Supplementary Materials:

1.Tetrahedral element Galerkin finite element equation discretization

The calculation area was modeled, tetrahedral meshing with the Gmsh 3D finite element mesh generator was conducted, and the results are shown in Figure S1.

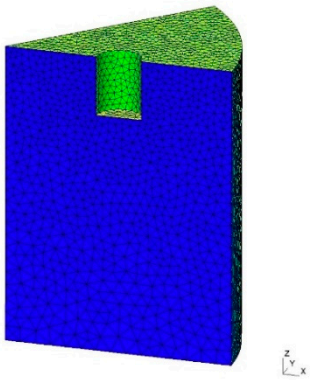

Figure S1. Tetrahedral dissection results.

Galerkin finite element method was used to solve the three-dimensional soil hydrothermal movement model [16](see main article), the time derivative term in the equation was treated by implicit backward difference, and the water content term $\frac{\Delta\theta}{\Delta t}$ was treated based on the conservation of mass [60]. The solution results are as follows:

$$\left(\frac{[F][C]_{j_{0+1}}^{k_0}}{\Delta t_{j_0}} + [A_W]_{j_{0+1}}^{k_0}\right)\{h\}_{j_{0+1}}^{k_{0+1}} = \frac{[F][C]_{j_{0+1}}^{k_0}}{\Delta t_{j_0}}\{h\}_{j_{0+1}}^{k_0} - [F]\frac{\{\theta\}_{j_{0+1}}^{k_0} - \{\theta\}_{j_0}}{\Delta t_{j_0}} + \{Q_W\}_{j_0} - \{B\}_{j_{0+1}}^{k_0} - \{D\}_{j_0} \tag{S1}$$

$$\left(\frac{[E]_{j_{0+1}}^{k_{0+1}}}{\Delta t_{j_0}} + [A_T]_{j_{0+1}}^{k_{0+1}} + [G]_{j_{0+1}}^{k_{0+1}}\right)\{T\}_{j_{0+1}} = \{Q_T\}_{j_0} + \frac{[E]_{j_{0+1}}^{k_{0+1}}}{\Delta t_{j_0}}\{T\}_{j_0} \tag{S2}$$

where j_{0+1} is the current time level, j_0 is the previous time level, Δt_{j_0} is the time interval, Δt_{j_0}=$t_{j0+1} - t_{j0}$; k_{0+1} is the current iteration, and k_0 is the previous iteration.

$$\{h\} = [h_1, h_2, \dots, h_n]^T \tag{S3}$$

$$\{T\} = [T_1, T_2, \cdots, T_n]^T \tag{S4}$$

$$[A_p] = \sum_e \frac{K}{36V^e}\begin{bmatrix} b_i^2 + c_i^2 + d_i^2 & b_ib_j + c_ic_j + d_id_j & b_ib_k + c_ic_k + d_id_k & b_ib_m + c_ic_m + d_id_m \\ b_jb_i + c_jc_i + d_jd_i & b_j^2 + c_j^2 + d_j^2 & b_jb_k + c_jc_k + d_jd_k & b_jb_m + c_jc_m + d_jd_m \\ b_kb_i + c_kc_i + d_kd_i & b_kb_j + c_kc_j + d_kd_j & b_k^2 + c_k^2 + d_k^2 & b_kb_m + c_kc_m + d_kd_m \\ b_mb_i + c_mc_i + d_md_i & b_mb_j + c_mc_j + d_md_j & b_mb_k + c_mc_k + d_md_k & b_m^2 + c_m^2 + d_m^2 \end{bmatrix} \tag{S5}$$

where $K = \overline{K}$ and $\overline{K} = \frac{1}{4}[K_i + K_j + K_k + K_m]$ if $A_p = A_w$, $K = \overline{K_h}$ and $\overline{K_h} = \frac{1}{4}(K_{hi} + K_{hj} + K_{hk} + K_{hm})$ if $A_p = A_T$.

$$V^e = \frac{1}{6}[(x_m - x_i)b_m + (y_m - y_i)c_m + (z_m - z_i)d_m] \tag{S6}$$

$$b_i = -\begin{vmatrix} 1 & y_j & z_j \\ 1 & y_k & z_k \\ 1 & y_m & z_m \end{vmatrix}, b_j = \begin{vmatrix} 1 & y_i & z_i \\ 1 & y_k & z_k \\ 1 & y_m & z_m \end{vmatrix}, b_k = -\begin{vmatrix} 1 & y_i & z_i \\ 1 & y_j & z_j \\ 1 & y_m & z_m \end{vmatrix}, b_m = \begin{vmatrix} 1 & y_i & z_i \\ 1 & y_j & z_j \\ 1 & y_k & z_k \end{vmatrix}$$

$$c_i = \begin{vmatrix} 1 & x_j & z_j \\ 1 & x_k & z_k \\ 1 & x_m & z_m \end{vmatrix}, c_j = -\begin{vmatrix} 1 & x_i & z_i \\ 1 & x_k & z_k \\ 1 & x_m & z_m \end{vmatrix}, c_k = \begin{vmatrix} 1 & x_i & z_i \\ 1 & x_j & z_j \\ 1 & x_m & z_m \end{vmatrix}, c_m = -\begin{vmatrix} 1 & x_i & z_i \\ 1 & x_j & z_j \\ 1 & x_k & z_k \end{vmatrix}$$

$$d_i = -\begin{vmatrix} 1 & x_j & y_j \\ 1 & x_k & y_k \\ 1 & x_m & y_m \end{vmatrix}, d_j = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_k & y_k \\ 1 & x_m & y_m \end{vmatrix}, d_k = -\begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_m & y_m \end{vmatrix}, d_m = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} \quad \text{(S7)}$$

where $i, j, k,$ and m are the node numbers of the tetrahedral element e, V^e is the volume of element e, and (x_i, y_i, z_i), (x_j, y_j, z_j), (x_k, y_k, z_k), and (x_m, y_m, z_m) are the corresponding node coordinates.

$$[F] = \sum_e \frac{V^e}{4} \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \quad \text{(S8)}$$

$$\{Q_p\} = -\sum_e q_n \frac{\Delta^e}{3} \begin{Bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{Bmatrix} \quad \text{(S9)}$$

where $q_n = q$ when $Q_p = Q_W$, and $q_n = q_h$ when $Q_p = Q_T$.

$$\{B\} = \sum_e \frac{\bar{K}}{6} \begin{Bmatrix} d_i \\ d_j \\ d_k \\ d_m \end{Bmatrix} \quad \text{(S10)}$$

$$\{D\} = \sum_e \frac{V^e}{20} \begin{bmatrix} 2S_i + S_j + S_k + S_m \\ S_i + 2S_j + S_k + S_m \\ S_i + S_j + 2S_k + S_m \\ S_i + S_j + S_k + 2S_m \end{bmatrix} \quad \text{(S11)}$$

$$[G] = \sum_e \frac{c_w}{24} \begin{bmatrix} q_x b_i + q_y c_i + q_z d_i & q_x b_j + q_y c_j + q_z d_j & q_x b_k + q_y c_k + q_z d_k & q_x b_m + q_y c_m + q_z d_m \\ q_x b_i + q_y c_i + q_z d_i & q_x b_j + q_y c_j + q_z d_j & q_x b_k + q_y c_k + q_z d_k & q_x b_m + q_y c_m + q_z d_m \\ q_x b_i + q_y c_i + q_z d_i & q_x b_j + q_y c_j + q_z d_j & q_x b_k + q_y c_k + q_z d_k & q_x b_m + q_y c_m + q_z d_m \\ q_x b_i + q_y c_i + q_z d_i & q_x b_j + q_y c_j + q_z d_j & q_x b_k + q_y c_k + q_z d_k & q_x b_m + q_y c_m + q_z d_m \end{bmatrix} \quad \text{(S12)}$$

where, $q_x = -\frac{\bar{K}}{6V^e}(b_i h_i + b_j h_j + b_k h_k + b_m h_m)$; $q_y = -\frac{\bar{K}}{6V^e}(c_i h_i + c_j h_j + c_k h_k + c_m h_m)$; $q_z = -\frac{\bar{K}}{6V^e}(d_i h_i + d_j h_j + d_k h_k + d_m h_m + 6V^e)$;

$$[E] = \sum_e \frac{V^e}{20} \begin{bmatrix} 2c_{v_i} + c_{v_j} + c_{v_k} + c_{v_m} & & & \\ & c_{v_i} + 2c_{v_j} + c_{v_k} + c_{v_m} & & \\ & & c_{v_i} + c_{v_j} + 2c_{v_k} + c_{v_m} & \\ & & & c_{v_i} + c_{v_j} + c_{v_k} + 2c_{v_m} \end{bmatrix} \quad \text{(S13)}$$

Equation (S9) is only significant to the flux boundaries. The internal and Dirichlet boundary nodes are 0. Suppose that the boundary in the tetrahedral element $ijkm$ is jkm, and Δ^e is the area of the interface jkm.

2. Programming

According to the governing equation and the definite conditions, the calculation program was compiled using MATLAB, defined irrigation at t=0. Considering that the influence of temperature on water movement was ignored, the soil moisture content could be directly solved using Equation (S1). The thermal properties of the soil were af-fected by the change of moisture content. Therefore, Equation (S2) needed to be solved simultaneously with Equation (S1). In the water movement equation, the moisture content θ of each node was solved and

substituted into the heat transfer Equation (Eq. (S2)), and the linear equation system was solved to obtain the corresponding soil tem-perature T. For the calculation, the initial moisture content of the period was set as the calculated value of the first iteration, the equation system in Equation (S1) was solved, the moisture content and water potential of each node at the end of the period were obtained, and the first iteration was completed. Then, the first iteration results were substituted into the relevant parameter equations, the correction values of the re-quired parameters could be obtained, the values were used as the next forecast values for calculation, the equation system was solved to obtain the moisture content and wa-ter potential after the period, and the second iteration was completed. The above steps were repeated until the soil water potential error obtained by the two iterations before and after reached the specified allowable error range, and Equation (S14) was satisfied. Finally, the resulting node water potential was substituted into the heat transfer equa-tion (Eq. (S2)) to obtain the soil temperature of each node.

$$\left\{ max \left| \frac{h_i^{j_{0+1}(k_{0+1})} - h_i^{j_{0+1}(k_0)}}{h_i^{j_{0+1}(k_{0-1})}} \right| \leq esp \right. \tag{S14}$$

where esp is the permissible relative error with a value of 0.01.

References

60. Celia, M.A.; Bouloutas, E.T.; Zarba, R.L. A general mass-conservative numerical solution for the unsaturated flow equation. *Water Resour. Res.* **1990**, 26, 1483-1496. [CrossRef]