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Mapmaking Process Reading from Local Distortions in Historical Maps: A Geographically Weighted Bidimensional Regression Analysis of a Japanese Castle Map

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Abstract: Shoho Castle Maps are maps of castle towns throughout Japan drawn by Kano School painters on the order of the shogun in 1644. The Shoho Castle Map of Takada, Joetsu City, Niigata Prefecture was used to visualize local distortions in historical maps and to scrutinize the mapmaking process. A novel method, geographically weighted bidimensional regression, was developed and applied to visualize the local distortions of the map. Exaggerated expressions by mapmakers that have not been identified in previous studies were revealed. That is, in addition to the castle being drawn enlarged, the town where the merchants and artisans lived was drawn larger than the castle. Therefore, the Takada Shoho Castle Map reflects mapmakers' intentions, besides enlarging military facilities, which appear to have emphasized the pictorial composition of the map by placing the main gate to the castle at the center and drawing the map area evenly from the center in a well-balanced layout.

Keywords: historical map; planimetric accuracy; geographically weighted regression; bidimensional regression; Shoho Castle Maps; Edo Shogunate; Kano School of painting

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1. Introduction

1.1. Background

In general, compared to modern maps, historical maps have various distortions that originate from differences in modern surveying techniques and ideas about cartography. The evaluation of the distortion or accuracy of historical maps is an important research subject [1–3], and studies of the distortion of historical maps using computer-aided quantitative methods, pioneered by Tobler's analysis [4], have been conducted [5–10]. Since the 2000s, the widespread use of geographic information systems (GIS) and quantitative methods has led to various studies of distortions in historical maps [11–16]. The debut of MapAnalyst [17–19] software, which has the ability to visualize distortions in historical maps, has been particularly influential and has found use in many studies.

By visualizing and interpreting historical map distortions, one can examine the process of creating historical maps, including the surveying methods of time and mapmakers' intentions [20–27]. The following section presents a brief review of existing studies that have read the surveying process or mapmakers' intentions from historical map distortions.

One factor that causes distortion during the surveying stage is the limitations of previous surveying techniques [28–30]. More specifically, by focusing on the topography and vegetation where historical maps are locally distorted, some studies have examined whether the distortion was due to surveying difficulties caused by topographical constraints, such as high mountains and cliffs, or by the lack of visibility in forests [23–27,31]. Most studies that examine the surveying process from the perspective of map distortion are based on historical maps from the 19th century. In addition to historical maps, there are other documents, such as field notes, that describe the surveying process, and the fact that the detailed surveying process can be examined through documents lies behind most of the research conducted during this period. For example, triangulation was used in military surveys of the Habsburg Empire in the 18th and 19th centuries [32,33]. Accurate city maps of Belgrade, created for urban redevelopment planning, were produced using a plane table and alidade [34]. These historical maps from the 18th and 19th centuries were based on surveying techniques similar to those used today, and they have a certain degree of accuracy compared to modern maps.

Historical maps are often intentionally exaggerated by mapmakers, causing distortion [2]. Ballon and Friedman [35] found that even in the plan of Imola by Leonardo da Vinci, the forerunner of accurate large-scale planimetric maps, showed distortions owing to the author's intention to improve the plan's formal composition. Specifically, the curves were drawn in an exaggerated manner for the main street to express organic coherence. In some large-scale historical maps, such as city maps, certain facilities and districts are represented as larger and at a more exaggerated scale than in reality [20,35–37]. What is depicted in an exaggerated manner depends on the purpose of the map; specifically, maps depicting enlarged military facilities, such as castles, have been created with military purposes in mind [35–37], for example, among the city maps of the first half of the 17th century in Cadiz, Spain, where maps were made for military purposes. In these maps, fortresses are depicted in accurate locations in plan view and often at exaggerated scales, whereas urban areas other than fortresses are depicted in perspective and their locations are not accurate [37]. The 16th- and 17th-century maps of cities for military purposes tended to focus on military facilities, such as castles, and not on urban areas other than military facilities. Studies discussing cartographers' intentions based on map distortion often focused on historical maps before the 18th century, which often show pictorial representations differing from contemporary scientific map practices [38].

1.2. Shoho Castle Maps

The abovementioned maps with intentionally enlarged military facilities are examples of large-scale maps from the 16th to 18th centuries, corresponding to present-day Italy, Spain, the Netherlands, and Belgium, whereas large-scale city maps from the 17th century with enlarged military facilities are also known in Japan. Shoho Castle Maps are one such example.

Shoho Castle Maps were prepared in 1644 by the Edo Shogunate during the reign of the third shogun, who ordered all feudal lords in the country to submit maps of their castle towns. Although the maps are named "castle maps", they also depict castles and towns around castles [39]. The purpose of the Shoho Castle Maps was to demonstrate the authority of the shogun. That is, by providing the shogun with detailed drawings of the interior of the castles, which were top secrets of each feudal lord, the shogun could confirm his control over the feudal lords and establish his authority [40–43]. When the Shoho Castle Maps were prepared, detailed instructions were provided regarding map specifications, such as including the numerical values of the lengths of streets, widths and depths of moats, and heights of stone walls [40–43]. These maps are generally two to three meters in length and width, and their scale varies depending on the castle town; however, large-scale maps are common [43]. In most cases, the task of drawing maps was assigned to painters from the Kano School [40,44]. Consequently, the style of the maps collected from feudal lords throughout Japan was almost uniform.

There were 131 Shoho Castle Maps at the end of the Edo period [43], but those currently held by the National Archives of Japan include 63 maps of Hiroshima, Okayama, and other areas. The other 68 maps are believed to have been scattered during the Meiji Restoration/Revolution, and maps believed to be Shoho Castle Maps remain in Aizu-Wakamatsu, Sendai, and Takada, although they are not held by the National Archives of Japan. The new government forces removed these maps as operational maps for castle attacks during the Meiji Restoration/Revolution [43,45,46].

1.3. Distortion of the Shoho Castle Maps

The distortion of the Shoho Castle Maps, wherein the castles are drawn larger than the towns around them, has been noted regarding many maps, including those of Hirosaki, Akita, Sendai, Kakegawa, Matsuzaka, Ogaki, Tanba-Sasayama, Tsuyama, Hiroshima, Matsue, and Kokura [40,47–49]. For example, in the Shoho Castle Map of Hiroshima, the scale of the castle is 1200:1, whereas that of the town is 1800:1 [40]. This large difference in scale is clearly due to the mapmakers' intention to depict the castles on a larger scale. According to Takaya [50], the accuracy of the surveying undertaken to create Shoho Castle Maps was generally high, except for maps depicting mountainous areas. Thus, distortions in historical maps occur at various stages of mapmaking; however, in the case of the Shoho Castle Maps, the most significant distortion is the mapmakers' intentional representation, except for areas depicting mountains.

There are two opposing interpretations of the mapmakers' intention in enlarging the castle. One interpretation is that the enlarged depiction of the castle occurred because the Shoho Castle Maps were created to depict military information in detail [40]. However, other interpretations argue that areas associated with the samurai were depicted as larger and areas inhabited by merchants and artisans as smaller, reflecting the social hierarchy of the time [41]. The question of whether the Shoho Castle Maps depict large castles because of their emphasis on describing military information in detail or because they reflect the social hierarchy is unresolved and requires further study.

In addition to the enlargement of the castle, other distortions in the Shoho Castle Maps have been identified. According to Yuasa [49], regarding the Shoho Castle Maps of Kakegawa, Ogaki, and Tsuyama, the scale differs between the east–west and north–south directions, which was established by measuring the distances between the two ends of the castle town. Yuasa [49] did not use quantitative methods, such as regression analysis, but simply measured the length. In addition, Yuasa [49] did not examine local distortions within the map. Therefore, it is unknown why these scales differ. Ideta [51], although he did not visualize the distortion, pointed out that with respect to the Shoho Castle Maps of Kokura and Matsue, some streets are drawn at different angles and parallel to other streets. However, the mapmakers' intention in drawing the streets in parallel is unclear.

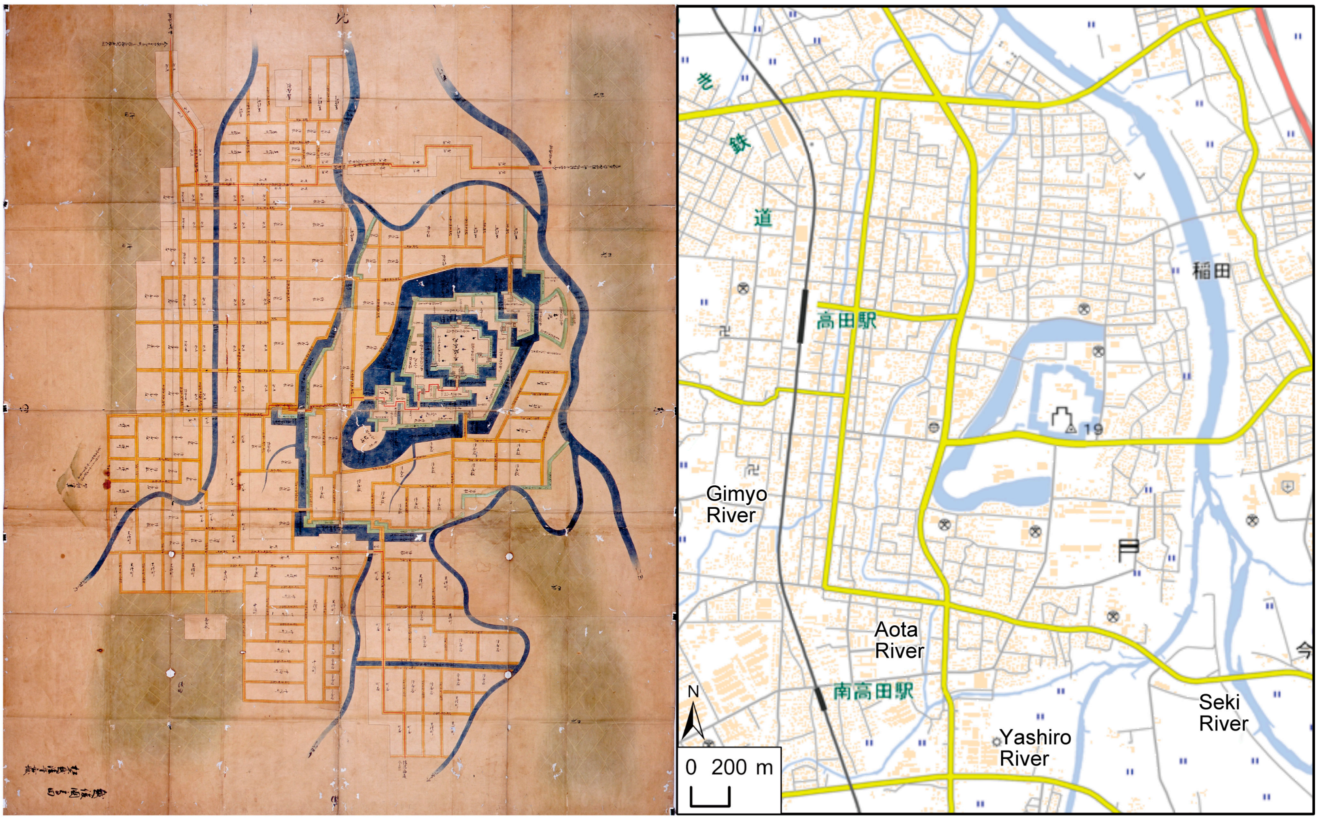
One reason for these unanswered questions regarding Shoho Castle Maps is that almost none of the existing studies of Shoho Castle Maps use quantitative methods to visualize the distortions, because most studies of Shoho Castle Maps were conducted before the widespread use of GIS. One exception is Goto [52], who analyzed the Shoho Castle Map of Sendai as an example of such distortion. However, the study simply compared the numerical lengths recorded on the Shoho Castle Map with the lengths on modern maps and examined only a limited number of streets. Goto [52] further demonstrated that the castle is represented on an enlarged scale, as other studies have pointed out. Yabe [53] visualized the distortions in the Takada Shoho Castle Map using quantitative methods, but did not use a local regression; as discussed later in the Methods section, only a global regression was used. Therefore, the local distortions were not accurately visualized.

As described above, in addition to enlarging the castles, some parts of the Shoho Castle Maps were distorted by the mapmakers' intentions. However, almost no research has been conducted to quantitatively visualize the distortions in Shoho Castle Maps. This study attempts to fill this research gap by visualizing the distortions of a Shoho Castle Map in order to understand the intention of mapmakers in creating the Shoho Castle Map.

2. Materials and Methods

2.1. Materials

This study analyzed the Shoho Castle Map depicting Takada Castle Town in Joetsu City, Niigata Prefecture, Japan (Figure 1). Within the Shoho Castle Map of Takada, the area inside the outer moat is defined as the castle and the area outside the outer moat as the castle town (Figure 2).



(a) Shoho Castle Map of Takada (b) Modern map of Takada

Figure 1. Historical and modern maps of Takada. Source: (**a**) "Takada Castle Town map (250 × 205 cm)" produced by Archives Center, Joetsu City; (**b**) "GSI Maps" produced by Geospatial Information Authority of Japan (GSI). Partly modified by the author.

Takada was selected from several Shoho Castle Maps for the following reasons: Yuasa [54] verified that the street length values noted in the Takada Shoho Castle Map are generally accurate and that the errors derived from surveying are small. This is because the surveying process was not conducted in areas prone to errors, as pointed out in previous studies [50]; that is, there are no high-elevation mountains in Takada, and its topography is flat. Therefore, we can proceed with our analysis of the Shoho Castle Map of Takada under the assumption that there were few errors in the mapping process in the survey. In addition, because the Shoho Castle Map is a large-scale map (1600:1 when measuring the length from the southern to northern ends of the castle town), we do not need to consider the differences in projection methods from modern maps [19,36]. Shoho Castle Maps were drawn manually on paper, and when examining the distortion of the map, it is necessary to consider whether there is any effect of paper expansion and contraction [55], for which it is necessary to assess whether the paper is coated with animal glue as a measure against paper expansion and contraction [56]. In the general procedure of map-making in the Edo period, paper was coated with animal glue as a base [57]. Shoho Castle Maps as official maps to be submitted to the Edo Shogunate are considered to have been coated with animal glue using the same general map-making procedure. The Takada Shoho Castle Map does not show any fine waving owing to the expansion and contraction of the paper. Therefore, the effects of paper expansion and contraction, if any, were considered small. The Takada Shoho Castle Map allows us to focus on the mapmakers' intentional representations as reflected in the distortions.

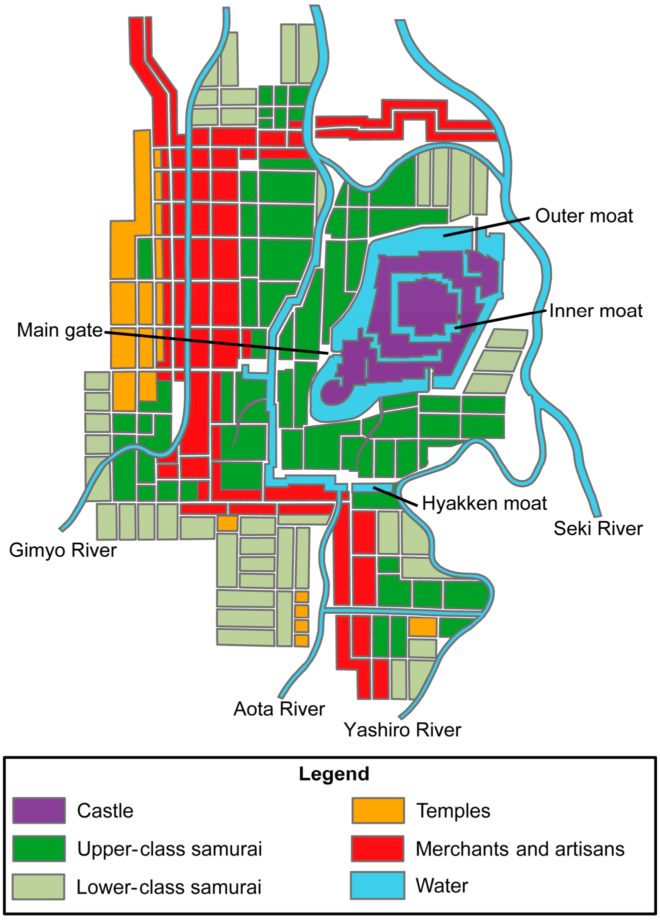


Figure 2. Trace drawing of the Shoho Castle Map of Takada.

Takada Castle Town was built in 1614, but it has not been confirmed whether any other maps were produced before the Shoho Castle Map was produced in 1644. Maps of Takada Castle Town were produced after 1644, at least in the late 17th, 18th, and mid-19th centuries. From the 17th century onward, these maps were created by feudal lords to manage the samurai houses in the castle town, and the names of the samurai were written on the houses. Even if the maps were made during the same Edo period, the purpose of these maps is different from that of the Shoho Castle Map submitted to the Edo Shogunate. The details of the surveying techniques used to produce large-scale city maps in 17th-century Japan are not known [55]. An 18th-century book on Japanese surveying techniques recommended the use of a surveying method similar to today's plane table surveys for measuring distances of approximately 12 km or less [58]. Later, in early 19th-century Japan, trigonometric functions were introduced to surveying, and machines with high accuracy were used [58,59]. Modern, triangulated, and accurate maps of Takada were not published until 1914.

In analyzing historical map distortion, the distortion that occurs when a modern map is superimposed on a historical map is analyzed by visualizing the distortion on the historical map. The GSI Maps produced by the Geospatial Information Authority of Japan (GSI) were used as the modern map to be superimposed on the historical map. The GSI Maps were projected onto a plane rectangular coordinate system (EPSG: 6676) in GIS and used as a modern map overlaid on the Shoho Castle Map (Figure 1).

When analyzing historical map distortions, it is necessary to establish control points at the corresponding points on both historical and modern maps. The following criteria were used to set the control points: (1) take control points as evenly as possible across the entire map; (2) use a variety of geographic features as control points; (3) use unambiguous objects as control points, such as intersections and fortifications; (4) do not use objects that change over time; and (5) use a large number of control points [36,60].

In this study, control points were set by comparing the Shoho Castle Map with GSI Maps for the following features: street intersections, corners of the castle, and bridges over the river. In setting the control points, their validity was verified through field surveys, comparisons with other historical maps of Takada from the 17th to 19th centuries, and discussions with researchers familiar with local history. Consequently, 115 control points were established. Although the streets depicted in the Takada Shoho Castle Map have generally retained their shapes compared to the modern map, there are areas on the outer edges of the castle town where the shape of the streets has changed, and control points cannot be established. One reason for changes in street shape is the impact of natural disasters. In the eastern and southern parts of the castle, a large river flows nearby, and the area is frequently damaged by flooding. In addition, Takada experienced major earthquakes during the Edo period, and street shapes changed during the reconstruction process following these natural disasters. Furthermore, Takada frequently changed feudal lords during the Edo period, thereby decreasing the number of samurai. Consequently, the number of samurai houses, specifically in the outer part of the castle town, decreased, and some areas fell into disuse, making it impossible to set control points. Moreover, when the railroad was constructed in the Meiji period, the streets on the west side of the castle town were erased where the railroad was laid. Therefore, there were locations in this area where control points could not be set because there were no corresponding intersections on the modern map, although they were depicted as intersections on the Shoho Castle Map.

2.2. Methods

The most commonly used quantitative method for analyzing historical map distortion is bidimensional regression [61–64]. Euclidean transformation (also called Hermart transformation or similarity transformation, depending on the field) and affine transformation are both representative methods included in bidimensional regression. There are two types of historical map distortion analyses, namely global and local bidimensional regressions [13], with global bidimensional regression being the most commonly used.

Displacement vectors, which are used to visualize local distortions, are derived from global bidimensional regression. The displacement vectors are the residuals after applying a global bidimensional regression using the control points of the entire map. In MapAnalyst software ver.1.3.34, a distortion grid, which is also often used to visualize local distortions, is created using the values of the displacement vector [19]. However, the use of displacement vectors has been criticized as not being directly indicative of the local distortion of the map, as it is the distortion that remains after removing the global distortion acting on the entire map [65–67]. Local bidimensional regression can be used instead of global bidimensional regression to accurately visualize local distortions [60,66,67].

MapAnalyst implements a local bidimensional regression to visualize local distortions [18]. Local bidimensional regression is performed using only control points within a certain bandwidth from a point, rather than using control points from the entire map, to reveal distortions at that point. However, one drawback of the MapAnalyst method is that the researcher must set an arbitrary bandwidth [66,67]. If an appropriate bandwidth corresponding to the map distortion is not set, important distortions may be missed.

Therefore, in this study, we apply a new method, geographically weighted bidimensional regression (GWBR), which is an extension of one of the local regression methods, geographically weighted regression (GWR) [68]. For more information on GWBR, see Appendix A.

GWR is a method for analyzing spatial heterogeneity that can determine whether different processes are at work in an area of interest, which has been applied in various fields [69,70]. Distortions, such as the enlargement of certain parts of a map by mapmakers, can be described as spatial heterogeneity, in which each part of the map is drawn differently. A study that visualized local distortions on a historical map of Spain noted that distortions were distributed heterogeneously in space [71]. However, researchers have not applied GWR to bidimensional regression despite the advantage of applying GWR to bidimensional regression in that it can automatically set an appropriate bandwidth independent of the subjectivity of the researcher. GWR and GWBR set the bandwidth that minimizes the corrected Akaike information criterion (AICc), a modified Akaike information criterion (AIC). In other words, GWBR sets the bandwidth corresponding to the scale that best explains the map distortion, and important distortions can be extracted and visualized. In the Takada Shoho Castle Map, intentional distortion introduced by the mapmaker is considered the most significant distortion, and the bandwidth setting by GWBR can capture the scale at which this distortion is at play. However, control points could not be set on the east and south sides of the castle, particularly at the outer edge of the castle town. Therefore, the bandwidth obtained by the AICc is based on the control points that could be set. The limitation of this method is that it cannot analyze distortion in areas where control points could not be set.

The GWBR used in this study employs an affine transformation instead of a Euclidean transformation, as the former can capture scaling and shearing on the x- and y-axes separately (Figure 3) and thus identify complex distortions such as enlargement in the x-axis direction and shrinkage in the y-axis direction. Notably, the affine transformation used did not consider translations but only scaling and shearing. Translation is a distortion related to geodetic accuracy [3,36] and could be considered if a coordinate system had been set in the Shoho Castle Map. However, the Shoho Castle Map does not contain information indicating its location on Earth. Therefore, translations related to geodetic accuracy were not considered in this study, which solely analyzed planimetric accuracy [3,36].

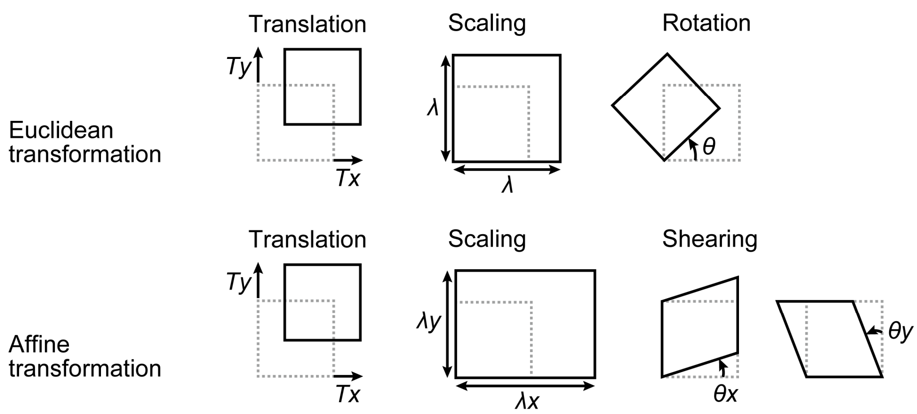


Figure 3. Distortions in Euclidean and affine transformations.

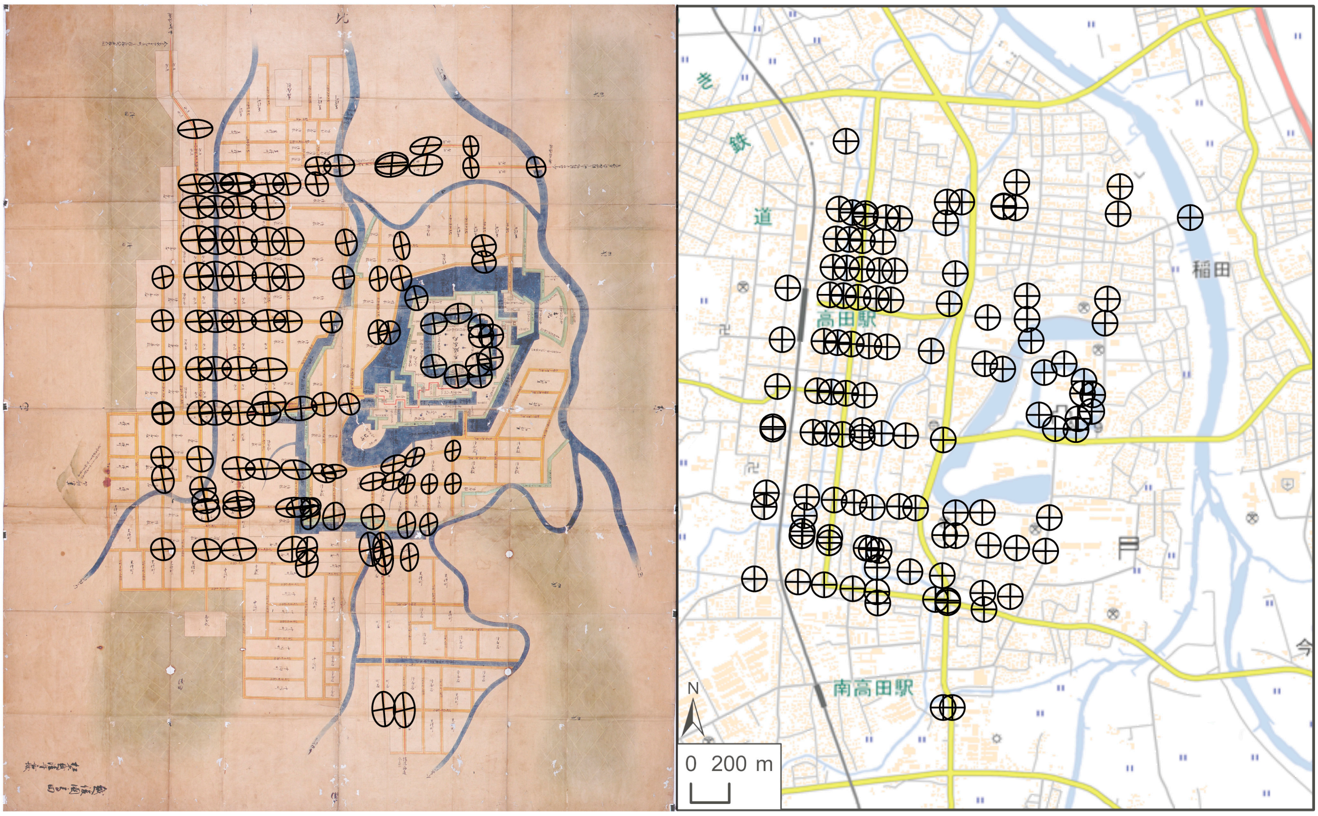
GWBR can also set points for regression at locations other than those where the control points are set, which allows points for regression to be set in the form of a grid and distortions at points where there are no control points to be interpolated. However, because the Shoho Castle Map of Takada has parts where the streets differ from the modern map, it was not possible to set control points throughout the map. Under these circumstances, regression with a grid of points lacks reliability for distortions in parts without control points. Therefore, in this study, the regression was conducted only at the control points.

3. Results

3.1. Visualization of Local Distortions

GWBR was applied to 115 control points, resulting in a bandwidth of 11 control points. Important distortions occurred at the scale of the 11 adjacent control points.

Local distortions were visualized in two ways. The first was to show the Tissot's indicatrices at the control points. This method shows perfect circles of the same size and angle over all the control points on the modern map (Figure 4), and the projections of these circles are drawn in a shape that reflects the resulting distortion when the modern map is overlaid on the historical map. Another visualization method shows the scaling and shearing above the control points of the historical map (Figure 5). For scaling, the x- and y-axes are shown separately. Because the range of the class intervals was set to the same value, the scaling of the x- and y-axes can be compared. The x- and y-axes for the shearing are shown in a single figure. If the shearing along the x- and y-axes shows almost the same angle, it can be interpreted as rotation rather than shearing. To highlight the locations where shearing, but not rotation, occurred, the differences between the shear angles along the x- and y-axes were mapped (Figure 6). A large clockwise angle indicates the presence of x-axis shearing, whereas a large counterclockwise angle indicates the presence of y-axis shearing.



(a) Shoho Castle Map of Takada (b) Modern map of Takada

Figure 4. Tissot's indicatrices at 115 control points. The background maps are displayed with 70% transparency. Source: (**a**) "Takada Castle Town map (250 × 205 cm)" produced by Archives Center, Joetsu City. (**b**) "GSI Maps" produced by Geospatial Information Authority of Japan (GSI). Partly modified by the author.

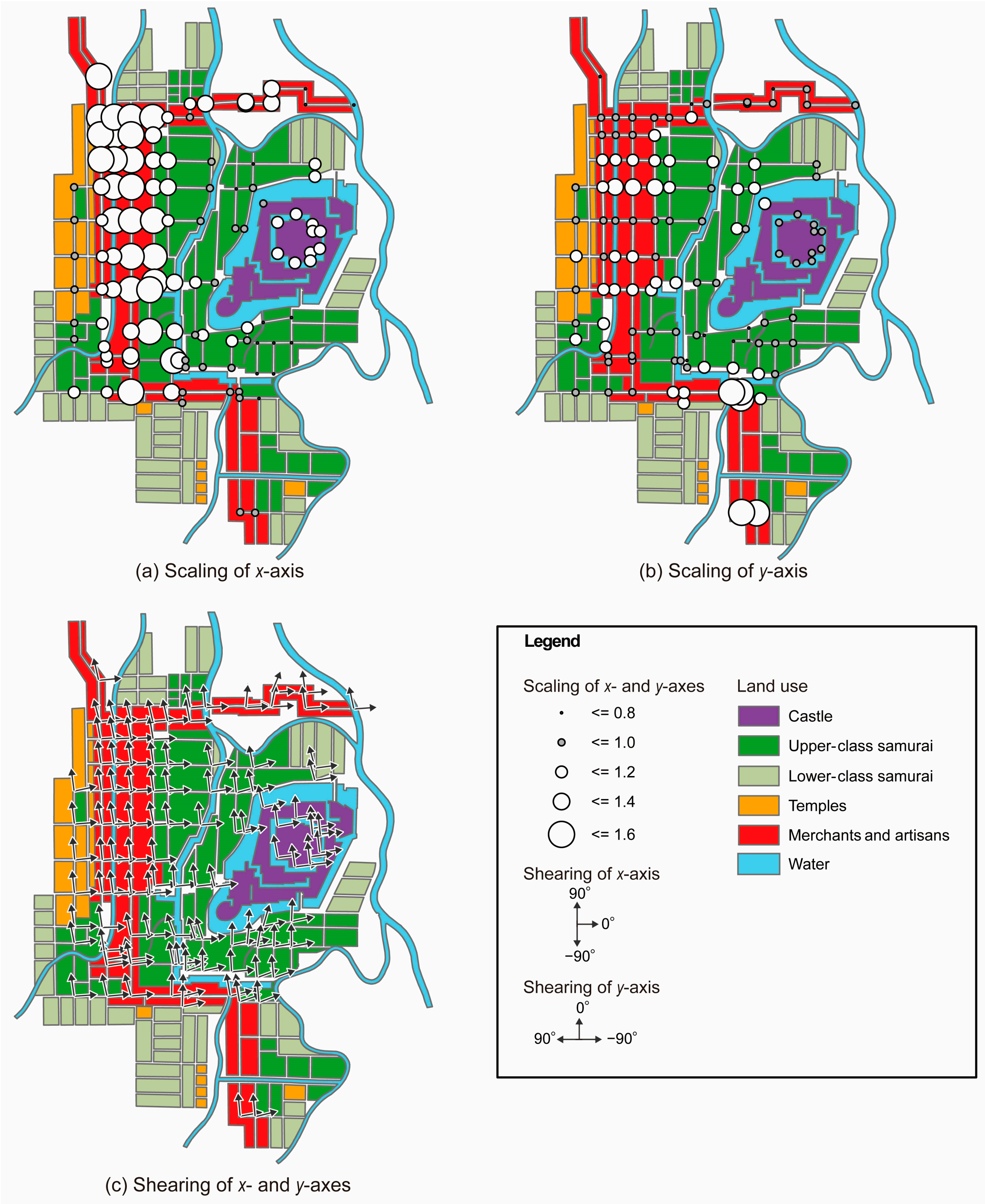


Figure 5. Local distortions at 115 control points in the Shoho Castle Map of Takada.

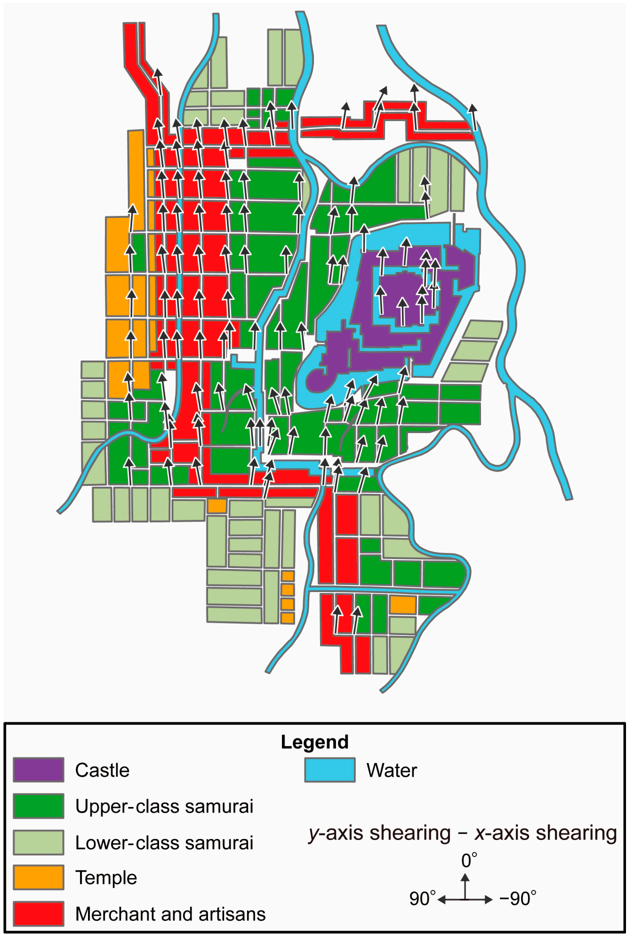


Figure 6. Differences between the shear angles along the x- and y-axes.

3.2. Scaling

Along the x-axis, the merchant/artisan housing areas in the castle town were enlarged by a factor of 1.5. These enlarged areas stand out because they are contiguous in the north–south direction. However, the enlarged areas were concentrated between the Gimyo and Aota Rivers, and the merchant/artisan areas in the southern part of the castle town were not enlarged. The castle inside the inner moat was also enlarged by a factor of approximately 1.2, which is more modest than the expansion of the merchant/artisan areas between the Gimyo and Aota Rivers. In addition, the western part of the convex-shaped merchant/artisan area north of the castle was enlarged by a factor of approximately 1.4, whereas its eastern part was shrunk by a factor of approximately 0.7. Consequently, the convex-shaped merchant/artisan area was drawn to the east.

For the points depicted as shrunken along the x-axis, shrinkage was noticeable south of the outer moat. The size of this area is reduced by a factor of approximately 0.6. Shrinking is also noticeable to the west of the castle, between the outer moat and Aota River. The temple area, which is the westernmost part of the castle town, is also depicted as shrunken.

The most obvious enlargement in the y-axis direction occurred around the Hyakken moat and at the southern end of the castle town. The area around the Hyakken moat was enlarged by a factor of approximately 1.4, and the southern end of the castle town by a factor of approximately 1.5. In contrast, the most obvious shrinkage occurred at the northern end of the castle town and southern part of the outer moat. The northern end of the castle town was shrunk by a factor of approximately 0.8, and the southern area of the outer moat by a factor of approximately 0.7.

Other minor shrinkage along the y-axis was observed in the inner moat. As a result, in addition to the enlargement in the x-axis direction, the inner moat was depicted as a slightly horizontal rectangle on the Shoho Castle Map, whereas it was actually a slightly vertical rectangle.

To summarize the distortions identified so far, the ratios of the lengths from the center of the maps are shown (Figure 7). In the Takada Shoho Castle Map, the main gate is almost at the center of the map (Figure 8), and the ratio of the west side to the east side of the main gate is 1.0:0.9 along the x-axis (Figure 7). However, the ratio of the west side to the east side of the main gate in the x-axis direction in the GSI Maps is 1.0:1.1, and the east side is longer than in the Shoho Castle Map (Figure 7). This is because the area between the Gimyo and Aota Rivers is drawn with a large expansion in the Shoho Castle Map (Figure 5). However, if the area between the Gimyo and Aota Rivers were enlarged, the main gate would be far from the center of the map. To avoid this, the areas between the Aota River and the outer moat and the temple area at the western end of the castle town were shrunk to keep the main gate at the center of the map (Figure 5). The ratios of the lengths between the Gimyo and Aota Rivers to that between the Aota River and the outer moat are 1.0:0.8 in the Shoho Castle Map and 1.0:1.5 in the GSI Maps (Figure 7). The ratios of the lengths between the Gimyo and Aota Rivers to that between the Gimyo River and the western edge of the castle town are 1.0:1.4 in the Shoho Castle Map and 1.0:1.7 in the GSI Maps (Figure 7). The layout was thus designed to keep the main gate at the center of the map while widening the distance between the Gimyo and Aota Rivers.

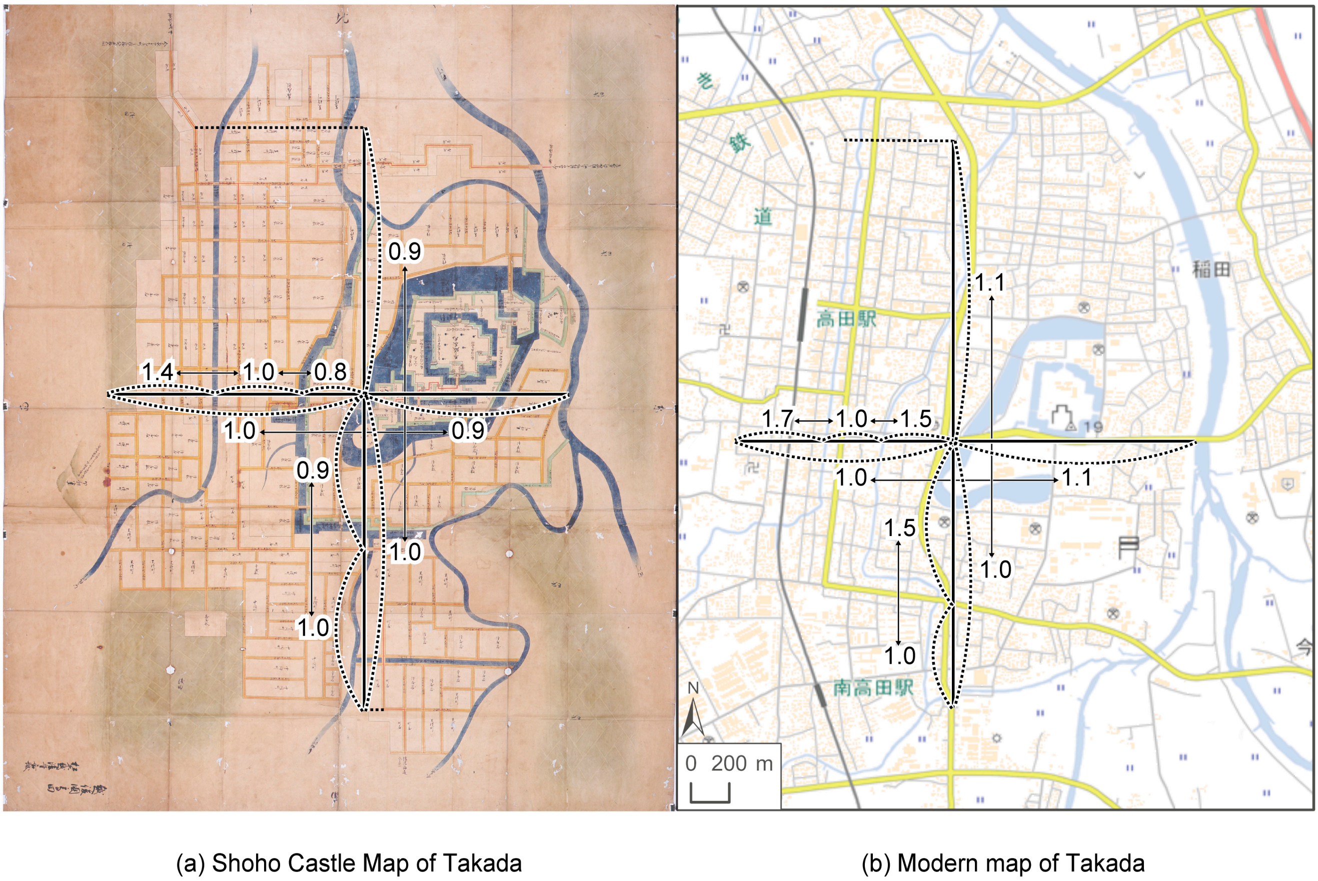
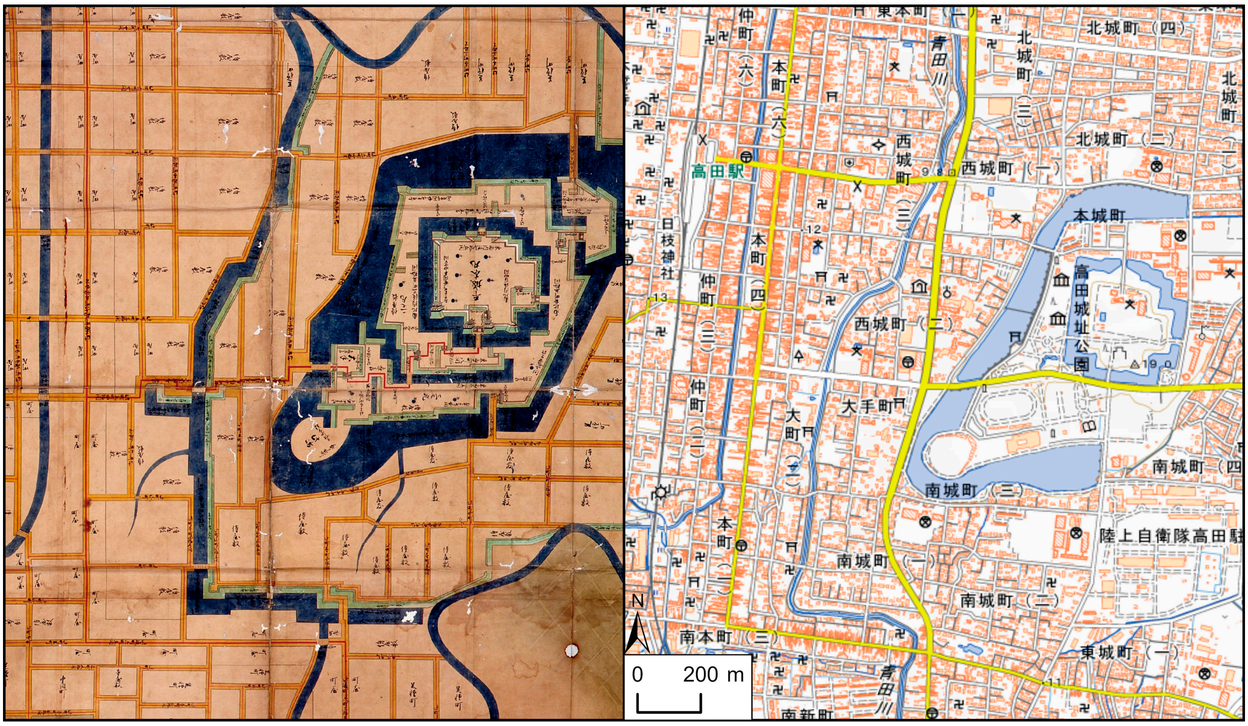


Figure 7. Ratios of the lengths of line segments from the center (main gate) of the maps. The numbers connected by arrows indicate ratios calculated with respect to the line segment with a value of 1.0. The line segments connect the corresponding edges on the two maps. The background maps are displayed with 70% transparency. Source: (**a**) "Takada Castle Town map (250 × 205 cm)" produced by Archives Center, Joetsu City. (**b**) "GSI Maps" produced by Geospatial Information Authority of Japan (GSI). Partly modified by the author.



(a) Shoho Castle Map of Takada (b) Modern map of Takada

Figure 8. Magnified view of the area around the main gate of historical and modern maps of Takada. Source: (**a**) "Takada Castle Town map (250 × 205 cm)" produced by Archives Center, Joetsu City; (**b**) "GSI Maps" produced by Geospatial Information Authority of Japan (GSI). Partly modified by the author.

In the y-axis direction of the Shoho Castle Map, the main gate is located near the center. The ratio of the length south of the main gate to that to the north is 1.0:0.9 (Figure 7). However, in the y-axis direction of the GSI Maps, the ratio of the length south of the main gate to that to the north is 1.0:1.1, with the north side longer than in the Shoho Castle Map (Figure 7). In the Shoho Castle Map, the northern end of the castle town is shrunk, and the southern end is enlarged (Figure 5). The ratios of the lengths between the southern end of the castle town and the Hyakken moat to that between the Hyakken moat and the main gate are 1.0:0.9 in the Shoho Castle Map and 1.0:1.5 in the GSI Maps (Figure 7). If the southern end of the castle town had simply been enlarged, the main gate would be off-center in the y-axis direction. To avoid this, the area between the main gate and the Hyakken moat was shrunk (Figure 5). This operation keeps the main gate at the center of the map.

3.3. Shearing

The trend for shearing was common to many control points throughout the map and is therefore described first. At many control points, the x- and y-axis shearing exhibited counterclockwise angles of approximately 4°. If the x- and y-axis shearing show the same angle, they are not shearing but rotation. As can be observed on the modern map, throughout the castle town of Takada, the streets in the north–south direction are tilted clockwise by approximately 4° from true north. However, in the Shoho Castle Map, the north–south streets are depicted as extending almost true north, resulting in a counterclockwise rotation of the entire map.

This counterclockwise rotation throughout the map is attributed to geomagnetic declination. Takada Castle Town was constructed in 1614, and the magnetic north near Japan was shifted clockwise by approximately 4° from true north [72]. The streets were constructed based on magnetic north, and the entire castle town of Takada was thus built with a clockwise rotation. Around 1644, when the Shoho Castle Map was drawn, not much time had passed since its construction, and the geomagnetic declination had not changed significantly [72]. Therefore, when the map was surveyed based on magnetic north, streets in the north–south direction pointed almost true north. Therefore, a counterclockwise rotation is required to superimpose the modern map.

Although a counterclockwise rotation acts on the entire map, local shearing occurred in addition to the overall trend. Shearing was considered when there was a difference between the shear angles along the x- and y-axes. Focusing on the area south of the outer moat, there was y-axis shearing on the west side of the area and x-axis shearing on the east side. In the merchant/artisan area north of the castle, x-axis shearing is observed.

The shearing observed in the southern part of the outer moat, particularly in the eastern part, occurred because this part was drawn with shrinkage along the y-axis. As this area was drawn at a reduced scale, it was necessary to draw the street, which originally stretched horizontally east–west, at a tilted angle, causing shearing. Shearing was also observed in the southwestern part of the outer moat and the convex merchant/artisan area north of the castle. This shearing can be attributed to the drawing of the tilted streets as orthogonalized. The Takada Shoho Castle Map tends to avoid depicting the tilted streets completely and depicts streets in the east–west and north–south directions as orthogonal to each other. Ideta [51] noted that some parts of the Shoho Castle Maps in Matsue and Kokura were rotated to draw streets parallel to each other. The shearing of streets observed in this study was similar to the findings of Ideta [51]. The mapmakers' intention behind this is thought to have been to draw the east–west and north–south streets as orthogonally as possible.

4. Discussion

4.1. Composition of the Takada Shoho Castle Map

Regarding scaling, new facts were found in addition to those pointed out in existing studies. As pointed out in previous studies of other Shoho Castle Maps [40,47–49], the castle, specifically inside the inner moat, is enlarged on the Takada Shoho Castle Map. However, the castle was not simply enlarged, but rather the interior of the inner moat was shaped into a slightly horizontal rectangle.

A newly discovered fact not noted in existing studies is that the castle town is depicted at a larger scale than the castle. This is particularly evident in the x-axis expansion of the area between the Gimyo and Aota Rivers. However, not all the castle town has been enlarged; rather, the temple area at the western end of the castle town has shrunk. This indicates that the mapmakers' intention was not simply to depict the entire castle town as enlarged.

In contrast to the new findings of this study and earlier results, the following points can be made: The view that the Shoho Castle Maps depict the samurai part as larger and the merchant/artisan part as smaller, reflecting the social hierarchy [41], does not apply, at least to the Shoho Castle Map of Takada, as some parts of the map depict the castle town as more enlarged than the castle. Although the intention of enlarging the castle to depict military facilities in detail [40] has been confirmed, another intention might have lain behind the enlarged depiction of the castle town.

What other mapmakers' intentions, then, could there have been for the map other than enlarging military facilities? One possible intention was to set the main gate at the center of the map to create a composition with a well-balanced map layout. In the Takada Shoho Castle map, the castle town area is evenly drawn from the center. The southern end of the castle town was drawn enlarged, apparently so that the area of the castle town would be distributed as evenly as possible on the north and south sides of the main gate. The actual Takada castle town has a smaller area south of the main gate, especially its southern end, than the area north of the gate; therefore, if the main gate were depicted as the center of the town, the town would be unbalanced. By enlarging the southern end of the castle town, the composition was balanced between the southern and northern areas centered on the main gate. This pictorial intention emphasizes the composition of the map, as in Leonardo da Vinci's plan for Imola [35]. The composition was limited by the size of the paper. The mapmakers also considered the need to depict the entire castle town within a limited paper size and reduce blank spaces.

Regarding the Shoho Castle Map of Takada, its composition, which emphasizes balance, seems to be related to the fact that it was submitted to the Edo Shogunate. Comparing the Shoho Castle Map with other Takada castle town maps of the Edo period, only the Shoho Castle Map is clearly different in composition. The Shoho Castle Maps were drawn to show the castle ruled by the shogun to the shogun. It may be that the Shoho Castle Maps were made with the shogun in mind, as the castles ruled by the shogun are beautiful and well-balanced. Because maps and paintings were undifferentiated during this period [38], it is possible to discuss the compositions related to paintings, which are a characteristic of the maps of this period.

Few studies have visualized local map distortions and discussed them in relation to their composition. One of the reasons for the lack of such studies is that local distortions have not been accurately visualized. Studies on the distortion of large-scale maps before the 18th century did not visualize the distortion using local regression [20,36,37]. In these studies, displacement vectors were used to visualize local distortions, but this interpretation is not easy. One reason is that displacement vectors show the distortions that remain after global distortions are removed [65–67]. Therefore, displacement vectors could not represent local distortions accurately. Another reason is that distortions, such as scaling and shearing, are all mixed in the displacement vectors. The GWBR used in this study can visualize local distortions separately for scaling and shearing, so interpretation is straightforward. MapAnalyst, which also implements the local regression method, visualizes the results as contour lines [18]. Contour lines are not always easy to use when interpreting the results, especially regarding the rotation angle. In addition, MapAnalyst contours cannot be displayed separately on the x- and y-axes. Therefore, it is not possible to visualize complex distortions such as the one shown in this study, where the x-axis is enlarged while the y-axis is shrunk.

The use of GWBR to visualize local distortions has allowed us to discuss the cartographer's intention to focus on composition, which is an achievement of this study. Yuasa [49] pointed out the difference in scale between the east–west and north–south directions for the Shoho Castle Maps of Kakegawa, Ogaki, and Tsuyama, but the factors that caused this difference were not clear. By visualizing and analyzing the local distortions, it is possible to examine the factors behind this difference in scale. In other words, there is a possibility that the scale of the maps differs between the east–west and north–south directions because the maps were locally distorted in consideration of the overall composition.

4.2. Limitations and Future Plan

The limitations of this study are as follows. This study focused on the Shoho Castle Map of Takada; however, there are more than 60 extant Shoho Castle Maps. By visualizing the local distortions of the other Shoho Castle Maps, the common characteristics of the Shoho Castle Maps could be identified. Testing the hypothesis that there is a mapmakers' intention to improve the composition is a future task. GWR and GWBR assume that geographic processes are smooth and continuous. However, this assumption may not apply to all the local distortions in historical maps. Future studies should apply local regression methods that allow for spatial discontinuities. In addition, it was not possible to determine whether the style of drawing large-scale maps identified in this study was influenced by foreign countries, such as European ones. Regarding surveying techniques, it has been pointed out that European surveying techniques were introduced to Japan during the Edo period [73,74]. Conversely, regarding the style of map drawing, Sugimoto [75] adduced the influence of China on small-scale maps of the Edo period; however, there are unknown points regarding the Shoho Castle Maps. For example, the Kano School's style of painting, which emphasized balance [76], might have influenced the style of map drawing. It is unclear whether the map-drawing style of the Kano School was influenced by foreign countries. There is room for research on drawing large-scale maps, including pictorial intent and the types of foreign influence that they might or might not have experienced.

5. Conclusions

In this study, local distortions were visualized for the Shoho Castle Map of Takada using GWBR, which, unlike the method implemented in MapAnalyst, allowed us to automatically set the bandwidth corresponding to significant distortions. It was found that parts of the castle town were enlarged, which has not been noted in previous studies. After considering the results together with other local distortions, the mapmakers' intention in drawing the Shoho Castle Map appeared to have been to set the composition of the entire map based on the center, such as the main gate, and to draw the area of the map extending evenly from the center. Thus, the Takada Shoho Castle Map emphasizes balance in composition. The visualization of local distortions via GWBR has made it possible to discuss the composition of the historical map, which is the achievement of this study.

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Data Availability Statement: The MATLAB codes of GWBR are available in FigShare at: https://doi.org/10.6084/m9.figshare.24525007 (accessed on 5 April 2024).

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Appendix A

This section describes geographically weighted bidimensional regression (GWBR), an extension of bidimensional regression to geographically weighted regression (GWR). Before explaining GWBR, it is important to understand global bidimensional regression.

Appendix A.1. Global Bidimensional Regression

Affine transformation comprises commonly used transformations in bidimensional regression, as shown below.

$$\begin{pmatrix} u_i \\ v_i \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} + \begin{pmatrix} \beta_1 & \beta_2 \\ \beta_3 & \beta_4 \end{pmatrix} \begin{pmatrix} x_i \\ y_i \end{pmatrix} + \begin{pmatrix} \varepsilon_i \\ \eta_i \end{pmatrix} \quad \text{(A1)}$$

To perform bidimensional regression, it is necessary to find a common point between the historical map and the modern map and set it as a control point. u and v are the coordinates of the control point set on the historical map, and x and y are the coordinates of the control point set on the modern map. α and β are parameters and ε and η are error terms. i represents each control point and takes values from 1 to n, where n is the number of control points. The formula is expressed in matrix form as follows.

$$Y = Da + \varepsilon \quad \text{(A2)}$$

Y is a vector summarizing the coordinates of the control points on the historical map. D is the design matrix and a is the vector of the parameters. Y is as follows.

$$Y = (u_1, \ldots, u_n, v_1, \ldots, v_n)^T \quad \text{(A3)}$$

D and a are shown below.

$$D = \begin{pmatrix} 1 & 0 & x_1 & y_1 & 0 & 0 \\ & & & \cdots & & \\ 1 & 0 & x_n & y_n & 0 & 0 \\ 0 & 1 & 0 & 0 & y_1 & x_1 \\ & & & \cdots & & \\ 0 & 1 & 0 & 0 & y_n & x_n \end{pmatrix} \tag{A4}$$

$$a = (\alpha_1 \ \ \alpha_2 \ \ \beta_1 \ \ \beta_2 \ \ \beta_3 \ \ \beta_4)^T \tag{A5}$$

The parameter a can be obtained by the least squares method as follows.

$$\hat{a} = \left(D^T D\right)^{-1} D^T Y \tag{A6}$$

From the obtained parameters, the distortions of the historical map, such as translation, scaling, and shearing are calculated as follows.

$$\begin{gathered}\text{Translation of } x\text{-axis} \\ T_x = \alpha_1\end{gathered} \tag{A7}$$

$$\begin{gathered}\text{Translation of } y\text{-axis} \\ T_y = \alpha_2\end{gathered} \tag{A8}$$

$$\begin{gathered}x\text{-axis scaling} \\ \lambda_x = \sqrt{{\beta_1}^2 + {\beta_3}^2}\end{gathered} \tag{A9}$$

$$\begin{gathered}y\text{-axis scaling} \\ \lambda_y = \sqrt{{\beta_2}^2 + {\beta_4}^2}\end{gathered} \tag{A10}$$

$$\begin{gathered}x\text{-axis shearing} \\ \theta_x = \tan^{-1}(\beta_3/\beta_1)\end{gathered} \tag{A11}$$

$$\begin{gathered}y\text{-axis shearing} \\ \theta_y = -\tan^{-1}(\beta_2/\beta_4)\end{gathered} \tag{A12}$$

Appendix A.2. Geographically Weighted Bidimensional Regression (GWBR)

GWBR, an extension of bidimensional regression to GWR, is introduced as follows.

$$\begin{pmatrix} u_i \\ v_i \end{pmatrix} = \begin{pmatrix} \alpha_{1i} \\ \alpha_{2i} \end{pmatrix} + \begin{pmatrix} \beta_{1i} & \beta_{2i} \\ \beta_{3i} & \beta_{4i} \end{pmatrix} \begin{pmatrix} x_i \\ y_i \end{pmatrix} + \begin{pmatrix} \varepsilon_i \\ \eta_i \end{pmatrix} \tag{A13}$$

Compared to global regression, the GWBR parameters vary with each control point i. The above equation can be expressed in matrix form as follows.

$$Y = (a \otimes D)1 + \varepsilon \tag{A14}$$

where $\otimes$ is a logical multiplication operator where each element of a is multiplied by the corresponding element of D. D is the design matrix and a is the matrix of parameters. a is different from the global regression, as shown below.

$$a = \begin{pmatrix} \alpha_{11} & \alpha_{21} & \beta_{11} & \beta_{21} & \beta_{31} & \beta_{41} \\ & & \cdots & & & \\ \alpha_{1n} & \alpha_{2n} & \beta_{1n} & \beta_{2n} & \beta_{3n} & \beta_{4n} \\ \alpha_{11} & \alpha_{21} & \beta_{11} & \beta_{21} & \beta_{31} & \beta_{41} \\ & & \cdots & & & \\ \alpha_{1n} & \alpha_{2n} & \beta_{1n} & \beta_{2n} & \beta_{3n} & \beta_{4n} \end{pmatrix} \tag{A15}$$

Each row of parameter a can be obtained as follows.

$$\hat{a}_i = \left(D^T w_i D\right)^{-1} D^T w_i Y \tag{A16}$$

The weight matrix w_i is shown below.

$$w_i = \begin{pmatrix} w_{i1} & 0 & \ldots & 0 \\ 0 & w_{i2} & \ldots & 0 \\ & & \ldots & \\ 0 & 0 & \ldots & w_{in} \\ w_{i1} & 0 & \ldots & 0 \\ 0 & w_{i2} & \ldots & 0 \\ & & \ldots & \\ 0 & 0 & \ldots & w_{in} \end{pmatrix} \tag{A17}$$

In this article, the bi-square function [68] was used as weight.

$$\begin{gathered} w_{ij} = \left[1 - \left(d_{ij}/b\right)^2\right]^2 \text{ if } d_{ij} < b \\ w_{ij} = 0 \ \text{ otherwise} \end{gathered} \tag{A18}$$

Modern map distances were used to calculate bandwidth b and the distance between control points d_{ij}. The number of control points used in the analysis was fixed and the bandwidth was adaptive. The optimal number of control points was determined by using the AICc [68].

$$\text{AICc} = 4n \log_e(\hat{\sigma}) + 2n \log_e(2\pi) + n\left\{\frac{2n + \text{tr}(S)}{2n - 2 - \text{tr}(S)}\right\} \tag{A19}$$

S is the hat matrix, which transforms the objective variable into its estimated value.

$$\hat{Y} = SY \tag{A20}$$

MATLAB R2022b was used for the calculations. The code used in the calculations was a heavily modified and appended version of the MATLAB code found in the supplementary material of Wu et al. [77].

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