

Supplementary Information

The Standard Oxygen Transfer Rate (SOTR), commonly expressed in kg O₂/h, represents the capacity of the system to diffuse an oxygen mass (in kg) per time unit (in h) into the water bulk. The determination of the SOTR is done using equation S1,

$$SOTR = K_{La20} \cdot C_L^* \cdot V, \quad (S1)$$

where K_{La20} (h⁻¹) is the system volumetric transfer ratio at 20 °C reference, C_L^* (kg dm⁻³) is the dissolved oxygen concentration at the same temperature, and V (dm³) is the specific volume. If one considers V as the liquid volume of the reactor, the SOTR is a system-specific SOTR, meaning that it represents the SOTR of the reactor under given conditions. For a comparison to other aeration techniques, Figure 4 uses process-specific SOTR values, where the volume used is assuming a one hour aeration process according to equation S2, where Q is the funnel flow rate expressed in dm³ min⁻¹.

$$SOTR = K_{La20} \cdot C_L^* \cdot (Q \cdot 60). \quad (S2)$$

In this approach, the volume is standardized by applying a hypothetical retention time of one hour (the inverse unit of the K_{La}) so that the oxygen transfer can be readily compared to other systems as it is independent from reactor, regime and flow rate. The energy estimation in appendix A also uses this approach and refers to a hypothetical funnel with one hour retention time.