum_{j=1}^{n}(LCI(\overline{r}_{ij}^q) + LCI(\overline{r}_{ji}^q)) \tag{26}$$

3. The LCI of an IVFPR $\overline{R}^q$ is obtained by taking the average of all LCI of alternatives x_i :

$$LCI(\overline{R}^q) = \frac{1}{n}\sum_{i=1}^{n} LCI(x_i) \tag{27}$$

As a result, utilizing the following relation, consistency weights can be allocated to experts.

$$Cw(E_q) = \frac{LCI(\overline{R}^q)}{\sum_{q=1}^{m} LCI(\overline{R}^q)} \tag{28}$$

Step-iv: The final priority weights for experts shall be assigned after evolving the respective predefined weights and consistency weights using the following relationship:

$$w(Eq) = \frac{\lambda_q \times Cw(E_q)}{\sum_{q=1}^{m} \lambda_q \times Cw(E_q)} \tag{29}$$

where $\sum_{q=1}^{m} w(E_q) = 1$. If λ_q is not given, then consistency weights will be taken as the priority weights of the experts.
Step-v: Construct the cumulative matrix $\overline{R}^C$ among all experts as shown below:

$$\overline{R}^C = \left(\overline{r}_{ij}^c\right)_{n\times n} = \left(\sum_{q=1}^{m} w(Eq) \times \widetilde{\overline{r}}_{ij}^q\right)_{n\times n}, \tag{30}$$

where $1 \le i \le n, 1 \le j \le n$.
Step-vi: Now, first, we calculate the average degree $\overline{av}_i$ of alternative x_i over all other alternatives by using the interval normalizing method

$$\overline{av}_i = \frac{\sum_{j=1,\, j\ne i}^{n} \overline{r}_{ij}^c}{\sum_{\substack{i=1,\\ i\ne j}}^{n} \sum_{j=1}^{n} \overline{r}_{ij}^c}, \quad i = 1, 2, 3, \ldots, n \tag{31}$$

then the possibility degree $d_{ij} = d(\overline{av}_i \geq \overline{av}_j)$ by using the following formula [19]:

$$d(\overline{av}_i \geq \overline{av}_j) = \min\left\{\max\left(\frac{av_i^+ - av_j^-}{av_i^+ - av_j^- + av_j^+ - av_i^-}, 0\right), 1\right\} \tag{32}$$

to construct the L-consistent complementary matrix $D = \left(d_{ij}\right)_{n\times n}$, where $d_{ij} \geq 0, d_{ij} + d_{ji} = 1, i, j = 1, 2, 3, \ldots, n$.

Step-vii: Finally, the ranking value $R_V(x_i)$ of alternative x_i can be determined by using the formula:

$$R_V(x_i) = \frac{2}{n(n-1)} \sum_{j=1,\, j\neq i}^{n} d_{ij}, \tag{33}$$

where $1 \leq i \leq n$ and $\sum_{i=1}^{n} R_V(x_i) = 1$.

Example 2. *A textile mill manager desires to select a suitable supplier to purchase the raw material for new products. Initially, the mill manager selects four suppliers, signified as $x_i (i = 1, 2, 3, 4)$, and presents them for further evaluation. A committee consisting of three experts $E_q (q = 1, 2, 3)$ with weights vector $\lambda_q = (1/2, 1/4, 1/4)$, respectively, from different departments, has been formed to assess the four suppliers $x_i (i = 1, 2, 3, 4)$. Suppose that the experts $E_q (q = 1, 2, 3)$ provide their assessments in the form of the following IIVFPRs:*

$$\overline{R}^1 = \begin{bmatrix} [0.5, 0.5] & \overline{r}_{12}^1 & [0.6, 0.8] & \overline{r}_{14}^1 \\ \overline{r}_{21}^1 & [0.5, 0.5] & [0.2, 0.7] & [0.4, 0.9] \\ [0.2, 0.4] & [0.3, 0.8] & [0.5, 0.5] & [0.6, 0.9] \\ \overline{r}_{41}^1 & [0.1, 0.6] & [0.1, 0.4] & [0.5, 0.5] \end{bmatrix},$$

$$\overline{R}^2 = \begin{bmatrix} [0.5, 0.5] & \overline{r}_{12} & [0.4, 0.6] & [0.2, 0.9] \\ \overline{r}_{21} & [0.5, 0.5] & [0.4, 0.8] & \overline{r}_{24} \\ [0.4, 0.6] & [0.2, 0.6] & [0.5, 0.5] & [0.3, 0.7] \\ [0.1, 0.8] & \overline{r}_{42} & [0.3, 0.7] & [0.5, 0.5] \end{bmatrix},$$

and

$$\overline{R}^3 = \begin{bmatrix} [0.5, 0.5] & [0.4, 0.8] & [0.3, 0.9] & \overline{r}_{14}^3 \\ [0.2, 0.6] & [0.5, 0.5] & \overline{r}_{23}^3 & [0.6, 0.9] \\ [0.1, 0.7] & \overline{r}_{32}^3 & [0.5, 0.5] & \overline{r}_{34}^3 \\ \overline{r}_{41}^3 & [0.1, 0.4] & \overline{r}_{43}^3 & [0.5, 0.5] \end{bmatrix}.$$

Step-i: By the utilization of (15) to (21), the unknown preference degrees for the IVFPR $\overline{R}^1$ given by expert E_1 are calculated, and $\overline{R}^1$ in complete form is:

$$\overline{R}^1 = \begin{bmatrix} [0.5, 0.5] & [0, 0.6] & [0.6, 0.8] & [0.2, 0.7] \\ [0.4, 1.0] & [0.5, 0.5] & [0.2, 0.7] & [0.4, 0.9] \\ [0.2, 0.4] & [0.3, 0.8] & [0.5, 0.5] & [0.6, 0.9] \\ [0.3, 0.8] & [0.1, 0.6] & [0.1, 0.4] & [0.5, 0.5] \end{bmatrix}.$$

Similarly, the missing preference values in $\overline{R}^2$ and $\overline{R}^3$ are estimated.

Step-ii: Now, applying (22) to get L-transitive IVFPR $\overset{\approx}{R}^1$ under the generalized t-norm T_L as:

$$\overset{\approx}{R}^1 = \begin{bmatrix} [0.5, 0.5] & [0, 0.6] & [0.2, 0.8] & [0.2, 0.7] \\ [0.4, 1.0] & [0.5, 0.5] & [0.2, 0.8] & [0.4, 0.9] \\ [0.2, 0.8] & [0.2, 0.8] & [0.5, 0.5] & [0.4, 0.9] \\ [0.3, 0.8] & [0.1, 0.6] & [0.1, 0.6] & [0.5, 0.5] \end{bmatrix}.$$

In a similar manner, L-transitive IVFPRs $\overline{\widetilde{R}}^{2}$ and $\overline{\widetilde{R}}^{3}$ against experts E_2 and E_3 are determined:

$$\overline{\widetilde{R}}^{2} = \begin{bmatrix} [0.5, 0.5] & [0, 0.9] & [0.3, 0.7] & [0, 0.9] \\ [0.1, 1.0] & [0.5, 0.5] & [0.3, 0.8] & [0, 0.9] \\ [0.3, 0.7] & [0.2, 0.7] & [0.5, 0.5] & [0.2, 0.7] \\ [0.1, 1.0] & [0.1, 1.0] & [0.3, 0.8] & [0.5, 0.5] \end{bmatrix},$$

$$\overline{\widetilde{R}}^{3} = \begin{bmatrix} [0.5, 0.5] & [0.1, 0.9] & [0.1, 0.9] & [0, 0.8] \\ [0.1, 0.9] & [0.5, 0.5] & [0, 0.8] & [0.1, 0.9] \\ [0.1, 0.9] & [0.2, 1.0] & [0.5, 0.5] & [0.1, 0.9] \\ [0.2, 1] & [0.1, 0.9] & [0.1, 0.9] & [0.5, 0.5] \end{bmatrix}.$$

Step-iii: To this end, we transform the IVFPRs $\overline{R}^{q}$ and $\overline{\widetilde{R}}^{q}(q = 1, 2, 3)$ into corresponding FPRs A^q and $\widetilde{A}^q(q = 1, 2, 3)$ by using (23) and (24):

$$A^1 = \begin{bmatrix} 0.50 & 0.30 & 0.70 & 0.45 \\ 0.70 & 0.50 & 0.45 & 0.65 \\ 0.30 & 0.55 & 0.50 & 0.75 \\ 0.55 & 0.35 & 0.25 & 0.50 \end{bmatrix} \text{ and } \widetilde{A}^1 = \begin{bmatrix} 0.50 & 0.30 & 0.50 & 0.45 \\ 0.70 & 0.50 & 0.50 & 0.65 \\ 0.50 & 0.50 & 0.50 & 0.65 \\ 0.55 & 0.35 & 0.35 & 0.5 \end{bmatrix},$$

$$A^2 = \begin{bmatrix} 0.50 & 0.10 & 0.50 & 0.55 \\ 0.90 & 0.50 & 0.60 & 0.45 \\ 0.50 & 0.40 & 0.50 & 0.50 \\ 0.45 & 0.55 & 0.50 & 0.50 \end{bmatrix} \text{ and } \widetilde{A}^2 = \begin{bmatrix} 0.50 & 0.45 & 0.50 & 0.45 \\ 0.55 & 0.50 & 0.55 & 0.45 \\ 0.50 & 0.45 & 0.50 & 0.45 \\ 0.55 & 0.55 & 0.55 & 0.50 \end{bmatrix},$$

$$A^3 = \begin{bmatrix} 0.50 & 0.60 & 0.60 & 0.35 \\ 0.40 & 0.50 & 0.25 & 0.75 \\ 0.40 & 0.75 & 0.50 & 0.50 \\ 0.65 & 0.25 & 0.50 & 0.50 \end{bmatrix} \text{ and } \widetilde{A}^3 = \begin{bmatrix} 0.50 & 0.50 & 0.50 & 0.40 \\ 0.50 & 0.50 & 0.40 & 0.50 \\ 0.50 & 0.60 & 0.50 & 0.50 \\ 0.60 & 0.50 & 0.50 & 0.5 \end{bmatrix},$$

Therefore, using (25)–(27) we can determine that

$$LCI(\overline{R}^{1}) = 0.94165, \ LCI(\overline{R}^{2}) = 0.90835 \text{ and } LCI(\overline{R}^{3}) = 0.89168.$$

The consistency weights of the experts are, therefore, calculated using (28) as:

$$Cw(E_1) = 0.3435, \ Cw(E_2) = 0.3313, \text{ and } Cw(E_3) = 0.3252$$

where $\sum_{q=1}^{3} Cw(E_q) = 1$.

Step-iv: Expression (29) results in the final priority weights of the experts as:

$$w(E_1) = 0.5114, \ w(E_2) = 0.2466, \text{ and } w(E_3) = 0.2420$$

Step-v: The collective matrix $\overline{R}^{C}$ against all experts is obtained by the use of (30)

$$\overline{R}^{C} = \begin{bmatrix} [0.5000, 0.5000] & [0.0242, 0.7466] & [0.2005, 0.7995] & [0.1023, 0.7735] \\ [0.2534, 0.9758] & [0.5000, 0.5000] & [0.1763, 0.8000] & [0.2288, 0.9000] \\ [0.2005, 0.7995] & [0.2000, 0.8237] & [0.5000, 0.5000] & [0.2781, 0.8507] \\ [0.2265, 0.8977] & [0.1000, 0.7712] & [0.1493, 0.7219] & [0.5000, 0.5000] \end{bmatrix}.$$

Step-vi: The average degree $\overline{av}_i,\ i = 1,2,3,4$, of each alternative is derived by using the interval normalizing Formula (31)

$$\overline{av}_1 = [0.033164, 1.083976];\ \overline{av}_2 = [0.066784, 1.250432];$$
$$\overline{av}_3 = [0.068823, 1.156082];\ \overline{av}_4 = [0.048255, 1.117248].$$

Therefore, the possibility degree matrix $D = (d_{ij})_{4\times 4}$ is obtained by using (32):

$$D = (d_{ij})_{4\times 4} = \begin{bmatrix} 0.5000 & 0.4552 & 0.4748 & 0.4886 \\ 0.5448 & 0.5000 & 0.5203 & 0.5337 \\ 0.5252 & 0.4797 & 0.5000 & 0.5138 \\ 0.5114 & 0.4663 & 0.4862 & 0.5000 \end{bmatrix}.$$

Step-vii: Expression (33) gives the ranking value $R_v(x_i)$ of alternative $x_i,\ 1 \le i \le 4$, as follows:

$$R_v(x_1) = 0.2364;\ R_v(x_2) = 0.2665;$$
$$R_v(x_3) = 0.2531;\ R_v(x_4) = 0.2440,$$

where $\sum_{i=1}^{4} R_v(x_i) = 1$. Thus, the final ranking of the alternatives is derived as follows:

$$x_2 \succ x_3 \succ x_4 \succ x_1.$$

Thus, the best alternative is x_2. The numerical illustration demonstrates the way in which the proposed technique for constructing a complete IVFPR based on L-consistency is implemented. In general, the proposed method is very straightforward to use when calculating uncertain preference values.

5. Comparison

Table 1 makes a comparison of the preference ranking of the alternatives of the proposed technique with the ones of Xu et al.'s method [11] under IIVFPRs and Zhang and Wang's method [26] under complete IVFPRs.

Table 1. Comparison of ranking of the alternatives.

Methods	Ranking Values/Scores	Preference Order
Xu et al.'s method [11]	ω_1 0.136, ω_2 0.250, ω_3 0.178, ω_4 0.222, ω_5 0.215	$x_2 \succ x_4 \succ x_5 \succ x_3 \succ x_1$
The proposed method	$R(x_1)$ 0.0468, $R(x_2)$ 0.2143, $R(x_3)$ 0.2016, $R(x_4)$ 0.2434, $R(x_5)$ 0.2939	$x_5 \succ x_4 \succ x_2 \succ x_3 \succ x_1$
Zhang and Wang's method [26]	Ranking values are calculated in compatible mode	$x_3 \succ x_2 \succ x_1 \succ x_4$
The proposed method	$R(x_1)$ 0.2054, $R(x_2)$ 0.2280, $R(x_3)$ 0.4321, $R(x_4)$ 0.1345	$x_3 \succ x_2 \succ x_1 \succ x_4$

From Table 1, we can observe that the preference order of the alternatives x_1, x_2, x_3, x_4, and x_5 obtained by the method of Xu et al. [11] is $x_2 \succ x_4 \succ x_5 \succ x_3 \succ x_1$, while the proposed method results in $x_5 \succ x_4 \succ x_2 \succ x_3 \succ x_1$. The positions of alternatives x_2 and x_5 have been interchanged while the order for others remained the same; this may be due to the inconsistency of complementary relation. Xu et al. [11] used additive transitivity, i.e., $r_{ik} = r_{ij} + r_{jk} - 0.5$, for all $i, j \in N$ to evaluate the final ranking. However, the complementary relation $P = [p_{ij}]$ constructed by Xu et al. as:

$$P = \begin{bmatrix} 0.5 & 0.217 & 0.389 & 0.284 & 0.311 \\ 0.783 & 0.5 & 0.685 & 0.572 & 0.581 \\ 0.611 & 0.315 & 0.5 & 0.387 & 0.409 \\ 0.716 & 0.428 & 0.613 & 0.5 & 0.514 \\ 0.689 & 0.419 & 0.591 & 0.486 & 0.5 \end{bmatrix}$$

is not additive consistent. For instance, if we take $i = 1$ and $k = 2$, then the transitivity $p_{12} = p_{1j} + p_{j2} - 0.5$ must be fulfilled for all $j = 1, 2, 3, 4, 5$, while from matrix P, we have: $p_{12} = 0.217$ and $p_{13} + p_{32} - 0.5 = 0.2024$, $p_{14} + p_{42} - 0.5 = 0.212$, $p_{15} + p_{52} - 0.5 = 0.23$. On the other hand, the possibility degree matrix $D = [d_{ij}]$ is given as follows:

$$D = \begin{bmatrix} 0.5000 & 0.1636 & 0.2099 & 0.0797 & 0.0145 \\ 0.8364 & 0.5000 & 0.5231 & 0.4446 & 0.3392 \\ 0.7901 & 0.4769 & 0.5000 & 0.4234 & 0.3259 \\ 0.9203 & 0.5554 & 0.5766 & 0.5000 & 0.3818 \\ 0.9855 & 0.6608 & 0.6741 & 0.6182 & 0.5000 \end{bmatrix}$$

based on L-consistency, i.e., $r_{ik} \geq (r_{ij} + r_{jk} - 1, 0)$ for all $i \neq j \neq k \in N$ (for whole process). One can easily validate the L-consistency for all the preference values evaluated in D. We believe that the outcomes under consistent information are more reliable than others.

However, the preference order of alternatives x_1, x_2, x_3, and x_4 in a practical example determined by the proposed method and Zhang and Wang's method [26] are the same, i.e., $x_3 > x_2 > x_1 > x_4$. Zhang and Wang [26] proposed a method to measure the proximity or compatibility between two interval numbers with values between 0 and 1. It uses a ratio-based compatibility degree and a compatibility measurement formula based on the geometric mean to evaluate the compatibility between two interval fuzzy preference relations. A procedure was developed to handle group decision-making problems with interval fuzzy preference relations.

6. Conclusions

This study develops a method for dealing with MPDM challenges when the data are provided in the form of IIVFPRs. The missing values have been estimated using the Lukasiewicz transitive property, and the information has been made completely consistent by applying the transitive closure procedure. The suggested method can be used with both complete and incomplete preference relations; in MPDM with complete IVFPRs, we just need to skip the stages that are needed to figure out the missing values. After adjusting the relevant consistency weights and predetermined weights, the priority weights for the experts were estimated. In order to be given more weight in the aggregation process, experts with high levels of consistency should logically be required to offer significant weights. The interval normalization technique was used to calculate the average priority level of each alternative compared to the others. After using possibility degrees to build the consistent, complementary fuzzy preference relation, the selection process started to rate each alternative and choose the best one. To demonstrate the effectiveness and viability of the suggested approach, some numerical examples were presented. Additionally, some comparisons with models proposed by Xu et al. [11] under IIVFPRs and Zhang and Wang [26] were presented. The outcomes proved the method's viability and showed how, using the interval valued data, we may better understand the MPDM process. To sum up, the following are some of the main advantages of the suggested technique:

(1). When compared to existing consistency-based methods, the suggested method efficiently achieves the consistency of IVFPRs by estimating the missing preference values using Lukasiewicz's transitivity; (2). Combining consistency weights and predetermined weights increased the reliability of expert weights, which were then used to quantify how consistently accurate the experts' estimates were; (3). The suggested method is equally effective for preference relations that are both complete and incomplete; (4). When compared to other models [11], the suggested strategy produced substantially consistent preference relations.

However, there are a few limitations that require further study: (1). Because some aspects must be taken into account, MPDM may have too many criteria for the decision-making process, such as behavioral sciences, national discourse, risk level, etc. (2). When expressing their preferred comparisons, experts sometimes hesitate. It would be interesting to develop coping mechanisms for MPDM in the framework of type-2 interval fuzzy preference relations.

To the authors' knowledge, there are not many approaches of this kind that have been suggested in the literature to address MPDM issues in the environment of IIVFPRs. This approach, in our opinion, deals with MPDM challenges more effectively.

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