Supporting Information for

Revisiting the Stabilization Value of Groundwater

Contents of this file

SI1, SI2, SI3, SI4, and SI5.

Introduction

We supplement the solution process of the two-stage model and the proof of Propositions 1 and 3, We also show the derivations of the generalized DRV and the set of sample parameter values used in the numerical illustrations.

SI1. Solution of the Two-stage Model

In the following discussion, we use the notations in Table S1 for the variables derived from these solutions:

Table S1. Notations for the variables derived from the solutions.

(a) Single decision-maker regime

Notation	Description
$\pi_u^{\text{single}} = \pi_{u1}^{\text{single}} + \pi_{u2}^{\text{single}}$	Aggregate expected net benefit (and its temporal decomposition) in the uncertain case
$\pi_c^{\text{single}} = \pi_{c1}^{\text{single}} + \pi_{c2}^{\text{single}}$	Aggregate expected net benefit (and its temporal decomposition) in the certain case
$w_{u1}^{\text{single}}(S_1)$	Aggregate water use at the first stage after observing S_1 in the uncertain case
$w_{c1}^{\text{single}}(\bar{S})$	Aggregate water use at the first stage after observing $\bar{S}$ in the certain case
$g_{u1}^{\text{single}}(S_1)$	Aggregate groundwater intake at the first stage after observing S_1 in the uncertain case
$g_{c1}^{\text{single}}(\bar{S})$	Aggregate groundwater intake at the first stage after observing $\bar{S}$ in the certain case

(b) Multiple-user regime

Notation	Description
$\pi_u^{\text{multi}} = \pi_{u1}^{\text{multi}} + \pi_{u2}^{\text{multi}}$	Aggregate expected net benefit (and its temporal decomposition) in the uncertain case
$\pi_c^{\text{multi}} = \pi_{c1}^{\text{multi}} + \pi_{c2}^{\text{multi}}$	Aggregate expected net benefit (and its temporal decomposition) in the certain case
$w_{u1}^{\text{multi}}(S_1)$	Aggregate water use at the first stage after observing S_1 in the uncertain case
$w_{c1}^{\text{multi}}(\bar{S})$	Aggregate water use at the first stage after observing $\bar{S}$ in the certain case
$g_{u1}^{\text{multi}}(S_1)$	Aggregate groundwater intake at the first stage after observing S_1 in the uncertain case
$g_{c1}^{\text{multi}}(\bar{S})$	Aggregate groundwater intake at the first stage after observing $\bar{S}$ in the certain case

[Note] The expected net benefits in the table are the expected values evaluated before the realization of surface water in the first period, whereas the water use and groundwater intake are the values that users determine after observing it. In addition, we don't use the discount factor to evaluate the expected net benefits, although the solutions used here, that is, $g_{u1}^{\text{single}}(S_1)$, $g_{c1}^{\text{single}}(\bar{S})$, $g_{u1}^{\text{multi}}(S_1)$ and $g_{c1}^{\text{multi}}(\bar{S})$, are the results of users' decisions with time discounting. Summing up the discounted net benefits is another option for evaluating the economic value of groundwater in dynamic environments and sometimes may be more appropriate for resource management practices. However, as researchers, we take a different approach for our analytical purpose to evaluate users' behaviours equally throughout the period. This is the same approach as the relevant previous studies such as Msangi & Hejazi (2022).

1. Single decision-maker regime

Since the intertemporal net benefit function is additively separable with respect to the instantaneous sum of the users' net benefits, we can solve the problem in two steps: the determination of the total water intake g_t for each period, where $g_t = \sum_{i\in\mathcal{N}} g_{it}$, and the allocation of water pumping to each user within period t taking the total water intake g_t as given.

Consider the problem of the second step first:

$$\max_{g_{1,2},\dots,g_{N,2}} \sum\nolimits_{i\in\mathcal{N}} [F_i(g_{i2} + \varepsilon_i S_2) - C_i(G_1)g_{i2}],$$

subject to $\sum_{i\in\mathcal{N}} g_{it} = g_t$. By solving the problem, we get

$$\begin{aligned} g_{it} &= \frac{\tilde{a}_i}{2b} + \frac{S_t}{N} + \frac{g_t}{N} - \varepsilon_i S_t \\ w_{it} &= \frac{\tilde{a}_i}{2b} + \frac{S_t}{N} + \frac{g_t}{N}, \end{aligned} \tag{A.1}$$

for all $i \in \mathcal{N}$ and for all $t \in \{1, 2\}$, where $\tilde{a}_i \triangleq a_i - a/N$ and $a \triangleq \sum_{i\in\mathcal{N}} a_i$. Therefore, the maximized instantaneous aggregate net benefit for given S_t, G_{t-1}, and g_t is given by

$$\pi_t(g_t, S_t, G_{t-1}) \triangleq W(S_t) + (H(S_t) + dG_{t-1})g_t - \frac{b}{N} g_t^2, \tag{A.2}$$

where

$$W(S_t) \triangleq \sum_{i=1}^{N} \left[a_i \left(\frac{\tilde{a}_i}{2b} + \frac{S_t}{N}\right)\right] - b \sum_{i=1}^{N} \left(\frac{\tilde{a}_i}{2b} + \frac{S_t}{N}\right)^2,$$

$$H(S_t) \triangleq \frac{a - Nc - 2bS_t}{N}.$$

Next, we consider the problem of determining the total water intake g_t for each period. By solving backword from period 2, we obtain the following solution:

$$g_2(G_1, S_2) = \frac{a - Nc + NdG_1}{2b} - S_2. \tag{A. 3}$$

The problem of the first period in the uncertain case is then given by:

$$\max_{g_1} W(S_1) + (H(S_1) + dG_0)g_1 - \frac{b}{N}g_1^2 + \beta E_1[\pi_2(g_2(G_1, S_2), S_2, G_0)|S_1].$$

Subsequently, we get

$$g_{u1}^{\text{single}}(S_1) = \frac{1}{4b^2 - N^2d^2\beta}[(2b - Nd\beta)X - 2b(2bS_1 - Nd\beta\bar{S}) - N^2d^2\beta R], \tag{A. 4}$$

where $X \triangleq a - Nc + NdG_0$ and $w_{u1}^{\text{single}}(S_1)$ is given by $w_{u1}^{\text{single}}(S_1) = g_{u1}^{\text{single}}(S_1) + S_1$.

For the above solution to satisfy the necessary and sufficient conditions, we further require the following from the second-order condition:

$$4b^2 - N^2d^2\beta > 0. \tag{A. 5}$$

Using (A.1), (A.3), and (A.4), we get:

$$\begin{aligned}\pi_{u1}^{\text{single}} = {} & \left(\sum_{i\in\mathcal{N}} a_i\varepsilon_i\right)\bar{S} - b\left(\sum_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{1}{4b}\sum_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 - \frac{Nd^2\beta^2(2b - Nd)^2}{4b(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})^2 \\ & - \frac{N^3bd^4\beta^2}{(4b^2 - N^2d^2\beta)^2}R^2 - \frac{N^2d^3\beta^2(2b - Nd)}{(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})R - \frac{N^3bd^4\beta^2}{(4b^2 - N^2d^2\beta)^2}\sigma^2,\end{aligned} \tag{A. 6}$$

$$\begin{aligned}\pi_{u2}^{\text{single}} = {} & (c - dG_0)\bar{S} + \frac{1}{4b}\sum_{i\in\mathcal{N}} X_i^2 - \frac{d(2b - Nd\beta)(8b^2 - N^2d^2\beta - 2Nbd)}{4b(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})^2 \\ & + \frac{4Nb^3d^2}{(4b^2 - N^2d^2\beta)^2}R^2 + \frac{4b^2d(2b - Nd)}{(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})R + \frac{4Nb^3d^2}{(4b^2 - N^2d^2\beta)^2}\sigma^2,\end{aligned} \tag{A. 7}$$

$$\begin{aligned}\pi_u^{\text{single}} = {} & \left(\sum_{i\in\mathcal{N}} a_i\varepsilon_i + c - dG_0\right)\bar{S} - b\left(\sum_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{1}{4b}\sum_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 + \frac{1}{4b}\sum_{i\in\mathcal{N}} X_i^2 \\ & - \frac{d[Nd\beta^2(2b^2 + N^2d^2 - 2Nbd) + 2b^2(4b - Nd - 2Nd\beta)]}{2b(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})^2 \\ & + \frac{Nbd^2(4b^2 - N^2d^2\beta^2)}{(4b^2 - N^2d^2\beta)^2}R^2 + \frac{d(2b - Nd)(4b^2 - N^2d^2\beta^2)}{(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})R \\ & + \frac{Nbd^2(4b^2 - N^2d^2\beta^2)}{(4b^2 - N^2d^2\beta)^2}\sigma^2,\end{aligned} \tag{A. 8}$$

where $X_i \triangleq a_i - c + dG_0$.

Similarly, the problem of the first period in the certain case is given by:

$$\max_{g_1} W(\bar{S}) + (H(\bar{S}) + dG_0)g_1 - \frac{b}{N}g_1^2 + \beta\pi_2(g_2(G_1, \bar{S}), \bar{S}, G_0).$$

Subsequently, we get:

$$g_{c1}^{\text{single}}(\bar{S}) = \frac{1}{4b^2 - N^2d^2\beta}[(2b - Nd\beta)(X - 2b\bar{S}) - N^2d^2\beta R]. \tag{A.9}$$

In addition, $w_{c1}^{\text{single}}(\bar{S})$ is given by $w_{c1}^{\text{single}}(\bar{S}) = g_{c1}^{\text{single}}(\bar{S}) + \bar{S}$. Using solutions (A.1), (A.3), and (A.9), we obtain:

$$\begin{aligned}\pi_{c1}^{\text{single}} = &\left(\sum_{i\in\mathcal{N}} a_i\varepsilon_i\right)\bar{S} - b\left(\sum_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{1}{4b}\sum_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 \\ &- \frac{Nd^2\beta^2(2b - Nd)^2}{4b(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})^2 - \frac{N^3bd^4\beta^2}{(4b^2 - N^2d^2\beta)^2}R^2 - \frac{N^2d^3\beta^2(2b - Nd)}{(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})R,\end{aligned} \tag{A.10}$$

$$\begin{aligned}\pi_{c2}^{\text{single}} = &(c - dG_0)\bar{S} + \frac{1}{4b}\sum_{i\in\mathcal{N}} X_i^2 - \frac{d(2b - Nd\beta)(8b^2 - N^2d^2\beta - 2Nbd)}{4b(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})^2 \\ &+ \frac{4Nb^3d^2}{(4b^2 - N^2d^2\beta)^2}R^2 + \frac{4b^2d(2b - Nd)}{(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})R,\end{aligned} \tag{A.11}$$

$$\begin{aligned}\pi_{c}^{\text{single}} = &\left(\sum_{i\in\mathcal{N}} a_i\varepsilon_i + c - dG_0\right)\bar{S} - b\left(\sum_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{1}{4b}\sum_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 + \frac{1}{4b}\sum_{i\in\mathcal{N}} X_i^2 \\ &- \frac{d[Nd\beta^2(2b^2 + N^2d^2 - 2Nbd) + 2b^2(4b - Nd - 2Nd\beta)]}{2b(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})^2 \\ &+ \frac{Nbd^2(4b^2 - N^2d^2\beta^2)}{(4b^2 - N^2d^2\beta)^2}R^2 + \frac{d(2b - Nd)(4b^2 - N^2d^2\beta^2)}{(4b^2 - N^2d^2\beta)^2}(X - 2b\bar{S})R.\end{aligned} \tag{A.12}$$

2. Multiple-user regime

User i's problem of the second period for given S_2 and G_1 is:

$$\max_{g_{i2}} F_i(g_{i2} + \varepsilon_i S_2) - C_i(G_1)g_{i2}.$$

Hence, the solution for this is:

$$g_{i2}(G_1, S_2) = \frac{a_i - c + dG_1}{2b} - \varepsilon_i S_2. \tag{A.13}$$

User i's problem of the first period in the uncertain case is given by

$$\max_{g_{i1}} F_i(g_{i1} + \varepsilon_i S_1) - C_i(G_0)g_{i1} + \beta E_1[\pi_i(g_{i2}(G_1, S_2), G_1, S_2)|S_1].$$

Subsequently, we obtain:

$$g_{u1}^{\text{multi}}(S_1) = \frac{1}{4b^2 - Nd^2\beta}[2b(X - 2bS_1) - d\beta(X - 2b\bar{S}) - Nd^2\beta R], \tag{A.14}$$

and $w_{u1}^{\text{multi}}(S_1)$ is given by $w_{u1}^{\text{multi}}(S_1) = g_{u1}^{\text{multi}}(S_1) + S_1$.

For the above solution to satisfy the necessary and sufficient conditions, we require the following from the second-order condition:

$$4b^2 - d^2\beta > 0 \tag{A.15}$$

Using the solutions (A.13) and (A.14), we get:

$$\begin{aligned}\pi_{u1}^{\text{multi}} = &\left(\sum_{i\in\mathcal{N}} a_i\varepsilon_i\right)\bar{S} - b\left(\sum_{i\in\mathcal{N}}\varepsilon_i^2\right)\bar{S}^2 + \frac{4b^2-d^2\beta^2}{16b^3}\sum_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 \\ &+\frac{d^3\beta^2(2b-d\beta)(8\beta^2-Nd^2\beta-2Nbd)}{16b^3(4b^2-Nd^2\beta)^2}(X-2b\bar{S})^2 - \frac{Nbd^4\beta^2}{(4b^2-Nd^2\beta)^2}R^2 \\ &-\frac{d^3\beta^2(2b-Nd)}{(4b^2-Nd^2\beta)^2}(X-2b\bar{S})R - \frac{Nbd^4\beta^2}{(4b^2-Nd^2\beta)^2}\sigma^2,\end{aligned} \tag{A.16}$$

$$\begin{aligned}\pi_{u2}^{\text{multi}} = &(c-dG_0)\bar{S} - \frac{d(2b-d\beta)(8b^2-Nd^2\beta-2Nbd)}{4b(4b^2-Nd^2\beta)^2}(X-2b\bar{S})^2 + \frac{1}{4b}\sum_{i\in\mathcal{N}}X_i^2 \\ &+\frac{4Nb^3d^2}{(4b^2-Nd^2\beta)^2}R^2 + \frac{4b^2d(2b-Nd)}{(4b^2-Nd^2\beta)^2}(X-2b\bar{S})R + \frac{4Nb^3d^2}{(4b^2-Nd^2\beta)^2}\sigma^2,\end{aligned} \tag{A.17}$$

$$\begin{aligned}\pi_{u}^{\text{multi}} = &\left(\sum_{i\in\mathcal{N}} a_i\varepsilon_i + c - dG_0\right)\bar{S} - b\left(\sum_{i\in\mathcal{N}}\varepsilon_i^2\right)\bar{S}^2 + \frac{4b^2-d^2\beta^2}{16b^3}\sum_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 + \frac{1}{4b}\sum_{i\in\mathcal{N}}X_i^2 \\ &-\frac{d(2b-d\beta)(4b^2-d^2\beta^2)(8b^2-Nd^2\beta-2Nbd)}{16b^3(4b^2-Nd^2\beta)^2}(X-2b\bar{S})^2 + \frac{Nbd^2(4b^2-d^2\beta^2)}{(4b^2-Nd^2\beta)^2}R^2 \\ &+\frac{d(4b^2-d^2\beta^2)(2b-Nd)}{(4b^2-Nd^2\beta)^2}(X-2b\bar{S})R + \frac{Nbd^2(4b^2-d^2\beta^2)}{(4b^2-Nd^2\beta)^2}\sigma^2.\end{aligned} \tag{A.18}$$

Similarly, the problem of the first period in the certain case is given by:

$$\max_{g_{i1}} F_i(g_{i1}+\varepsilon_i\bar{S}) - C_i(G_0)g_{i1} + \beta\pi_i(g_{i2}(G_1,\bar{S}), G_1, \bar{S}).$$

Subsequently, we get:

$$g_{c1}^{\text{multi}}(\bar{S}) = \frac{1}{4b^2-Nd^2\beta}[(2b-d\beta)(X-2b\bar{S}) - Nd^2\beta R^g]. \tag{A.19}$$

In addition, $w_{c1}^{\text{multi}}(\bar{S})$ is given by $w_{c1}^{\text{multi}}(\bar{S}) = g_{c1}^{\text{multi}}(\bar{S}) + \bar{S}$. Using solutions (A.13) and (A.19), we obtain:

$$\begin{aligned}\pi_{c1}^{\text{multi}} = &\left(\sum_{i\in\mathcal{N}} a_i\varepsilon_i\right)\bar{S} - b\left(\sum_{i\in\mathcal{N}}\varepsilon_i^2\right)\bar{S}^2 + \frac{4b^2-d^2\beta^2}{16b^3}\sum_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 \\ &+\frac{d^3\beta^2(2b-d\beta)(8\beta^2-Nd^2\beta-2Nbd)}{16b^3(4b^2-Nd^2\beta)^2}(X-2b\bar{S})^2 - \frac{Nbd^4\beta^2}{(4b^2-Nd^2\beta)^2}R^2 \\ &-\frac{d^3\beta^2(2b-Nd)}{(4b^2-Nd^2\beta)^2}(X-2b\bar{S})R,\end{aligned} \tag{A.20}$$

$$\begin{aligned}\pi_{c2}^{\text{multi}} = &(c-dG_0)\bar{S} - \frac{d(2b-d\beta)(8b^2-Nd^2\beta-2Nbd)}{4b(4b^2-Nd^2\beta)^2}(X-2b\bar{S})^2 + \frac{1}{4b}\sum_{i\in\mathcal{N}}X_i^2 \\ &+\frac{4Nb^3d^2}{(4b^2-Nd^2\beta)^2}R^2 + \frac{4b^2d(2b-Nd)}{(4b^2-Nd^2\beta)^2}(X-2b\bar{S})R,\end{aligned} \tag{A.21}$$

$$\pi_c^{\text{multi}} = \left(\sum_{i\in\mathcal{N}} a_i\varepsilon_i + c - dG_0\right)\bar{S} - b\left(\sum_{i\in\mathcal{N}} \varepsilon_i^2\right)\bar{S}^2 + \frac{4b^2 - d^2\beta^2}{16b^3}\sum_{i\in\mathcal{N}}(X_i - 2b\varepsilon_i\bar{S})^2 + \frac{1}{4b}\sum_{i\in\mathcal{N}} X_i^2$$
$$-\frac{d(2b - d\beta)(4b^2 - d^2\beta^2)(8b^2 - Nd^2\beta - 2Nbd)}{16b^3(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})^2 + \frac{Nbd^2(4b^2 - d^2\beta^2)}{(4b^2 - Nd^2\beta)^2}R^2 \quad \text{(A. 22)}$$
$$+\frac{d(4b^2 - d^2\beta^2)(2b - Nd)}{(4b^2 - Nd^2\beta)^2}(X - 2b\bar{S})R.$$

SI2. **Proof of Proposition 1**

From (A.8) and (A.12), we obtain:

$$DRV_{\text{single}} = \pi_u^{\text{single}} - \pi_c^{\text{single}} = \frac{Nbd^2(4b^2 - N^2d^2\beta^2)}{(4b^2 - N^2d^2\beta)^2}\sigma^2. \quad \text{(B. 1)}$$

From (A.5), we can demonstrate $DRV_{\text{single}} > 0$.

Similarly, from (A.18) and (A.22), we obtain:

$$DRV_{\text{multi}} = \pi_u^{\text{multi}} - \pi_c^{\text{multi}} = \frac{Nbd^2(4b^2 - d^2\beta^2)}{(4b^2 - Nd^2\beta)^2}\sigma^2. \quad \text{(B. 2)}$$

From (A.15), we can demonstrate $DRV_{\text{multi}} > 0$.

SI3. **Proof of Proposition 3**

From (A.6) and (A.10), we obtain:

$$\pi_{u1}^{\text{single}} - \pi_{c1}^{\text{single}} = -\frac{N^3bd^4\beta^2}{(4b^2 - N^2d^2\beta)^2}\sigma^2 < 0.$$

From (A.16) and (A.20), we obtain:

$$\pi_{u1}^{\text{multi}} - \pi_{c1}^{\text{multi}} = -\frac{Nbd^4\beta^2}{(4b^2 - Nd^2\beta)^2}\sigma^2 < 0.$$

SI4. Generalization of DRV to Arbitrary Number of Stages

1. Single decision-maker regime

Proposition S1. In the single decision-maker regime, the dynamic reallocation value (DRV) in a dynamic problem of maximizing $\Pi: (\mathcal{G} \times \mathcal{S} \times U_{11} \times \ldots \times U_{N1}) \times \ldots \times (\mathcal{G} \times \mathcal{S} \times U_{1T} \times \ldots \times U_{NT}) \to \mathbb{R}_{\geq 0}$ *subject to (9), (10), and the initial stock level* G_0 *is given by* $DRV_{single} = \sum_T \Xi_t$*, where*

$$\Xi_t \triangleq -\frac{b}{N}\big(1+\Psi(t)\big)^2\sigma^2 + \frac{\Phi(t)\big(Nd-b\Phi(t)\big)}{N}\Bigg\{[\Psi(t-1)]^2 + \big[\Psi(t-2)\big(1-\Phi(t-1)\big)\big]^2 + \cdots + \Bigg[\Psi(1)\prod_{\tau=2}^{t-1}\big(1-\Phi(\tau)\big)\Bigg]^2\Bigg\}\sigma^2, \quad 4 \leq t \leq T,$$

$$\Xi_3 \triangleq -\frac{b}{N}\big(1+\Psi(t)\big)^2\sigma^2 + \frac{\Phi(t)\big(Nd-b\Phi(t)\big)}{N}\Big\{[\Psi(t-1)]^2 + \big[\Psi(t-2)\big(1-\Phi(t-1)\big)\big]^2\Big\}\sigma^2, \quad t=3,$$

$$\Xi_2 \triangleq -\frac{b}{N}\big(1+\Psi(t)\big)^2\sigma^2 + \frac{\Phi(t)\big(Nd-b\Phi(t)\big)}{N}[\Psi(t-1)]^2\sigma^2, \quad t=2,$$

$$\Xi_1 \triangleq -\frac{b}{N}\big(1+\Psi(t)\big)^2\sigma^2, \quad t=1.$$

Proof of Proposition S1

First we find a general solution for groundwater intake in the case of an arbitrary number of stages. Let $\gamma_t = (\gamma_{1t}, \ldots, \gamma_{Nt}) \in \Gamma_t = \Gamma_{1t} \times \ldots \times \Gamma_{Nt}$ denote an admissible action rule of the social planner, where Γ_{it} is the set of admissible action rules concerning user i in period t. Let $V(t, G_{t-1}, S_t)$ denote the optimal value function in period $t \in \mathrm{T}$ given the current groundwater stock G_{t-1} and the realization of surface flow S_t,

$$V(t, G_{t-1}, S_t) \triangleq \max_{\gamma_t \in \Gamma_t, \ldots, \gamma_T \in \Gamma_T} E_t\Big[\sum\nolimits_{t\in\mathrm{T}}\sum\nolimits_{i\in\mathrm{N}} \beta^{t-1}\left[F_i(\gamma_{i\tau}+\varepsilon_i S_\tau) - C_i(G_{\tau-1})\gamma_{i\tau}\right]\Big]. \tag{C.1}$$

The recursive structure of the returns leads to the following Bellman optimality equation (Bellman 1952; Basar, 2012):

$$V(t, G_{t-1}, S_t) = \max_{\gamma_t \in \Gamma_t} \sum\nolimits_{i\in\mathrm{N}} [F_i(\gamma_{i\tau}+\varepsilon_i S_\tau) - C_i(G_{\tau-1})\gamma_{i\tau}] + \beta E_{t+1}[V(t+1, G_t, S_{t+1})],$$
$$V(T+1, G_T, S_{T+1}) = 0. \tag{C.2}$$

Now we prove the following action rules constitute a unique solution for groundwater intake.

$$\gamma_{iT}^*(S_T, G_{T-1}) = \frac{1}{2b}[\Theta_i(S_T) - Nd^2\beta G_{T-1}],$$
$$\gamma_{it}^*(S_t, G_{t-1}) = \frac{1}{v_t}\Big[\frac{v_t}{2b}\Theta_i(S_t) + \frac{Nd^2\beta\rho_{t+1}}{2b}\Theta(S_t) - d\beta\rho_{t+1}\Theta(\bar{S}) - Nd^2\beta\eta_t R + d(v_{t+1} - Nd\beta\rho_{t+1})G_{t-1}\Big], \quad t \leq T-1, \tag{C.3}$$

where

$$\Theta_i(S_t) \triangleq a_i - c_i - 2b\varepsilon_i S_t, \qquad \Theta(S_t) \triangleq \sum_{i\in\mathcal{N}} (a_i - c_i) - 2bS_t,$$
$$\rho_t \triangleq \begin{cases} 1, & t = T \\ \upsilon_{t+1} - 2\beta\rho_{t+1}(Nd - b), & t \le T-1 \end{cases}$$
$$\upsilon_t \triangleq \begin{cases} 2b, & t = T \\ 2b\upsilon_{t+1} - N^2d^2\beta\rho_{t+1}, & t \le T-1 \end{cases}$$
$$\eta_t \triangleq \begin{cases} 0, & t = T \\ \beta\eta_{t+1}(2b - Nd) + \rho_{t+1}, & t \le T-1. \end{cases}$$

Also consider

$$E_{t-1}\left[\frac{\partial V(t, G_{t-1}, S_t)}{\partial G_{t-1}}\right] = \frac{d}{\upsilon_t}[\rho_t\Theta(\bar{S}) + Nd\beta\eta_t(2b - Nd)R + Nd\rho_t G_{t-1}]. \tag{C.4}$$

For $t = T$ and $T-1$, solving backward from T, we can easily show (C.3) and (C.4) are true. Assume that they also hold for some $t = k+1$ ($1 \le k \le T-2$):

$$\begin{aligned} \gamma^*_{ik+1}(S_{k+1}, G_k) = \frac{1}{\upsilon_{k+1}}\Bigg[\frac{\upsilon_{k+1}}{2b}\Theta_i(S_{k+1}) + \frac{Nd^2\beta\rho_{k+2}}{2b}\Theta(S_{k+1}) - d\beta\rho_{k+2}\Theta(\bar{S}) \\ -Nd^2\beta\eta_{k+1}R + d(\upsilon_{k+2} - Nd\beta\rho_{k+2})G_k\Bigg], \\ E_k\left[\frac{\partial V(k+1, G_k, S_{k+1})}{\partial G_k}\right] = \frac{d}{\upsilon_{k+1}}[\rho_{k+1}\Theta(\bar{S}) + Nd\beta\eta_{k+1}(2b - Nd)R + Nd\rho_{k+1}G_k]. \end{aligned} \tag{C.5}$$

Consider the problem for $t = k$:

$$\max_{g_{1,k},\dots,g_{N,k}} \Omega(S_k) + \sum_{i\in\mathcal{N}}[\Theta_i(S_k) + dG_{k-1}]g_{ik} - b\sum_{i\in\mathcal{N}} g_{ik}^2 + \beta E_k[V(k+1, G_k, S_{k+1})|S_k],$$

where $\Omega(S_t) \triangleq (\sum_{\mathcal{N}} a_i\varepsilon_i)S_t - b(\sum_{\mathcal{N}} \varepsilon_i^2)S_t^2$. By using (C.5), we obtain the following solution:

$$\begin{aligned} g_{ik} = \gamma^*_{ik}(S_t, G_{t-1}) = \frac{1}{\upsilon_k}\Bigg[\frac{\upsilon_k}{2b}\Theta_i(S_k) + \frac{Nd^2\beta\rho_{k+1}}{2b}\Theta(S_k) - d\beta\rho_{k+1}\Theta(\bar{S}) \\ -Nd^2\beta\eta_k R + d(\upsilon_{k+1} - Nd\beta\rho_{k+1})G_{k-1}\Bigg]. \end{aligned} \tag{C.6}$$

By using (C.6), we can demonstrate the following:

$$E_{k-1}\left[\frac{\partial V(k, G_{k-1}, S_k)}{\partial G_{k-1}}\right] = \frac{d}{\upsilon_k}[\rho_k\Theta(\bar{S}) + Nd\beta\eta_k(2b - Nd)R + Nd\rho_k G_{k-1}]. \tag{C.7}$$

From equation (C.6) and (C.7), equation (C.3) and (C.5) also holds for $t = k$. By mathematical induction, they are true for all $t \le T-1$.

Subsequently, we find the DRV. The aggregate groundwater intake is given by:

$$\begin{aligned} g_T &= \frac{1}{\upsilon_T}[\Theta(S_T) - N^2d^2\beta\eta_T R + NdG_{T-1}], \\ g_t &= \frac{1}{\upsilon_t}[\upsilon_{t+1}\Theta(S_t) - Nd\beta\rho_{t+1}\Theta(\bar{S}) - N^2d^2\beta\eta_t R + Nd(\upsilon_{t+1} - Nd\beta\rho_{t+1})G_{t-1}], \qquad t \le T-1. \end{aligned} \tag{C.8}$$

We rewrite (C.8) as:

$$g_t = \Lambda(t) + \Phi(t)G_{t-1} + \Psi(t)S_t, \tag{C.9}$$

where

$$\Lambda(t) \triangleq \begin{cases} \dfrac{1}{v_T}\left[\sum_{i\in\mathcal{N}}(a_i - c_i) - N^2 d^2 \beta \eta_T R\right], & t = T \\ \dfrac{1}{v_t}\left[v_{t+1}\sum_{i\in\mathcal{N}}(a_i - c_i) - Nd\beta\rho_{t+1}\Theta(\bar{S}) - N^2 d^2 \beta \eta_t R\right], & t \le T-1 \end{cases}$$

$$\Phi(t) \triangleq \begin{cases} \dfrac{Nd}{v_T}, & t = T \\ \dfrac{Nd(v_{t+1} - Nd\beta\rho_{t+1})}{v_t}, & t \le T-1 \end{cases}$$

$$\Psi(t) \triangleq \begin{cases} -\dfrac{2b}{v_T}, & t = T \\ -\dfrac{2bv_{t+1}}{v_t}, & t \le T-1. \end{cases}$$

Using this, the groundwater stock G_{t-1} can be transformed into:

$$\begin{aligned} G_{t-1} = {} & \left[\prod_{\tau=1}^{t-1}\big(1-\Phi(\tau)\big)\right]G_0 \\ & -\left\{\Psi(t-1)S_{t-1} + \Psi(t-2)\big(1-\Phi(t-1)\big)S_{t-2} + \cdots + \left[\Psi(1)\prod_{\tau=2}^{t-1}\big(1-\Phi(\tau)\big)\right]S_1\right\} \\ & +\left\{1 + \big(1-\Phi(t-1)\big) + \cdots + \left[\prod_{\tau=2}^{t-1}\big(1-\Phi(\tau)\big)\right]\right\}R \\ & -\left\{\Lambda(t-1) + \big(1-\Phi(t-1)\big)\Lambda(t-2) + \cdots + \left[\prod_{\tau=2}^{t-1}\big(1-\Phi(\tau)\big)\right]\Lambda(1)\right\}. \end{aligned} \tag{C.10}$$

In addition, the solutions (C.3) can be transformed into:

$$g_{it} = \gamma_{it}^*(S_t, G_{t-1}) = \frac{\hat{a}_i}{2b} + \frac{\Lambda(t)}{N} + \frac{\Phi(t)}{N}G_{t-1} + \frac{1}{N}\big(1 - N\varepsilon_i + \Psi(t)\big)S_t, \tag{C.11}$$

where $\hat{a}_i \triangleq a_i - c_i - \frac{1}{N}\sum_{i=1}^{N}(a_i - c_i)$. Substitute (C.11) into the aggregate instantaneous net benefit

$$\pi(g_{1t}, \dots, g_{Nt}, G_{t-1}, S_t) \triangleq \sum_{i\in\mathcal{N}}\left[F_i(g_{i2} + \varepsilon_i S_2) - C_i(G_1)g_{i2}\right]. \tag{C.12}$$

Extracting only the terms with $S_1^2, \dots, S_T^2$ from $\pi(g_{1t}, \dots, g_{Nt}, G_{t-1}, S_t)$ by using (C.10), we obtain

$$\begin{aligned} -\frac{b}{N}\big(1+\Psi(t)\big)^2 S_t^2 + \frac{\Phi(t)\big(Nd - b\Phi(t)\big)}{N}\Big[& \Psi(t-1)^2 S_{t-1}^2 + \Psi(t-2)^2\big(1-\Phi(t-1)\big)^2 S_{t-2}^2 \\ & + \cdots + \Psi(1)^2\prod_{\tau=2}^{t-1}\big(1-\Phi(\tau)\big)^2 S_1^2\Big]. \end{aligned} \tag{C.13}$$

If we take the expected value of $E_0[\pi(g_{1t}, \dots, g_{Nt}, G_{t-1}, S_t)]$, the terms with σ^2 are generated by replacing $S_1^2, \dots, S_T^2$ in (C.13) with σ^2. They give $\Xi_1, \dots, \Xi_T$ in Proposition 4.

2. Multiple-user regime

Proposition S2. In the multiple-user regime, the dynamic reallocation value (DRV) in a N-user discrete-time stochastic infinite dynamic noncooperative game of a finite horizon, $\{\mathcal{N}, \mathcal{T}, \mathcal{G}, \mathcal{S}, \{U_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}}, \{f_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}}, \{\Gamma_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}}, \{\Pi_i\}_{i\in\mathcal{T}}\}$*, is given by* $DRV_{multi} = \sum_{\mathcal{T}} \tilde{\Xi}_t$*, where*

$$\tilde{\Xi}_t \triangleq -\frac{b}{N}\left(1+\tilde{\Psi}(t)\right)^2\sigma^2 + \frac{\tilde{\Phi}(t)\left(Nd-b\tilde{\Phi}(t)\right)}{N}\left\{\left[\tilde{\Psi}(t-1)\right]^2+\left[\tilde{\Psi}(t-2)\left(1-\tilde{\Phi}(t-1)\right)\right]^2+\cdots + \left[\tilde{\Psi}(1)\prod_{\tau=2}^{t-1}\left(1-\tilde{\Phi}(\tau)\right)\right]^2\right\}\sigma^2, \quad 4\le t\le T,$$

$$\tilde{\Xi}_3 \triangleq -\frac{b}{N}\left(1+\tilde{\Psi}(t)\right)^2\sigma^2 + \frac{\tilde{\Phi}(t)\left(Nd-b\tilde{\Phi}(t)\right)}{N}\left\{\left[\tilde{\Psi}(t-1)\right]^2+\left[\tilde{\Psi}(t-2)\left(1-\tilde{\Phi}(t-1)\right)\right]^2\right\}\sigma^2, \quad t=3,$$

$$\tilde{\Xi}_2 \triangleq -\frac{b}{N}\left(1+\tilde{\Psi}(t)\right)^2\sigma^2 + \frac{\tilde{\Phi}(t)\left(Nd-b\tilde{\Phi}(t)\right)}{N}\left[\tilde{\Psi}(t-1)\right]^2\sigma^2, \quad t=2,$$

$$\tilde{\Xi}_1 \triangleq -\frac{b}{N}\left(1+\tilde{\Psi}(t)\right)^2\sigma^2, \quad t=1.$$

Proof of Proposition S2

The procedure is the same as in the proof of *the single decision-maker regime*. We first prove the following strategy constitutes a unique feedback Nash equilibrium solution for $\{\mathcal{N}, \mathcal{T}, \mathcal{G}, \mathcal{S}, \{U_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}}, \{f_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}}, \{\Gamma_{it}\}_{i\in\mathcal{N},t\in\mathcal{T}}, \{\Pi_i\}_{i\in\mathcal{T}}\}$.

$$\begin{aligned}\gamma_{iT}^{**}(G_T,S_T) &= \frac{1}{2b}[\Theta_i(S_T)+dG_{T-1}],\\ \gamma_{it}^{**}(G_{t-1},S_t) &= \frac{1}{\tilde{v}_t}\left[\frac{\tilde{v}_t}{2b}\Theta_i(S_t) - \frac{d\beta\tilde{v}_t(\tilde{\rho}_{t+1}+N\tilde{\varphi}_{t+1})}{2b\tilde{v}_{t+1}}\Theta_i(\bar{S}) + \frac{d^2\beta\tilde{\rho}_{t+1}}{2b}\Theta(S_t)\right.\\ &\left. - \frac{d\beta(d^2\beta\tilde{\rho}_{t+1}^2-\tilde{v}_t\tilde{\varphi}_{t+1})}{2b\tilde{v}_{t+1}}\Theta(\bar{S}) - d^2\beta\tilde{\eta}_t R + d(\tilde{v}_{t+1}-d\beta\tilde{\rho}_{t+1})G_{t-1}\right], \quad t\le T-1,\end{aligned} \tag{D.1}$$

where

$$\tilde{\rho}_t \triangleq \begin{cases}1, & t=T\\ \tilde{v}_{t+1}-\beta\tilde{\rho}_{t+1}(Nd+d-2b)-\dfrac{(N-1)d\beta\tilde{\rho}_{t+1}(2b-Nd)(\tilde{v}_{t+1}-d\gamma\tilde{\rho}_{t+1})}{\tilde{v}_t}, & t\le T-1\end{cases}$$

$$\tilde{v}_t \triangleq \begin{cases}2b, & t=T\\ 2b\tilde{v}_{t+1}-Nd^2\beta\tilde{\rho}_{t+1}, & t\le T-1\end{cases}$$

$$\tilde{\eta}_t \triangleq \begin{cases}0, & t=T\\ \beta\tilde{\eta}_{t+1}\tilde{\mu}_{t+1}(2b-Nd)+\tilde{\rho}_{t+1}, & t\le T-1\end{cases}$$

$$\tilde{\mu}_t \triangleq \begin{cases}1, & t=T\\ \dfrac{2b\tilde{v}_{t+1}-d^2\beta\tilde{\rho}_{t+1}}{\tilde{v}_t}, & t\le T-1\end{cases}$$

$$\tilde{\varphi}_t \triangleq \begin{cases} 0, & t = T \\ \dfrac{\beta\tilde{\mu}_t(2b - Nd)[d\tilde{\rho}_{t+1}(\tilde{v}_{t+1} - d\beta\tilde{\rho}_{t+1}) + \tilde{v}_t\tilde{\varphi}_{t+1}]}{2b\tilde{v}_{t+1}}, & t \leq T-1. \end{cases}$$

Moreover, consider

$$\begin{aligned} E_{t-1}\left[\frac{\partial V^i(t, G_{t-1}, S_t)}{\partial G_{t-1}}\right] = \frac{d}{\tilde{v}_t}[(\tilde{\rho}_t + N\tilde{\varphi}_t)\Theta_i(\bar{S}) - \tilde{\varphi}_t\Theta(\bar{S}) \\ + d\beta\tilde{\eta}_t\tilde{\mu}_t(2b - Nd)R + d\tilde{\rho}_t G_{t-1}]. \end{aligned} \tag{D.2}$$

For $t = T$ and $T-1$, solving backward from T, we can show that (D.1) and (D.2) are true. Assume they hold for $t = k+1$ $(1 \leq k \leq T-2)$, and we can prove they are also true for all $t \leq T-1$ in the same way as the single decision-maker regime.

The aggregate groundwater intake is given by:

$$\begin{aligned} g_T &= \frac{1}{\tilde{v}_T}[\Theta(S_T) - Nd^2\beta\tilde{\eta}_T R + NdG_{T-1}], \\ g_t &= \frac{1}{\tilde{v}_t}[\tilde{v}_{t+1}\Theta(S_t) - d\beta\tilde{\rho}_{t+1}\Theta(\bar{S}) - Nd^2\beta\tilde{\eta}_t R + Nd(\tilde{v}_{t+1} - d\beta\tilde{\rho}_{t+1})G_{t-1}], \qquad t \leq T-1. \end{aligned} \tag{D.3}$$

Hence, we rewrite (D.3) as:

$$g_t = \widetilde{\Lambda}(t) + \widetilde{\Phi}(t)G_{t-1} + \widetilde{\Psi}(t)S_t, \tag{D.4}$$

where

$$\widetilde{\Lambda}(t) \triangleq \begin{cases} \dfrac{1}{\tilde{v}_T}\left[\sum_{i=1}^{N}(a_i - c_i) - Nd^2\beta\tilde{\eta}_T R\right], & t = T \\ \dfrac{1}{\tilde{v}_t}\left[\tilde{v}_{t+1}\sum_{i=1}^{N}(a_i - c_i) - d\beta\tilde{\rho}_{t+1}\Theta(\bar{S}) - Nd^2\beta\tilde{\eta}_t R\right], & t \leq T-1 \end{cases}$$

$$\widetilde{\Phi}(t) \triangleq \begin{cases} \dfrac{Nd}{\tilde{v}_T}, & t = T \\ \dfrac{Nd(\tilde{v}_{t+1} - d\beta\tilde{\rho}_{t+1})}{\tilde{v}_t}, & t \leq T-1 \end{cases}$$

$$\widetilde{\Psi}(t) \triangleq \begin{cases} -\dfrac{2b}{\tilde{v}_T}, & t = T \\ -\dfrac{2b\tilde{v}_{t+1}}{\tilde{v}_t}, & t \leq T-1. \end{cases}$$

Using this, the groundwater stock G_{t-1} can be transformed into:

$$
\begin{aligned}
G_{t-1} = & \left[\prod_{\tau=1}^{t-1}\left(1-\widetilde{\Phi}(\tau)\right)\right]G_0 \\
& -\left\{\widetilde{\Psi}(t-1)S_{t-1}+\widetilde{\Psi}(t-2)\left(1-\widetilde{\Phi}(t-1)\right)S_{t-2}+\cdots+\left[\widetilde{\Psi}(1)\prod_{\tau=2}^{t-1}\left(1-\widetilde{\Phi}(\tau)\right)\right]S_1\right\} \\
& +\left\{1+\left(1-\widetilde{\Phi}(t-1)\right)+\cdots+\left[\prod_{\tau=2}^{t-1}\left(1-\widetilde{\Phi}(\tau)\right)\right]\right\}R \\
& -\left\{\widetilde{\Lambda}(t-1)+\left(1-\widetilde{\Phi}(t-1)\right)\widetilde{\Lambda}(t-2)+\cdots+\left[\prod_{\tau=2}^{t-1}\left(1-\widetilde{\Phi}(\tau)\right)\right]\widetilde{\Lambda}(1)\right\}.
\end{aligned}
\tag{D.5}
$$

In addition, (D.1) can be transformed into:

$$
g_{it} = \gamma_{it}^{**}(S_t, G_{t-1}) = \frac{\hat{a}_i}{2b} + \frac{\widetilde{\Lambda}(t)}{N} + \frac{\widetilde{\Phi}(t)}{N}G_{t-1} - Z_{it} + \frac{1}{N}\left(1 - N\varepsilon_i + \widetilde{\Psi}(t)\right)S_t, \tag{D.6}
$$

where

$$
Z_{it} = \begin{cases} 0, & t = T \\ \dfrac{d\beta}{2b}\left[\dfrac{\tilde{\rho}_{t+1} + N\tilde{\varphi}_{t+1}}{\tilde{v}_{t+1}}\Theta_i(\bar{S}) + \dfrac{Nd^2\beta\tilde{\rho}_{t+1}^2 - N\tilde{v}_t\tilde{\varphi}_{t+1} - 2b\tilde{v}_{t+1}\tilde{\rho}_{t+1}}{N\tilde{v}_t\tilde{v}_{t+1}}\Theta(\bar{S})\right], & t \le T-2. \end{cases}
$$

Substituting (D.6) into the aggregate instantaneous net benefit $\pi(g_{1t}, \ldots, g_{Nt}, G_{t-1}, S_t)$ and extracting only the terms with $S_1^2, \ldots, S_T^2$ by using (D.5), we get:

$$
\begin{aligned}
& -\frac{b}{N}\left(1+\widetilde{\Psi}(t)\right)^2 S_t^2 + \frac{\widetilde{\Phi}(t)\left(Nd - b\widetilde{\Phi}(t)\right)}{N}\Big[\widetilde{\Psi}(t-1)^2 S_{t-1}^2 + \widetilde{\Psi}(t-2)^2\left(1-\widetilde{\Phi}(t-1)\right)^2 S_{t-2}^2 \\
& \qquad + \cdots + \widetilde{\Psi}(1)^2\prod_{\tau=2}^{t-1}\left(1-\widetilde{\Phi}(\tau)\right)^2 S_1^2\Big].
\end{aligned}
\tag{D.7}
$$

If we take the expected value of $E_0[\pi(g_{1t}, \ldots, g_{Nt}, G_{t-1}, S_t)]$, the terms with σ^2 are generated by replacing $S_1^2, \ldots, S_T^2$ in (D.7) with σ^2. They give $\widetilde{\Xi}_1, \ldots, \widetilde{\Xi}_T$ in Proposition 5.

SI5. Parameter Values Used in the Numerical Illustration

Table S2 shows the set of sample parameter values used in the numerical illustrations. Note that the purpose of this numerical illustration is not to simulate the concrete values of TEV using actual water data. Rather, we aim to examine the basic responses of each component to changes in major parameters in a theoretical setting. So the values in the table are arbitrarily chosen to allow clearer graphical demonstrations in the figures in the main text, and they do not have concrete physical and monetary units.

Table S2. Parameter Values Used in the Numerical Illustration

Parameter	Description	Value
a_i	First-order coefficient of instantaneous benefit function	12,200
b	Second-order coefficient of instantaneous benefit function	300
c_i	Pumping cost intercept	21,000
d	Pumping cost slope (pumping elasticity)	{15.0, 17.5, 20.0}
G_0	Initial groundwater stock	1,000
$\bar{S}$	Average surface water supply	100
σ^2	Variance of surface water supply	[0, 600]
R	Natural groundwater recharge	0.1
N	Number of users	10
ε_i	Share of water right	$1/N$
β	Discount factor	{0.47, 0.50, …, 0.98}
T	Number of stages	{3, 4, …, 20}

References in the Supporting Information

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