

Supplementary Materials

(1) Proof of the equilibrium results in Model NN

$$\pi^{NN} = (p^{NN} - w_n)(1 - p^{NN})$$

Since $\frac{\partial^2 \pi^{NN}}{\partial (p^{NN})^2} = -2 < 0$, π^{NN} is concave in p^{NN} .

Set $\frac{\partial \pi^{NN}}{\partial p^{NN}} = 0$, it derives that $\frac{\partial \pi^{NN}}{\partial p^{NN}} = (1 - p^{NN}) - (p^{NN} - w_n) = 0$. Accordingly, $p^{NN} = \frac{1 + w_n}{2}$,

$$q^{NN} = 1 - p^{NN} = \frac{1 - w_n}{2}.$$

$$\pi^{NN} = (p^{NN} - w_n)(1 - p^{NN}) = \frac{(1 - w_n)^2}{4}$$

Since $q^{NN} = \frac{1 - w_n}{2} \geq 0$, $w_n \leq 1$ is derived.

(2) Proof of Proposition 1

$$\frac{\partial p^{NN}}{\partial w_n} = \frac{1}{2} > 0;$$

$$\frac{\partial q^{NN}}{\partial w_n} = \frac{-1}{2} < 0;$$

$$\frac{\partial \pi^{NN}}{\partial w_n} = -\frac{(1 - w_n)}{2} < 0.$$

(3) Proof of the equilibrium results in Model NH

$$\pi^{NH} = (p^{NH} - w_n - \alpha_n p^{NH} - c_s)(1 + s - p^{NH})$$

Since $\frac{\partial^2 \pi^{NH}}{\partial (p^{NH})^2} = -2(1 - \alpha_n) < 0$, π^{NH} is concave in p^{NH} .

Set $\frac{\partial \pi^{NH}}{\partial p^{NH}} = 0$, it derives that $\frac{\partial \pi^{NH}}{\partial p^{NH}} = (1 - \alpha_n)(1 + s - p^{NH}) - (p^{NH} - w_n - \alpha_n p^{NH} - c_s) = 0$. Accordingly,

$$p^{NH} = \frac{(1 - \alpha_n)(1 + s) + w_n + c_s}{2(1 - \alpha_n)}, \quad q^{NH} = 1 + s - p^{NH} = \frac{(1 - \alpha_n)(1 + s) - w_n - c_s}{2(1 - \alpha_n)}.$$

$$\pi^{NH} = (p^{NH} - w_n - \alpha_n p^{NH} - c_s)(1 + s - p^{NH}) = \frac{((1 - \alpha_n)(1 + s) - w_n - c_s)^2}{4(1 - \alpha_n)}$$

Since $q^{NH} = \frac{(1 - \alpha_n)(1 + s) - w_n - c_s}{2(1 - \alpha_n)} \geq 0$, $w_n \leq (1 - \alpha_n)(1 + s) - c_s$ is derived.

(4) Proof of Proposition 2

$$\frac{\partial p^{NH}}{\partial s} = \frac{1}{2} > 0;$$

$$\frac{\partial p^{NH}}{\partial c_s} = \frac{1}{2(1 - \alpha_n)} > 0;$$

$$\frac{\partial p^{NH}}{\partial w_n} = \frac{1}{2(1 - \alpha_n)} > 0;$$

$$\frac{\partial p^{NH}}{\partial \alpha_n} = \frac{2(w_n + c_s)}{4(1 - \alpha_n)^2} > 0.$$

$$\frac{\partial q^{NH}}{\partial s} = \frac{1}{2} > 0;$$

$$\frac{\partial q^{NH}}{\partial c_s} = \frac{-1}{2(1 - \alpha_n)} < 0;$$

$$\frac{\partial q^{NH}}{\partial w_n} = \frac{-1}{2(1 - \alpha_n)} < 0;$$

$$\frac{\partial q^{NH}}{\partial \alpha_n} = \frac{-2(w_n + c_s)}{4(1 - \alpha_n)^2} < 0.$$

$$\frac{\partial \pi^{NH}}{\partial s} = \frac{(1 - \alpha_n)^2(2 + 2s) - 2(1 - \alpha_n)(w_n + c_s)}{4(1 - \alpha_n)} = \frac{(1 - \alpha_n)(1 + s) - (w_n + c_s)}{2} = \frac{q^{NH}}{(1 - \alpha_n)} > 0;$$

$$\frac{\partial \pi^{NH}}{\partial c_s} = \frac{-(1 - \alpha_n)(1 + s) + (w_n + c_s)}{2(1 - \alpha_n)} = -q^{NH} < 0;$$

$$\frac{\partial \pi^{NH}}{\partial w_n} = \frac{-(1 - \alpha_n)(1 + s) + (w_n + c_s)}{2(1 - \alpha_n)} = -q^{NH} < 0;$$

Since

$$q^{NH} = \frac{(1-\alpha_n)(1+s)-w_n-c_s}{2(1-\alpha_n)} > 0$$

$$\frac{\partial \pi^{NH}}{\partial \alpha_n} = \frac{(w_n+c_s)^2-4(1-\alpha_n)^2(1+s)^2}{4(1-\alpha_n)^2} = -\frac{(2(1-\alpha_n)(1+s)+w_n+c_s)(2(1-\alpha_n)(1+s)-w_n-c_s)}{4(1-\alpha_n)^2} < 0.$$

(5) Proof of the equilibrium results in Model RN

$$\pi^{RN} = (p^{RN} - w_r) \left(\frac{\delta - p^{RN}}{\delta} \right)$$

Since $\frac{\partial^2 \pi^{RN}}{\partial (p^{RN})^2} = -\frac{2}{\delta} < 0$, π^{RN} is concave in p^{RN} .

Set $\frac{\partial \pi^{RN}}{\partial p^{RN}} = 0$, it derives that $\frac{\partial \pi^{RN}}{\partial p^{RN}} = \left(\frac{\delta - p^{RN}}{\delta} \right) - \frac{1}{\delta} (p^{RN} - w_r) = 0$. Accordingly, $p^{RN} = \frac{\delta + w_r}{2}$.

$$q^{RN} = \frac{\delta - p^{RN}}{\delta} = \frac{\delta - w_r}{2\delta}.$$

$$\pi^{RN} = (p^{RN} - w_r) \left(\frac{\delta - p^{RN}}{\delta} \right) = \frac{(\delta - w_r)^2}{4\delta}$$

Since $q^{RN} = \frac{\delta - w_r}{2\delta} \geq 0$, $w_r \leq \delta$ is derived.

(6) Proof of Proposition 3

$$\frac{\partial p^{RN}}{\partial \delta} = \frac{\partial p^{RN}}{\partial w_r} = \frac{1}{2} > 0;$$

$$\frac{\partial q^{RN}}{\partial w_r} = \frac{-1}{2\delta} < 0.$$

$$\frac{\partial q^{RN}}{\partial \delta} = \frac{w_r}{2\delta^2} > 0.$$

$$\frac{\partial \pi^{RN}}{\partial w_r} = -\frac{\delta - w_r}{2\delta} < 0;$$

$$\frac{\partial \pi^{RN}}{\partial \delta} = \frac{(\delta + w_r)(\delta - w_r)}{4\delta^2} > 0.$$

(7) Proof of the equilibrium results in Model RH

$$\pi^{RH} = (p^{RH} - w_r - \alpha_r p^{RH} - c_s) \left(\frac{\delta + s - p^{RH}}{\delta} \right)$$

Since $\frac{\partial^2 \pi^{RH}}{\partial (p^{RH})^2} = -\frac{2(1-\alpha_r)}{\delta} < 0$, π^{RH} is concave in p^{RH} .

Set $\frac{\partial \pi^{RH}}{\partial p^{RH}} = 0$, it derives that $\frac{\partial \pi^{RH}}{\partial p^{RH}} = (1-\alpha_r) \left(\frac{\delta+s-p^{RH}}{\delta} \right) - \frac{1}{\delta} (p^{RH} - w_r - \alpha_r p^{RH} - c_s) = 0$.

Accordingly,
$$p^{RH} = \frac{(1-\alpha_r)(\delta+s) + w_r + c_s}{2(1-\alpha_r)},$$

$$q^{RH} = \frac{\delta+s-p^{RH}}{\delta} = \frac{(1-\alpha_r)(\delta+s) - w_r - c_s}{2\delta(1-\alpha_r)}.$$

$$\pi^{RH} = (p^{RH} - w_r - \alpha_r p^{RH} - c_s) \left(\frac{\delta+s-p^{RH}}{\delta} \right) = \frac{((1-\alpha_r)(\delta+s) - w_r - c_s)^2}{4\delta(1-\alpha_r)}$$

Since $q^{RH} = \frac{(1-\alpha_r)(\delta+s) - w_r - c_s}{2\delta(1-\alpha_r)} \geq 0$, $w_r \leq (1-\alpha_r)(\delta+s) - c_s$ is derived.

(8) Proof of Proposition 4

$$\frac{\partial p^{RH}}{\partial s} = \frac{1}{2} > 0;$$

$$\frac{\partial p^{RH}}{\partial \delta} = \frac{1}{2} > 0;$$

$$\frac{\partial p^{RH}}{\partial c_s} = \frac{1}{2(1-\alpha_r)} > 0;$$

$$\frac{\partial p^{RH}}{\partial w_r} = \frac{1}{2(1-\alpha_r)} > 0;$$

$$\frac{\partial p^{RH}}{\partial \alpha_r} = \frac{2(w_r + c_s)}{4(1-\alpha_r)^2} > 0.$$

$$\frac{\partial q^{RH}}{\partial s} = \frac{1}{2\delta} > 0;$$

When $s < \frac{w_r + c_s}{1-\alpha_r}$, then $\frac{\partial q^{RH}}{\partial \delta} = \frac{w_n + c_s - (1-\alpha_r)s}{2\delta^2(1-\alpha_r)} > 0$; When $s > \frac{w_r + c_s}{1-\alpha_r}$, then

$$\frac{\partial q^{RH}}{\partial \delta} = \frac{w_n + c_s - (1-\alpha_r)s}{2\delta^2(1-\alpha_r)} < 0.$$

$$\frac{\partial q^{RH}}{\partial c_s} = \frac{-1}{2\delta(1-\alpha_n)} < 0;$$

$$\frac{\partial q^{RH}}{\partial w_r} = \frac{-1}{2\delta(1-\alpha_r)} < 0;$$

$$\frac{\partial q^{RH}}{\partial \alpha_r} = \frac{-2(w_r + c_s)}{4\delta(1-\alpha_r)^2} < 0.$$

$$\frac{\partial \pi^{RH}}{\partial s} = \frac{(1-\alpha_r)(\delta+s)-(w_r+c_s)}{2\delta} = \frac{q^{RH}}{(1-\alpha_r)} > 0;$$

$$\frac{\partial \pi^{RH}}{\partial c_s} = \frac{-(1-\alpha_r)(\delta+s)+(w_r+c_s)}{2\delta(1-\alpha_r)} = -q^{RH} < 0;$$

$$\frac{\partial \pi^{RH}}{\partial w_r} = \frac{-(1-\alpha_r)(\delta+s)+(w_r+c_s)}{2(1-\alpha_r)} = -q^{RH} < 0;$$

Since $q^{RH} = \frac{(1-\alpha_r)(\delta+s)-w_r-c_s}{2\delta(1-\alpha_r)} > 0$,

$$\frac{\partial \pi^{RH}}{\partial \alpha_r} = \frac{-(\delta+s)^2}{4\delta} + \frac{(w_r+c_s)^2}{4\delta(1-\alpha_r)^2} = \frac{(w_r+c_s)^2 - (1-\alpha_r)^2(\delta+s)^2}{4\delta(1-\alpha_r)^2} = \frac{((1-\alpha_r)(\delta+s)+w_r+c_s)((1-\alpha_r)(\delta+s)-w_r-c_s)}{4\delta(1-\alpha_r)^2} < 0$$

When $(1-\alpha_r)^2(\delta^2-s^2)-2s(1-\alpha_r)(w_r+c_s)-(w_r+c_s)^2 > 0$, then

$$\frac{\partial \pi^{RH}}{\partial \delta} = \frac{(1-\alpha_r)^2(\delta^2-s^2)-2s(1-\alpha_r)(w_r+c_s)-(w_r+c_s)^2}{4\delta^2(1-\alpha_r)} > 0; \quad \text{When}$$

$(1-\alpha_r)^2(\delta^2-s^2)-2s(1-\alpha_r)(w_r+c_s)-(w_r+c_s)^2 < 0$, then

$$\frac{\partial \pi^{RH}}{\partial \delta} = \frac{(1-\alpha_r)^2(\delta^2-s^2)-2s(1-\alpha_r)(w_r+c_s)-(w_r+c_s)^2}{4\delta^2(1-\alpha_r)} < 0.$$

(9) Proof of the equilibrium results in Model NRN

$$\pi^{NRN} = \left[p^{NRN} - (1-a)w_n - aw_r \right] \frac{\beta - p^{NRN}}{\beta}$$

Since $\frac{\partial^2 \pi^{NRN}}{\partial (p^{NRN})^2} = -\frac{2}{\beta} < 0$, π^{NRN} is concave in p^{NRN} .

Set $\frac{\partial \pi^{NRN}}{\partial p^{NRN}} = 0$, it derives that $\frac{\partial \pi^{NRN}}{\partial p^{NRN}} = \frac{\beta - p^{NRN}}{\beta} - \frac{1}{\beta} [p^{NRN} - (1-a)w_n - aw_r] = 0$. This

study sets $A = [1 - \alpha_n(1-a) - \alpha_r a]$, $B = [(1-a)w_n + aw_r]$ in the equilibrium results.

Accordingly, $p^{NRN} = \frac{\beta + B}{2}$, $q^{NRN} = \frac{\beta - B}{2\beta}$.

$$\pi^{NRN} = [p^{NRN} - (1-a)w_n - aw_r] q^{NRN} = \beta (q^{NRN})^2 = \frac{(\beta - B)^2}{4\beta}$$

Since $q^{NRN} = \frac{\beta - B}{2\beta} \geq 0$, $\beta \geq B$ is derived.

(10) Proof of Proposition 5

$$\frac{\partial p^{NRN}}{\partial \beta} = \frac{1}{2} > 0;$$

$$\frac{\partial p^{NRN}}{\partial w_r} = \frac{a}{2} > 0;$$

;

$$\frac{\partial p^{NRN}}{\partial w_n} = \frac{1-a}{2} > 0 \quad \frac{\partial p^{NRN}}{\partial a} = \frac{-w_n + w_r}{2} < 0.$$

$$\frac{\partial q^{NRN}}{\partial w_r} = \frac{-a}{2\beta} < 0;$$

$$\frac{\partial q^{NRN}}{\partial w_n} = \frac{-(1-a)}{2\beta} < 0;$$

$$\frac{\partial q^{NRN}}{\partial a} = \frac{w_n - w_r}{2\beta} > 0;$$

$$\frac{\partial q^{NRN}}{\partial \beta} = \frac{-((1-a)w_n + aw_r)}{2\beta^2} < 0.$$

$$\frac{\partial \pi^{NRN}}{\partial w_r} = -\frac{a(\beta - [(1-a)w_n + aw_r])}{2\beta} = -aq^{NRN} < 0;$$

$$\frac{\partial \pi^{NRN}}{\partial w_n} = -\frac{(1-a)(\beta - [(1-a)w_n + aw_r])}{2\beta} = -(1-a)q^{NRN} < 0;$$

$$\frac{\partial \pi^{NRN}}{\partial a} = \frac{(w_n - w_r)(\beta - [(1-a)w_n + aw_r])}{2\beta} = (w_n - w_r)q^{NRN} > 0;$$

$$\frac{\partial \pi^{NRN}}{\partial \beta} = \frac{(\beta - B)(\beta + B)}{4\beta^2} > 0.$$

(11) Proof of the equilibrium results in Model NRH

$$\pi^{NRH} = \left[p^{NRH} - (1-a)w_n - aw_r - \alpha_n(1-a)p^{NRH} - \alpha_r ap^{NRH} - c_s \right] \frac{\beta + s - p^{NRH}}{\beta}$$

Since $\frac{\partial^2 \pi^{NRH}}{\partial (p^{NRH})^2} = -\frac{2[1 - \alpha_n(1-a) - \alpha_r a]}{\beta} < 0$, π^{NRH} is concave in p^{NRH} .

Set $\frac{\partial \pi^{NRH}}{\partial p^{NRH}} = 0$, it derives that $\frac{\partial \pi^{NRH}}{\partial p^{NRH}} = A \frac{\beta + s - p^{NRH}}{\beta} - \frac{1}{\beta} [Ap^{NRH} - B - c_s] = 0$.

Accordingly, $p^{NRH} = \frac{A(\beta + s) + B + c_s}{2A}$, $q^{NRH} = \frac{A(\beta + s) - B - c_s}{2A\beta}$.

$$\pi^{NRH} = \left[p^{NRH} - (1-a)w_n - aw_r - \alpha_n(1-a)p^{NRH} - \alpha_r ap^{NRH} - c_s \right] \frac{\beta + s - p^{NRH}}{\beta} = \frac{(A(\beta + s) - B - c_s)^2}{4A\beta}$$

Since $q^{NRH} = \frac{A(\beta + s) - B - c_s}{2A\beta} \geq 0$, $\beta \geq \frac{B + c_s - As}{A}$ is derived.

(12) Proof of Proposition 6

$$\frac{\partial p^{NRH}}{\partial s} = \frac{1}{2} > 0;$$

$$\frac{\partial p^{NRH}}{\partial c_s} = \frac{1}{2A} > 0;$$

$$\frac{\partial p^{NRH}}{\partial w_n} = \frac{1-a}{2A} > 0;$$

$$\frac{\partial p^{NRH}}{\partial w_r} = \frac{a}{2A} > 0;$$

$$\frac{\partial p^{NRH}}{\partial \alpha_n} = (1-a) \frac{B + c_s}{2A^2} > 0;$$

$$\frac{\partial p^{NRH}}{\partial \alpha_r} = a \frac{B + c_s}{2A^2} > 0;$$

$$\frac{\partial p^{NRH}}{\partial a} = \frac{(1 - \alpha_n)w_r - (1 - \alpha_r)w_n - (\alpha_n - \alpha_r)c_s}{2A^2}$$

When $(1 - \alpha_n)w_r - (1 - \alpha_r)w_n - (\alpha_n - \alpha_r)c_s > 0$, then $\frac{\partial p^{NRH}}{\partial a} > 0$; When

$(1 - \alpha_n)w_r - (1 - \alpha_r)w_n - (\alpha_n - \alpha_r)c_s < 0$, then $\frac{\partial p^{NRH}}{\partial a} < 0$.

$$\frac{\partial p^{NRH}}{\partial \beta} = \frac{1}{2} > 0 .$$

$$\frac{\partial q^{NRH}}{\partial s} = \frac{1}{2\beta} > 0 ;$$

$$\frac{\partial q^{NRH}}{\partial c_s} = -\frac{1}{2A\beta} < 0 ;$$

$$\frac{\partial q^{NRH}}{\partial w_n} = -\frac{1-a}{2A\beta} < 0 ;$$

$$\frac{\partial q^{NRH}}{\partial w_r} = -\frac{a}{2A\beta} < 0 ;$$

$$\frac{\partial q^{NRH}}{\partial \alpha_n} = -(1-a)\frac{B+c_s}{2\beta A^2} < 0 ;$$

$$\frac{\partial q^{NRH}}{\partial \alpha_r} = -a\frac{B+c_s}{2\beta A^2} < 0 ;$$

$$\frac{\partial q^{NRH}}{\partial a} = \frac{-(1-\alpha_n)w_r + (1-\alpha_r)w_n + (\alpha_n - \alpha_r)c_s}{2\beta A^2} ;$$

When $-(1 - \alpha_n)w_r + (1 - \alpha_r)w_n + (\alpha_n - \alpha_r)c_s > 0$, then $\frac{\partial q^{NRH}}{\partial a} > 0$; When

$-(1 - \alpha_n)w_r + (1 - \alpha_r)w_n + (\alpha_n - \alpha_r)c_s < 0$, then $\frac{\partial q^{NRH}}{\partial a} < 0$.

$$\text{Since } \frac{\partial q^{NRH}}{\partial \beta} = -\frac{As - B - c_s}{2A\beta^2}$$

When $s < \frac{B+c_s}{A}$, then $\frac{\partial q^{NRH}}{\partial \beta} = -\frac{As-B-c_s}{2A\beta^2} > 0$; When $s > \frac{B+c_s}{A}$, then

$$\frac{\partial q^{NRH}}{\partial \beta} = -\frac{As-B-c_s}{2A\beta^2} < 0.$$

$$\pi^{NRH} = \frac{(A(\beta+s)-B-c_s)^2}{4A\beta} = A\beta(q^{NRH})^2$$

Since $\frac{\partial q^{NRH}}{\partial s} = \frac{1}{2\beta} > 0$, then $\pi^{NRH} = \frac{(A(\beta+s)-B-c_s)^2}{4A\beta} = A\beta(q^{NRH})^2$ rises in s .

$$\frac{\partial \pi^{NRH}}{\partial c_s} = -\frac{A(\beta+s)-B-c_s}{2A\beta} = -q^{NRH} < 0;$$

$$\frac{\partial \pi^{NRH}}{\partial w_n} = -(1-a)\frac{(A(\beta+s)-B-c_s)}{2A\beta} < 0;$$

$$\frac{\partial \pi^{NRH}}{\partial w_r} = -a\frac{(A(\beta+s)-B-c_s)}{2A\beta} < 0;$$

$$\frac{\partial \pi^{NRH}}{\partial \beta} = -\frac{(As-B-c_s)^2}{4A\beta^2} < 0;$$

$$\frac{\partial \pi^{NRH}}{\partial \alpha_n} = -(1-a)\frac{(A(\beta+s)+B+c_s)(A(\beta+s)-B-c_s)}{4A^2\beta} < 0;$$

$$\frac{\partial \pi^{NRH}}{\partial \alpha_r} = -a\frac{(A(\beta+s)+B+c_s)(A(\beta+s)-B-c_s)}{4A^2\beta} < 0;$$

$$\frac{\partial \pi^{NRH}}{\partial a} = (m+na)\frac{2n[1-\alpha_n+(\alpha_n-\alpha_r)a]-(\alpha_n-\alpha_r)(m+na)}{4[1-\alpha_n+(\alpha_n-\alpha_r)a]^2\beta}$$

$$m = (1-\alpha_n)(\beta+s)-w_n-c_s$$

$$n = (\alpha_n-\alpha_r)(\beta+s)+w_n-w_r$$

$$m+na = A(\beta+s)-B-c_s > 0$$

When $2nA - (\alpha_n - \alpha_r)(m + na) > 0$, then $\frac{\partial \pi^{NRH}}{\partial a} > 0$; When $2nA - (\alpha_n - \alpha_r)(m + na) < 0$,

then $\frac{\partial \pi^{NRH}}{\partial a} < 0$.

In summary, the following constraints must be satisfied to guarantee economically meaningful non-negative equilibrium outcomes.

$$w_n \leq 1$$

$$w_n \leq (1 - \alpha_n)(1 + s) - c_s$$

$$w_r \leq \delta$$

$$w_r \leq (1 - \alpha_r)(\delta + s) - c_s$$

$$\beta \geq B$$

$$\beta \geq \frac{B + c_s - As}{A}$$

(13) Proof of Proposition 7

Let

$$q^{NH} - q^{NN} = \frac{(1 - \alpha_n)(1 + s) - w_n - c_s}{2(1 - \alpha_n)} - \frac{1 - w_n}{2} = \frac{(1 - \alpha_n)(1 + s) - w_n - c_s - (1 - \alpha_n)(1 - w_n)}{2(1 - \alpha_n)} > 0$$

, it derives that $s > \frac{\alpha_n w_n + c_s}{1 - \alpha_n}$.

Let

$$q^{RH} - q^{RN} = \frac{(1 - \alpha_r)(\delta + s) - w_r - c_s}{2\delta(1 - \alpha_r)} - \frac{\delta - w_r}{2\delta} = \frac{(1 - \alpha_r)(\delta + s) - w_r - c_s - (1 - \alpha_r)(\delta - w_r)}{2\delta(1 - \alpha_r)} > 0$$

, it derives that $s > \frac{\alpha_r w_r + c_s}{1 - \alpha_r}$.

(14) Proof of Proposition 8

$$q^{NRH} = \frac{A(\beta + s) - B - c_s}{2A\beta}$$

$$q^{NRN} = \frac{\beta - B - c_s}{2\beta}$$

Let $q^{NRH} - q^{NRN} = \frac{As - (1-A)(B+c_s)}{2A\beta} > 0$, it derives that $s > \frac{(1-A)(B+c_s)}{A}$.

(15) Proof of Proposition 9

$$\pi^{NH} = (p^{NH} - w_n - \alpha_n p^{NH} - c_s)(1+s-p^{NH}) = \frac{((1-\alpha_n)(1+s)-w_n-c_s)^2}{4(1-\alpha_n)}$$

$$\pi^{RH} = (p^{RH} - w_r - \alpha_r p^{RH} - c_s)\left(\frac{\delta+s-p^{RH}}{\delta}\right) = \frac{((1-\alpha_r)(\delta+s)-w_r-c_s)^2}{4\delta(1-\alpha_r)}$$

Let $\pi^{RH} - \pi^{NH} = \frac{((1-\alpha_r)(\delta+s)-w_r-c_s)^2}{4\delta(1-\alpha_r)} - \frac{((1-\alpha_n)(1+s)-w_n-c_s)^2}{4(1-\alpha_n)} = 0$, it derives that

$$s_1 = \frac{\sqrt{\delta(1-\alpha_n)(1-\alpha_r)}(\sqrt{1-\alpha_n} - \sqrt{\delta(1-\alpha_r)}) + (w_r + c_s)\sqrt{1-\alpha_n} - (w_n + c_s)\sqrt{\delta(1-\alpha_r)}}{\sqrt{(1-\alpha_n)(1-\alpha_r)}(\sqrt{1-\alpha_r} - \sqrt{\delta(1-\alpha_n)})}$$

①When $\sqrt{1-\alpha_r} > \sqrt{\delta(1-\alpha_n)}$, only if the refund guarantee strength s surpasses s_1

$\pi^{RH} > \pi^{NH}$. Then procuring remanufactured core components becomes the optimal strategy ;

②When $\sqrt{1-\alpha_r} < \sqrt{\delta(1-\alpha_n)}$, only if the refund guarantee strength s is lower than s_1 ,

$\pi^{RH} > \pi^{NH}$. Then procuring remanufactured core components becomes the optimal strategy.

(16) Proof of Proposition 10

$$\pi^{NRH} = \frac{(A(\beta+s) - B - c_s)^2}{4A\beta}$$

$$\pi^{NH} = \frac{((1-\alpha_n)(1+s)-w_n-c_s)^2}{4(1-\alpha_n)}$$

$$\pi^{NRH} - \pi^{NH} = \frac{(1-\alpha_n)(A(\beta+s) - B - c_s)^2 - A\beta((1-\alpha_n)(1+s) - w_n - c_s)^2}{4A\beta(1-\alpha_n)}$$

$$\pi^{NRH} - \pi^{NH} = \frac{(\sqrt{(1-\alpha_n)}(A(\beta+s) - B - c_s) + \sqrt{A\beta}((1-\alpha_n)(1+s) - w_n - c_s))(\sqrt{(1-\alpha_n)}(A(\beta+s) - B - c_s) - \sqrt{A\beta}((1-\alpha_n)(1+s) - w_n - c_s))}{4A\beta(1-\alpha_n)}$$

$$\pi^{NRH} - \pi^{NH} > 0$$

Let , then

$$\sqrt{(1-\alpha_n)}(A(\beta+s) - B - c_s) - \sqrt{A\beta}((1-\alpha_n)(1+s) - w_n - c_s) > 0$$

$$As\sqrt{(1-\alpha_n)} + \sqrt{(1-\alpha_n)}(A\beta - B - c_s) - s(1-\alpha_n)\sqrt{A\beta} - \sqrt{A\beta}((1-\alpha_n) - w_n - c_s) > 0$$

$$(A\sqrt{1-\alpha_n} - (1-\alpha_n)\sqrt{A\beta})s > (B+c_s)\sqrt{1-\alpha_n} - (w_n+c_s)\sqrt{A\beta} - (A\beta\sqrt{1-\alpha_n} - (1-\alpha_n)\sqrt{A\beta})$$

Since

$$A\sqrt{1-\alpha_n} - (1-\alpha_n)\sqrt{A\beta} = \sqrt{A(1-\alpha_n)}(\sqrt{A} - \sqrt{\beta(1-\alpha_n)})$$

$$A = [1 - \alpha_n(1-a) - \alpha_r a]$$

Therefore,

$$(1) \text{ when } \sqrt{A} > \sqrt{(1-\alpha_n)\beta}, \text{ only if } s > s_2, \quad \pi^{NRH} > \pi^{NH}; (2) \text{ when } \sqrt{A} < \sqrt{(1-\alpha_n)\beta},$$

$$\text{only if } s < s_2, \quad \pi^{NRH} > \pi^{NH}. \quad \text{Among,}$$

$$s_2 = \frac{(B+c_s)\sqrt{1-\alpha_n} - (w_n+c_s)\sqrt{A\beta} - (A\beta\sqrt{1-\alpha_n} - (1-\alpha_n)\sqrt{A\beta})}{A\sqrt{1-\alpha_n} - (1-\alpha_n)\sqrt{A\beta}}$$

(17) Proof of Proposition 11

$$\pi^{NRH} = \frac{(A(\beta+s) - B - c_s)^2}{4A\beta}$$

$$\pi^{RH} = (p^{RH} - w_r - \alpha_r p^{RH} - c_s) \left(\frac{\delta + s - p^{RH}}{\delta} \right) = \frac{((1-\alpha_r)(\delta+s) - w_r - c_s)^2}{4\delta(1-\alpha_r)}$$

$$\pi^{NRH} - \pi^{RH} = \frac{(A(\beta+s) - B - c_s)^2}{4A\beta} - \frac{((1-\alpha_r)(\delta+s) - w_r - c_s)^2}{4\delta(1-\alpha_r)}$$

$$\pi^{NRH} - \pi^{RH} = \frac{\delta(1-\alpha_r)(A(\beta+s) - B - c_s)^2 - A\beta((1-\alpha_r)(\delta+s) - w_r - c_s)^2}{4A\beta\delta(1-\alpha_r)}$$

$$\pi^{NRH} - \pi^{RH} > 0$$

Let , then

$$\pi^{NRH} - \pi^{RH} = \frac{(\sqrt{\delta(1-\alpha_r)}(A(\beta+s) - B - c_s) + \sqrt{A\beta}((1-\alpha_r)(\delta+s) - w_r - c_s))(\sqrt{\delta(1-\alpha_r)}(A(\beta+s) - B - c_s) - \sqrt{A\beta}((1-\alpha_r)(\delta+s) - w_r - c_s))}{4A\beta\delta(1-\alpha_r)}$$

$$(\sqrt{\delta(1-\alpha_r)}(A(\beta+s) - B - c_s) - \sqrt{A\beta}((1-\alpha_r)(\delta+s) - w_r - c_s)) > 0$$

$$As\sqrt{\delta(1-\alpha_r)} + \sqrt{\delta(1-\alpha_r)}(A\beta - B - c_s) - s(1-\alpha_r)\sqrt{A\beta} - \sqrt{A\beta}((1-\alpha_r)\delta - w_r - c_s) > 0$$

$$s(A\sqrt{\delta(1-\alpha_r)} - (1-\alpha_r)\sqrt{A\beta}) > (B+c_s)\sqrt{\delta(1-\alpha_r)} - (w_r+c_s)\sqrt{A\beta} - (A\beta\sqrt{\delta(1-\alpha_r)} - \delta(1-\alpha_r)\sqrt{A\beta})$$

Since

$$A\sqrt{\delta(1-\alpha_r)} - (1-\alpha_r)\sqrt{A\beta} = \sqrt{A(1-\alpha_r)}(\sqrt{A\delta} - \sqrt{\beta(1-\alpha_r)})$$

Therefore,

$$A = [1 - \alpha_n(1-a) - \alpha_r a]$$

$$(1) \text{ when } \sqrt{A} > \sqrt{\frac{(1-\alpha_r)\beta}{\delta}}, \text{ only if } s > s_3, \quad \pi^{NRH} > \pi^{RH}; (2) \text{ when } \sqrt{A} < \sqrt{\frac{(1-\alpha_r)\beta}{\delta}},$$

only if $s < s_3$, $\pi^{NRH} > \pi^{RH}$; $\therefore$ Among,

$$s_3 = \frac{(B + c_s)\sqrt{\delta(1-\alpha_r)} - (w_r + c_s)\sqrt{A\beta} - (A\beta\sqrt{\delta(1-\alpha_r)} - \delta(1-\alpha_r)\sqrt{A\beta})}{A\sqrt{\delta(1-\alpha_r)} - (1-\alpha_r)\sqrt{A\beta}}$$

(18) Proof of Consumers Surplus (CS) for Table 5

$$\text{Model NN: } CS^{NN} = \int_{p^{NN}}^1 (\theta - p^{NN}) d\theta = \frac{1}{2}(1 - p^{NN})^2 = \frac{(1 - w_n)^2}{8}$$

$$ES^{NN} = \tau Ee_n q^{NN} = \tau Ee_n \frac{1 - w_n}{2} = \frac{\tau Ee_n (1 - w_n)}{2}$$

$$W^{NN} = CS^{NN} + \pi^{NN} + ES^{NN} = \int_{p^{NN}}^1 (\theta - p^{NN}) d\theta + \pi^{NN} + \tau e_n q^{NN} = \frac{3(1 - w_n)^2 + 4\tau Ee_n (1 - w_n)}{8}$$

$$\text{Model NH: } CS^{NH} = \int_{p^{NH}-s}^1 (\theta + s - p^{NH}) d\theta = \frac{1}{2}(1 + s - p^{NH})^2 = \frac{((1 - \alpha_n)(1 + s) - w_n - c_s)^2}{8(1 - \alpha_n)^2}$$

$$ES^{NH} = \tau Ee_n q^{NH} = \tau Ee_n \frac{((1 - \alpha_n)(1 + s) - w_n - c_s)}{2(1 - \alpha_n)} = \frac{\tau Ee_n ((1 - \alpha_n)(1 + s) - w_n - c_s)}{2(1 - \alpha_n)}$$

$$W^{NH} = CS^{NH} + \pi^{NH} + ES^{NH} = \frac{(3 - 2\alpha_n)((1 - \alpha_n)(1 + s) - w_n - c_s)^2}{8(1 - \alpha_n)^2} + \frac{\tau Ee_n ((1 - \alpha_n)(1 + s) - w_n - c_s)}{2(1 - \alpha_n)}$$

$$W^{NH} = CS^{NH} + \pi^{NH} + ES^{NH} = \frac{(3 - 2\alpha_n)((1 - \alpha_n)(1 + s) - w_n - c_s)^2 + 4\tau Ee_n (1 - \alpha_n)((1 - \alpha_n)(1 + s) - w_n - c_s)}{8(1 - \alpha_n)^2}$$

$$\text{Model RN: } CS^{RN} = \int_{\frac{p^{RN}}{\delta}}^1 (\delta\theta - p^{RN}) d\theta = \frac{\delta}{2} \left(\frac{\delta - p^{RN}}{\delta} \right)^2 = \frac{(\delta - w_r)^2}{8\delta}$$

$$ES^{RN} = \tau Ee_r q^{RN} = \tau Ee_r \frac{(\delta - w_r)}{2\delta} = \frac{\tau Ee_r (\delta - w_r)}{2\delta}$$

$$W^{RN} = CS^{RN} + \pi^{RN} + ES^{RN} = \int_{\frac{p^{RN}}{\delta}}^1 (\delta\theta - p^{RN}) d\theta + \pi^{RN} + \frac{\tau Ee_r(\delta - w_r)}{2\delta} = \frac{3(\delta - w_r)^2}{8\delta} + \frac{\tau Ee_r(\delta - w_r)}{2\delta}$$

$$W^{RN} = CS^{RN} + \pi^{RN} + ES^{RN} = \frac{3(\delta - w_r)^2 + 4\tau Ee_r(\delta - w_r)}{8\delta}$$

$$\text{Model RH: } CS^{RH} = \int_{\frac{p^{RH}-s}{\delta}}^1 (\delta\theta + s - p^{RH}) d\theta = \frac{\delta}{2} \left(\frac{\delta + s - p^{RH}}{\delta} \right)^2 = \frac{((1-\alpha_r)(\delta + s) - w_r - c_s)^2}{8\delta(1-\alpha_r)^2}$$

$$ES^{RH} = \tau Ee_r q^{RH} = \tau Ee_r \frac{((1-\alpha_r)(\delta + s) - w_r - c_s)}{2\delta(1-\alpha_r)} = \frac{\tau Ee_r ((1-\alpha_r)(\delta + s) - w_r - c_s)}{2\delta(1-\alpha_r)}$$

$$W^{RH} = CS^{RH} + \pi^{RH} + ES^{RH} = \frac{(3-2\alpha_r)((1-\alpha_r)(\delta + s) - w_r - c_s)^2}{8\delta(1-\alpha_r)^2} + \frac{\tau Ee_r ((1-\alpha_r)(\delta + s) - w_r - c_s)}{2\delta(1-\alpha_r)}$$

$$W^{RH} = CS^{RH} + \pi^{RH} + ES^{RH} = \frac{(3-2\alpha_r)((1-\alpha_r)(\delta + s) - w_r - c_s)^2 + 4\tau Ee_r (1-\alpha_r)((1-\alpha_r)(\delta + s) - w_r - c_s)}{8\delta(1-\alpha_r)^2}$$

$$\text{Model NRN: } CS^{NRN} = \int_{\frac{p^{NRN}}{\beta}}^1 (\beta\theta - p^{NRN}) d\theta = \frac{\beta}{2} \left(\frac{\beta - p^{NRN}}{\beta} \right)^2 = \frac{(\beta - B)^2}{8\beta}$$

$$ES^{NRN} = \tau E((1-a)e_n + ae_r)q^{NRN} = \tau EC \frac{(\beta - B)}{2\beta} = \frac{\tau EC(\beta - B)}{2\beta}$$

$$W^{NRN} = CS^{NRN} + \pi^{NRN} + ES^{NRN} = \int_{\frac{p^{NRN}}{\beta}}^1 (\beta\theta - p^{NRN}) d\theta + \pi^{NRN} = \frac{3(\beta - B)^2}{8\beta} + \frac{\tau EC(\beta - B)}{2\beta} = \frac{3(\beta - B)^2 + 4\tau EC(\beta - B)}{8\beta}$$

$$\text{Model NRH: } CS^{NRH} = \int_{\frac{p^{NRH}-s}{\beta}}^1 (\beta\theta + s - p^{NRH}) d\theta = \frac{\beta}{2} \left(\frac{\beta + s - p^{NRH}}{\beta} \right)^2 = \frac{(A(\beta + s) - B - c_s)^2}{8\beta A^2}$$

$$ES^{NRH} = \tau E((1-a)e_n + ae_r)q^{NRH} = \tau EC \frac{(A(\beta + s) - B - c_s)}{2\beta A} = \frac{\tau EC(A(\beta + s) - B - c_s)}{2\beta A}$$

$$W^{NRH} = CS^{NRH} + \pi^{NRH} + ES^{NRH} = \frac{(1+2A)(A(\beta + s) - B - c_s)^2}{8\beta A^2} + \frac{\tau EC(A(\beta + s) - B - c_s)}{2\beta A}$$

$$W^{NRH} = CS^{NRH} + \pi^{NRH} + ES^{NRH} = \frac{(1+2A)(A(\beta + s) - B - c_s)^2 + 4A\tau EC(A(\beta + s) - B - c_s)}{8\beta A^2}$$

(19) Data nondimensionalization for base case

In summary, the following constraints must be satisfied to guarantee economically meaningful non-negative equilibrium outcomes.

$$w_n \leq 1$$

$$w_n \leq (1 - \alpha_n)(1 + s) - c_s$$

$$w_r \leq \delta$$

$$w_r \leq (1 - \alpha_r)(\delta + s) - c_s$$

$$\beta \geq B$$

$$\beta \geq \frac{B + c_s - As}{A}$$

Since $w_n \leq 1$, we nondimensionalize $w_n = 0.5$ in the base case.

Therefore, $w_r = 0.5w_n = 0.25$.

Since $\tau E e_n = -60 \times 100 \times 74 \div 1000 = -444$ EUR, then $\tau E e_n$ is normalized as

$$\tau E e_n = \frac{\tau E e_n}{w_n} \times 0.5 = \frac{-444 \text{ EUR}}{4300 \text{ EUR}} \times 0.5 = -0.052.$$

Since $e_r = 0.1e_n$, then $\tau E e_r = 0.1\tau E e_n = -0.0052$.

According to the data, $\alpha_n = 0.002$, $\alpha_r = 5\alpha_n = 0.01$.

Since $w_r \leq \delta$, $\delta < 1$, $\delta < \beta < 1$, $0 \leq a \leq 1$, then the nondimensionalization data for base case is shown as follows Table S1:

Table S1 The nondimensionalization data for base case

Case	w_n	w_r	α_n	α_r	a	β	c_s	δ	τ	$\tau E e_n$	$\tau E e_r$
Data for Base Case	0.5	0.25	0.002	0.01	0.7	0.7	0.05	0.5	60	-0.052	-0.0052