Development and Application of a New Exponential Model for Hydraulic Conductivity with Depth of Rock Mass

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1. Other sensitive analysis

Sensitivity analyses were performed for the exponential and power-like models in the range 0 to 5km, setting the baseline: log K_s = -5, log K_r = -11, and α = 1.2. Figure S1 shows that the decay rate of both models increases as the residual hydraulic conductivity (log K_r) decreases. However, the decay rate of the exponential model is faster than that of the power-like model, and the hydraulic conductivity of the exponential model decays steadily up to log K_r easier. This reflects that for the same parameter like log K_s and α, the exponential model decays faster than the power-like model.

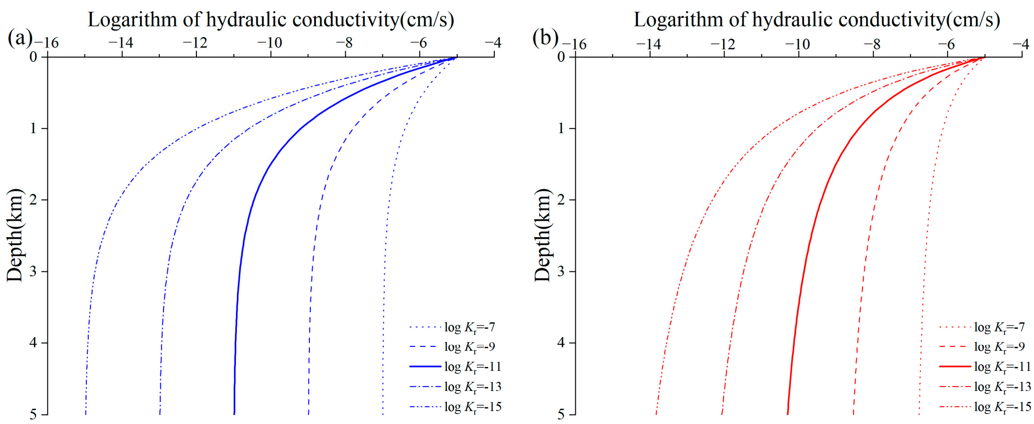

Figure S1 Sensitivity analysis of two models on log K_r, (a) proposed exponential model and (b) the power-like model.

Figure S2 focus on the effect of the decay coefficient α on the models. When the decay coefficient α increases, the hydraulic conductivity curves of two models shift to the left. The larger the decay coefficient α, the distance that the curve shifts to the left is shorter. For the exponential model, a smaller decay coefficient α (like α=0.6) allows the curve to approach the residual hydraulic conductivity log K_r, whereas for the power-like model, a larger decay coefficient α (like α=1.8) allows the curve to stabilize around the residual hydraulic conductivity log K_r. Thus the exponential model requires a smaller value of α for the permeability coefficient to stabilize. This further illustrates that the exponential model is more suitable for patterns where the distribution of hydraulic conductivities is more concentrated.

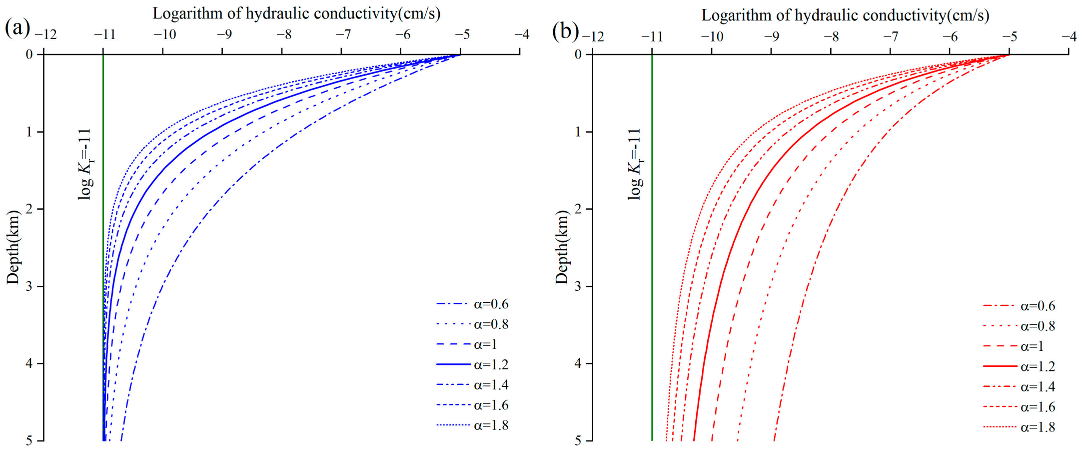

Figure S2 Sensitivity analysis of two models on α, (a) proposed exponential model and (b) the power-like model.

Based on above discussion, the actual decay rate is a function of the decay coefficient α, log K_r-log K_s. For the case where the decay mode is more concentrated (e.g. due to the influence of weathering unloading, the surface hydraulic conductivities fluctuate greatly while the deep hydraulic conductivities are more intense), the exponential model is more applicable due to the characteristics of easier attenuation to the stable log K_r. On the other hand, for the case where the decay mode is more dispersed (the deep hydraulic conductivities are not concentrated), the power-like model is more applicable.

2. The conversion method for hydraulic conductivity

The hydraulic conductivity K and the permeability k can be transformed standardly by the following equation, where ρ is the density of the fluid and μ is the viscosity of the fluid:

$$K = k\rho g/\mu \quad \text{(S1)}$$

The relationship is not consistent. It is acceptable to use the constants at 25°C for estimation, due to the low depth of data points in the database. The approximate Eq. 10 is expressed as:

$$K = 1.1 \times 10^9 k \quad \text{(S2)}$$

where the unit of the hydraulic conductivity K is cm/s and the permeability k is in m^2.

The lugeon value is determined by conducting the packer test. The packer test is conducted at three levels of pressure. The process can be divided into five stages about pressure application and pressure release. In the pressure application stage, the pressure increases from P_1 to P_2 and then to P_3. In the pressure release stage, it decreases again from P_3 to P_2 and then to P_1(P_1=0.3MPa, P_2=0.6MPa, P_3=1MPa).The lugeon value can be expressed as:

$$q = Q_3/(\mathrm{P}_3 * L) \tag{S3}$$

where q is the lugeon value of the test section, Q_3 is the flow rate of the third stage, P_3 is the test pressure of the third stage, and L is the length of the test section.

There is no precise conversion relationship between the lugeon value and the hydraulic conductivity. If the water flow is laminar and the lugeon value is less than 10, the conversion can be made using the following equation:

$$K = \frac{Q_3}{2\pi HL} \ln \frac{1}{r} \tag{S4}$$

where K is the hydraulic conductivity of the test section, Q_3 is the flow rate of the third stage, H is the water head of the third stage, L is the length of the test section, and r is the borehole radius.

If there is insufficient information about the packer test, take the third stage flow rate Q_3 as 1L/min, the head difference H as 100m, the length of the test section as 1m, then the lugeon value is 1Lu by Eq. 12. For common boreholes with diameters of 56 to 150mm, the following equation can be used as an approximation:

$$K = 1.24 \times 10^{-5} q \tag{S5}$$

where the unit of the hydraulic conductivity K is cm/s and the lugeon value q is in Lu.

3. Normalization method

Data normalization is a method of scaling characteristics and is a key step in data pre-processing. Evaluation indicators often have large numerical differences with the difference in magnitude, which can affect the results of data analysis. To eliminate these effects, data normalization is required to address the comparability of data indicators. After the raw data have been normalized the indicators will be of the same order of magnitude, making them suitable for comprehensive comparative evaluation. Normalization is essentially a linear transformation, and there are many good properties of linear transformations that ensure that changes to the data do not "invalidate" the data, but rather improve its performance, which is a prerequisite for normalization. Common mapping ranges are [0, 1] and [-1, 1]. The most common normalization method is Min-Max normalization, also known as divergence normalization, which is a linear transformation of the original data such that the resultant values map between [0, 1]. The transformation function is as follows:

$$X_{new} = \frac{X - X_{min}}{X_{max} - X_{min}} \quad \text{(S6)}$$

where X_{max} is the maximum value of the sample data and X_{min} is the minimum value of the sample data.

The decay rate β in the paper were normalized using the method described above so that the compared decay rates β were all in the interval (0, 1). The closer the decay rates are to 0, the closer the hydraulic conductivities converge to residual hydraulic conductivity.

4. The engineering project area data

In Chapter "Application of the exponential model to engineering" of the paper, we apply the exponential model to a specific project area. Several major faults pass through the engineering area as Table S1. A number of boreholes were drilled in this project area and a lot of permeability data was obtained as Table S2.

Table S1 Major faults information for the engineering project

Fault name	Length(Km)	Fault strike	Fault dip	Property
F2	81	NW	S	strike-slip
F3	105	NWW	SW	Thrust tectonics
F5	170	EW-NWW	N	strike-slip
F6	180	NWW	S	strike-slip

Table S2 The borehole hydraulic conductivity information

Borehole Description	Depth(m)	Log K(cm/s)
QZK01-02	1.5	-3.77995
Mean burial depth is 115m	5.4	-3.23209
Main lithology is granodiorite/	6.8	-4.04141

without fault crossing	15	-3.17215
	24	-3.42969
	24.5	-3.20273
	27.1	-3.89449
	29.5	-3.93554
	32.1	-4.25493
	34.5	-3.97881
	37.1	-4.5544
	39.5	-3.29671
	42.1	-3.80382
	44.5	-3.4034
	47.1	-4.11805
	49.5	-3.59176
	52.1	-3.22475
	54.5	-3.31515
	57.1	-3.07469
	59.5	-3.42022
	62.1	-3.73518
	64.5	-4.08092
	67.1	-3.17134
	69.5	-4.42022
	72.1	-3.47756
	74.5	-3.54212
	77.1	-3.37366
	79.5	-3.40012
	82.1	-3.37161
	84.5	-3.19654
	87.1	-3.61618
	89.5	-3.02919

	92.1	-4.03152
	94.5	-3.08566
	97.3	-3.90309
	99.5	-3.28651
	104.5	-3.28819
	109.5	-3.45223
	114.5	-3.64207
	119.5	-3.31247
	124.5	-3.47366
	128.5	-3.54821
QZK03~12 Mean burial depth is 90m Main lithology is granodiorite/ without fault crossing	1	-3.99823
	1.8	-3.98352
	2	-4.00941
	2	-4.17309
	3	-4.04487
	3.1	-4.00211
	4.9	-4.17851
	4.9	-4.09136
	5	-4.07182
	5	-4.11077
	5	-4.03882
	5.6	-4.28971
	6.1	-4.20846
	7.5	-4.07665
	7.5	-4.23597
	7.5	-4.35223
	8.3	-4.27258
	8.5	-4.10516
	9	-4.16391

	9.6	-4.40666
	9.6	-4.26715
	10	-4.26793
	11	-4.25852
	11.5	-4.25618
	12.1	-4.27376
	12.5	-4.2426
	12.5	-4.19368
	12.5	-4.42205
	14.8	-4.16276
	16.5	-4.31186
	16.5	-4.28651
	17	-4.39911
	17.1	-4.42205
	17.5	-4.33951
	17.5	-4.34266
	17.5	-4.10491
	17.5	-4.33124
	17.5	-4.47573
	19.9	-4.33002
	21.5	-4.32818
	22.1	-4.55721
	22.5	-4.36091
	22.5	-4.41454
	22.5	-4.29964
	22.5	-4.09173
	22.5	-4.54363
	22.5	-4.80024
	25	-4.37965

26	-4.4005
26.5	-4.26664
27.5	-4.33124
27.5	-4.51456
27.5	-4.14972
27.5	-4.20121
27.5	-4.05988
27.5	-4.44009
27.5	-4.53047
27.5	-4.56767
31.5	-4.33329
32.1	-4.64474
32.5	-4.60836
32.5	-4.43227
32.5	-4.42713
32.5	-4.42458
32.5	-4.14972
32.5	-4.39513
32.5	-4.65758
33	-4.52272
36.5	-4.3999
37.1	-4.67081
37.5	-4.45346
37.5	-4.55377
37.5	-4.36311
37.5	-4.50836
37.5	-4.15652
37.5	-4.55377
37.5	-4.66194

40	-4.60944
41.5	-4.43227
42.1	-4.74909
42.5	-4.75449
42.5	-4.41953
42.5	-4.49026
42.5	-4.31319
42.5	-4.26664
42.5	-4.19219
42.5	-4.45346
42.5	-4.55377
46.5	-4.48439
47.1	-4.75449
47.5	-5.04479
47.5	-4.47289
47.5	-4.45892
47.5	-4.49921
47.5	-4.42205
47.5	-4.15927
47.5	-4.56067
47.5	-4.52724
51.5	-4.36311
52.1	-4.71806
52.5	-4.94157
52.5	-4.60836
52.5	-4.48149
52.5	-4.46725
52.5	-4.32514
52.5	-4.25964

	52.5	-4.50224
	52.5	-4.56767
	56.5	-4.55721
	57.1	-4.71309
	57.5	-4.83803
	57.5	-4.31417
	57.5	-4.44806
	57.5	-4.35438
	57.5	-4.37202
	57.5	-4.3292
	57.5	-4.52724
	57.5	-4.71309
	61.5	-4.48149
	62.1	-4.69379
	62.5	-4.9017
	62.5	-4.56067
	62.5	-4.37428
	62.5	-4.34795
	62.5	-4.46445
	62.5	-4.39041
	62.5	-4.41953
	62.5	-5.10127
	66.5	-4.54699
	67.1	-4.74376
	67.5	-4.80632
	67.5	-4.68909
	67.5	-4.47006
	67.5	-4.51456
	67.5	-4.35438

	67.5	-4.49026
	67.5	-4.81248
	68.4	-4.5857
	71.5	-4.64898
	72.1	-4.74909
	72.5	-4.96738
	72.5	-4.58939
	72.5	-4.67985
	72.5	-4.48149
	72.5	-4.41454
	72.5	-4.51145
	72.5	-4.4786
	72.5	-4.80024
	76.5	-4.7333
	77.1	-4.79425
	77.5	-5.04479
	77.5	-4.62819
	77.5	-4.77109
	77.5	-4.5784
	77.5	-4.39751
	77.5	-4.64898
	77.5	-4.55721
	77.5	-4.38112
	81.5	-4.5857
	82.1	-4.67985
	82.5	-5.11351
	82.5	-4.67985
	82.5	-4.56767
	82.5	-4.51145

	82.5	-4.41454
	82.5	-4.68909
	82.5	-4.29585
	86.5	-4.77676
	87.1	-4.69379
	87.5	-4.98548
	87.5	-4.82507
	87.5	-4.69144
	87.5	-4.36311
	87.5	-4.78252
	88	-4.75449
	90.3	-4.67985
	92	-4.75721
	92.1	-4.56767
	92.5	-4.81248
	92.9	-4.69616
	97.1	-4.63639
	102.1	-4.75449
	107.1	-4.76548
	112.1	-4.84466
	117.1	-4.84802
SZK01	4	-3.12867
Burial depth is 190m	7.8	-3.16883
Main lithology is granodiorite/	10.3	-3.27432
without fault crossing	15.9	-3.24427
	26	-3.35111
	36	-3.22308
	45	-3.49898
	55	-3.34227

	65	-3.39255
	75	-3.67438
	85	-3.45143
	95	-3.5497
	105	-3.60022
	114	-3.54314
	120	-3.4371
	122.5	-3.12537
	127.5	-3.35507
	132.5	-3.53546
	137.5	-3.68484
	142.5	-3.76494
	147.5	-3.79334
	152.5	-3.91438
	157.5	-3.83006
	162.5	-3.78751
	167.5	-2.98297
	172.5	-3.77047
	177.5	-3.83006
	182.5	-3.94656
	187.5	-4.03932
SZK03	7	-3.64584
Burial depth is 840m	15	-3.80859
Main lithology is granodiorite/	21	-4.11467
without fault crossing	26.3	-4.18847
	31.2	-4.32624
	39.8	-4.59552
	44.5	-4.56953
	51	-4.37822

57	-4.40762
61.8	-4.48874
78.5	-4.07823
84	-3.9038
103.5	-3.73507
120	-4.16909
130.1	-4.32754
140	-3.99354
150	-4.49025
160.9	-4.54049
175	-3.72969
190	-3.48895
205	-3.59027
221.8	-3.41396
243.5	-4.02668
254.5	-3.77958
294.6	-4.19866
345.9	-4.22573
376	-4.10453
405	-4.05507
437	-4.23838
469	-4.00503
499.3	-4.00841
530	-4.26847
558.7	-4.46112
570.9	-4.16284
586.9	-4.26034
619.4	-4.4977
648.5	-4.73919

	664.1	-4.30628
	712.1	-4.20579
	746	-4.03153
	772.5	-4.43726
	777.5	-4.16564
	782.5	-4.32212
	787.5	-4.99047
	792.5	-5.10627
	797.5	-4.86323
	802.5	-5.00936
	807.5	-4.50421
	812.5	-5.08279
	817.5	-5.10627
	822.5	-5.4073
	827.5	-5.38382
	832.5	-5.23121
	837.5	-4.74349
SZK04 Burial depth is 666m Main lithology is conglomerate, sandstone, mudstone/ without fault crossing	6.8	-3.41213
	22.7	-3.19105
	40	-3.24552
	56.4	-3.41052
	66	-3.46867
	84	-4.04962
	101	-3.32223
	111	-3.42735
	117.6	-3.56859
	131.5	-3.38186
	145.9	-3.89568
	165	-3.82768

186	-3.48173
208	-3.03041
223.8	-3.25761
239.7	-3.12027
252.7	-3.5561
294.6	-3.50813
320	-2.95892
343	-2.91218
363.6	-3.98045
397	-3.46642
431.6	-3.71094
450	-4.05774
476.6	-3.97145
499.7	-4.25372
515	-3.91031
542.1	-3.9268
559.8	-4.04442
586.8	-4.24615
602.5	-4.36372
607.5	-4.32411
612.5	-4.62514
617.5	-4.55536
622.5	-4.60187
627.5	-4.18113
632.5	-4.53546
637.5	-4.87018
642.5	-4.50421
647.5	-4.83649
652.5	-4.66693

	657.5	-4.59811
	662.5	-4.53224
SZK05 Burial depth is 430m Main lithology are schist, marble, phyllite/ without fault crossing	15.81	-4.01757
	36.61	-3.76966
	66.7	-3.5553
	104	-3.79802
	145	-3.8819
	182	-3.47141
	229	-4.2247
	271.4	-4.64124
	286.6	-4.16771
	294.8	-4.26936
	323	-4.06033
	338.5	-4.91664
	357	-4.5922
	362.5	-4.59069
	367.5	-4.55536
	372.5	-4.64973
	377.5	-4.55536
	382.5	-4.51955
	387.5	-4.66693
	392.5	-4.66257
	397.5	-4.59811
	402.5	-4.59069
SZK06~09 Burial depth is 100m Main lithology are sandstone, argillaceous siltstone, mudstone/ without fault	7	-5.29581
	9.5	-5.04928
	14	-5.04048
	17.5	-5.13616
	20	-4.928

crossing	22	-4.79418
	22.5	-4.64171
	25	-4.66114
	27.5	-4.72096
	27.5	-4.86707
	30	-4.75934
	30	-4.91892
	32.5	-4.70652
	32.5	-4.87494
	35	-4.70926
	35	-4.67686
	37.5	-4.75529
	37.5	-4.70572
	50	-4.52806
	53	-4.44156
	54	-5.91735
	56.5	-5.80656
	59	-5.5892
	60.6	-4.54746
	61.5	-5.53299
	64	-5.48798
	66.5	-5.52301
	69.2	-5.55559
	90	-5.26185
	93	-5.2294
	95	-5.30985
	97.5	-5.29325
	112.5	-5.08247
	115	-5.17167

	127.5	-4.87331
	130	-4.99555
	132.5	-5.04814
	135	-5.04674
	137.5	-5.07048
	150	-5.02278
	152.5	-5.05402
	155	-5.10204
SZK10 Burial depth is 580m Main lithology are phyllite, limestone/with F6 fault crossing	8	-3.10031
	16	-3.027
	24.7	-3.79857
	34	-4.20396
	47.7	-4.2173
	65	-4.29192
	86	-4.27137
	102.1	-4.25647
	120	-3.65683
	142	-3.47983
	165	-2.92971
	180	-3.3303
	199.6	-3.65812
	220	-3.48515
	241	-3.72566
	263	-3.90361
	285	-4.60896
	300	-4.14936
	319.1	-4.19691
	351.7	-4.60473
	358.9	-4.54919

	366.8	-5.01966
	382	-5.01325
	397	-4.96994
	411.1	-5.03581
	427.4	-4.98011
	439.6	-5.14985
	453	-4.97658
	468	-5.01823
	487.2	-5.01463
	503.6	-4.43615
	512.5	-3.53579
	517.5	-5.06051
	522.5	-5.20017
	527.5	-5.06051
	532.5	-4.9636
	537.5	-4.97237
	542.5	-4.89172
	547.5	-4.76494
	552.5	-4.76494
	557.5	-4.66257
	562.5	-5.18545
	567.5	-5.06051
	572.5	-4.88442
	577.5	-4.81747
SZK13	6.3	-3.32902
Burial depth is 980m	23	-3.88572
Main lithology are sandstone,	41	-4.18675
conglomerate/ with F3 fault	58	-3.98016
crossing	75	-4.18091

91.9	-4.08649
103	-4.0231
117	-3.96984
136.4	-3.46929
146	-3.68518
164	-3.56558
183.9	-3.59356
212.4	-3.01731
238	-3.92552
264	-4.40384
290	-4.54415
310.4	-4.82471
343	-4.99308
360	-5.04542
382	-4.9612
400.3	-4.56886
412.2	-5.13164
427	-5.09076
445.3	-5.17722
490.5	-5.06517
535.7	-5.22719
591.3	-5.01199
622.6	-4.42421
636.1	-4.35475
654	-4.12449
668.3	-5.15153
683	-4.27814
700.7	-4.02399
747.1	-4.74147

	760.6	-4.85811
	780	-4.25123
	802.4	-4.11018
	833.5	-4.66261
	846.5	-4.58685
	871.4	-4.71216
	905	-4.5769
	912.5	-4.31232
	917.5	-4.27696
	922.5	-4.46667
	927.5	-4.87018
	932.5	-4.39306
	937.5	-4.61725
	942.5	-4.87724
	947.5	-4.99047
	952.5	-5.23121
	957.5	-4.97237
	962.5	-5.04979
	967.5	-5.04979
	972.5	-4.81131
	977.5	-5.01912
SZK18 Burial depth is 910m Main lithology are sandstone, conglomerate, mudstone/ with F5 fault crossing	15	-4.291
	30.4	-4.28613
	39	-4.62542
	58	-4.0581
	77	-4.13857
	96	-4.44729
	115	-4.34728
	124	-3.59754

	144	-3.12263
	163	-4.34966
	184	-4.01858
	202	-4.3458
	220	-4.29064
	243	-4.71119
	265	-4.62796
	280	-4.75437
	300	-4.53998
	325.5	-4.50662
	346	-4.92395
	370	-5.07046
	396	-4.86189
	420	-4.89564
	446	-4.98083
	470	-4.93811
	496	-5.0282
	520	-5.06661
	548	-4.91881
	570	-4.90523
	589.4	-5.03884
	607	-4.18836
	625	-5.0168
	640	-5.07178
	655.3	-4.54538
	673	-4.22455
	690	-5.00681
	708	-5.03729
	725.7	-4.8588

	746	-4.76777
	765	-4.09975
	786	-4.76087
	824	-4.99507
	824	-4.99507
	842.5	-4.27308
	847.5	-4.29295
	852.5	-4.2107
	857.5	-4.02155
	862.5	-4.11624
	867.5	-4.05229
	872.5	-4.10531
	877.5	-4.08195
	882.5	-4.47368
	887.5	-4.03461
	892.5	-4.37398
	897.5	-4.40634
	902.5	-4.36298
	907.5	-4.33158
SZK27	44.5	-4.40556
Burial depth is 162m	50	-4.29669
Main lithology are sandstone,	55.5	-4.17105
argillaceous siltstone,	60.5	-4.16102
mudstone/ without fault	65.5	-4.17721
crossing	72.5	-4.19978
	77.5	-4.19978
	82.5	-4.22974
	87.5	-4.42163
	92.5	-4.44645

	97.5	-4.44137
	102.5	-4.37587
	107.5	-4.21151
	112.5	-4.42163
	117.5	-4.42163
	122.5	-4.42163
	127.5	-4.30399
	132.5	-4.33448
	137.5	-4.33448
	142.5	-4.47278
SZK30	6	-3.48122
Burial depth is 600m	15	-3.03758
Main lithology is granodiorite/	23.4	-2.93782
without fault crossing	38	-3.13659
	40	-3.75339
	55	-3.52453
	65.7	-3.66513
	74.8	-4.09014
	91.7	-3.96511
	116.6	-3.27173
	144.5	-3.17903
	173.4	-2.9048
	178.2	-2.82744
	200	-3.79609
	224.4	-3.66109
	253	-3.67221
	279	-3.65064
	308	-2.88294
	336	-4.03973

	365	-3.64937
	393	-3.40449
	424	-3.65871
	452	-3.59755
	480	-3.94735
	508	-3.96765
	530	-4.02534
	532.5	-4.2734
	537.5	-3.86392
	542.5	-3.8653
	547.5	-4.28966
	552.5	-4.05834
	557.5	-4.13752
	562.5	-4.06051
	567.5	-4.05619
	572.5	-4.28599
	577.5	-4.4073
	582.5	-4.2476
	587.5	-3.9474
	592.5	-4.30084
	597.5	-4.26812
GDH13,15,17 Burial depth is 100m Main lithology is argillaceous sandstone/ without fault crossing	3	-4.16025
	6	-4.09836
	10	-4.38029
	10.3	-4.20706
	10.6	-3.53169
	24	-3.64567
	25	-4.72198
	32.55	-4.59007

	37	-4.68503
	37.1	-4.32057
	38	-3.76862
	42.3	-4.8041
	45	-4.55523
	47.85	-4.91009
	52.5	-3.84466
	52.6	-5.05948
	54	-4.51512
	57.5	-3.8962
	60	-5.11861
	62.5	-3.93554
	62.5	-4.74854
	67.5	-3.93554
	69.2	-5.17456
	72.5	-3.98177
	80	-4.76929
	90	-5.69404
	100	-5.01328
	103.1	-5.50169
	108.9	-5.42946
	114.3	-5.4698
	120	-5.47756
	125	-5.31966
	128.75	-5.31605

5. Datasets collected from other literature

In the paper, the database is divided into the following datasets: igneous dataset, metamorphic dataset, sedimentary(sandstone) dataset, sedimentary(mudstone) dataset, stable dataset, unstable dataset, faulted dataset, non-faulted dataset.

Meanwhile, in order to verify the characteristics of the two models, dataset 1 and dataset 2 were extracted from the igneous dataset and metamorphic dataset. The numerical code used to generate all above datasets is available at:

https://github.com/Chaoqi445154070/Supporting-information-for-the-exponential-paper.