Basic Enveloping Model and Rigid Ring Model with Integrated Slip Model

The semi-empirical tire model under consideration consists of a rigid ring representing the tire belt and the tandem cam enveloping model for filtering the road surface. The ring is elastically suspended to the rim using spring-damper elements, representing the tire sidewalls with pressurized air. Two models describe the contact of the tire with the road surface: an enveloping model to generate an effective road surface and a slip model. The enveloping model, rigid ring model, and pragmatic slip model are derived based on the works of Schmitz, Pacejka, and P Zegelaar.

Enveloping Model:

Figure 1 shows a schematic of the tandem elliptical cam model, which moves over an obstacle. The capital letters represent the global coordinates, and the small letters represent the local coordinates. The effective height (w) shows the height of the midpoint of the lower tandem rod and can be calculated using Equation (1).

$$w_{global} = \frac{Z_{front} + Z_{rear}}{2} \tag{1}$$

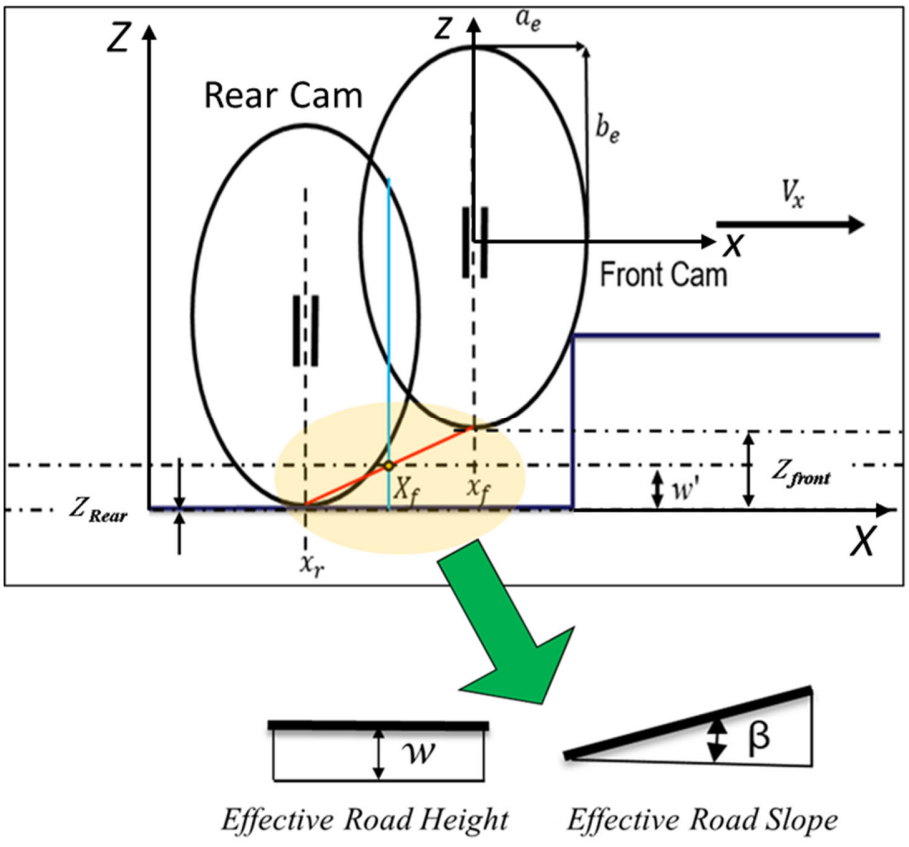


Figure 1. Tandem elliptical cam model [4]

The inclination angle of the tandem rod corresponds to the effective forward slope (tan βy) and is calculated using.

$$\tan \beta_y = \frac{Z_{front} + Z_{rear}}{l_s} \qquad (2)$$

The general equation of any given ellipse is given by

$$\left(\frac{x}{a_e}\right)^{c_e} + \left(\frac{z}{b_e}\right)^{c_e} = 1 \qquad (3)$$

These equations are used to filter the road data and prepare the enveloped road profile for the rigid ring model.

Rigid Ring Model with integrated slip model:

Figure 2 shows a schematic of rigid ring model. The ring is elastically suspended to the rim using spring-damper elements, which represents the tire sidewalls with pressurized air.

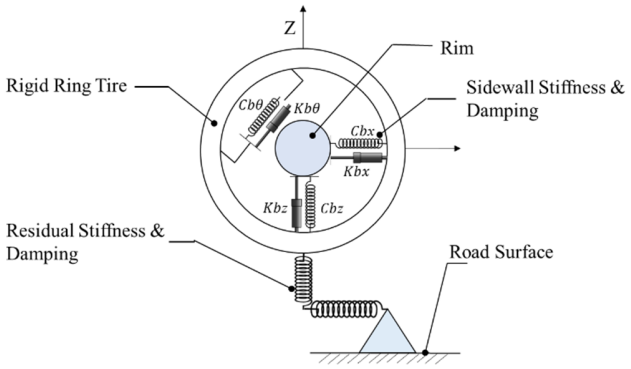


Figure 2. A schematic representation of rigid ring model [4]

The enveloping model and slip model are coupled with the rigid ring model to obtain the full-fledged tire model.

Equations for slip modeling with four pragmatic approaches was developed by Zegelaar's thesis [22]. Out of four different approaches namely 'force filtering', 'deformation filtering', 'velocity filtering' and 'mass in contact patch', the last approach is used because of the numerical stability of simulation.

"The equations are written for a circular rigid ring body with the mass m_b and mass moments of inertia of I_{bx}, I_{by} and I_{bz}. The contact model consists of a small mass m_c, residual stiffness and damping elements, and a slip model.

The final governing equations of motions for rigid ring model are given by Equations (4-7).

$$m_b\ddot{x}_b + k_{bx}\dot{x}_b + c_{bx}x_b + k_{bz}\dot{\theta}_a(z_b - z_{a0}) = F_{bx}^b \tag{4}$$

$$m_b\ddot{z}_b + k_{bz}\dot{z}_b + c_{bz}(z_b - z_{a0}) - k_{bx}\dot{\theta}_a x_b = F_{bz}^b \tag{5}$$

$$I_{by}\ddot{\theta}_b + k_{b\theta}(\dot{\theta}_b - \dot{\theta}_a) + c_{b\theta}(\theta_b - \theta_a) = M_{by}^b \tag{6}$$

$$I_{ay}\ddot{\theta}_a + k_{b\theta}(\dot{\theta}_a - \dot{\theta}_b) + c_{b\theta}(\theta_a - \theta_b) = M_{ay}^b \tag{7}$$

Where x_b is belt deflection in X-axis (longitudinal direction), z_b is belt delfection in Z-axis (vertical direction), θ_b is belt rotation about Y axis and $and\ \theta_a$ is the axle roation about Y axis. Forces in X and Z directions and moment about Y direction are defined in belt axis system for belt and axle."

$$\sigma_c \frac{d\xi_x}{dt} + |V|\xi_x = -V_{c,sx} \tag{8}$$

$$V_{c,sx} = V_{c,sx}{}^{C} = V_{b,cx}{}^{e} + \dot{x}_c - r_e\dot{\theta}_b + p_e\dot{\theta}_b \tag{9}$$

Relaxation length of contact patch model is given as

$$\sigma_c = \sigma_{C0}\frac{C_K}{C_{K0}} \tag{10}$$

The dynamic contact patch equation is in the form of

$$m\ddot{u} + k\dot{u} + cu = F_x \tag{11}$$

Contact patch equation which is coupled with all the rigid ring equations is given as follows

$$m_c\ddot{x}_c + k_{rx}(\dot{x}_c - \dot{x}_b) + c_{rx}(x_c - x_b) = C_{cK} * K_c' \tag{12}$$

where σ_c is the relaxation length, ξ_x is the longitudinal slip, and $V_{c,sx}$ is contact patch slip velocity,
σ_{c0} is the relaxation length at no slip condition.
u, c and k denote carcass deflection, stiffness, and damping respectively.

These equations are based on the works of Zegelaar 1998 [22] and Schmeitz [4] where pragmatic slip model and flexible tire model combination was simulated against different road surfaces and for braking maneuvers."

The algorithm used to calculate the transient longitudinal slip is shown in The algorithm used to calculate the transient longitudinal slip is shown in Figure S1. A mass-spring system represents the carcass deformation, and the transient response of the slip model is represented by the slip model with longitudinal slip filtering, where the relaxation length is calculated based on the first-order differential equation given below.

The additional mass is added to the contact patch spring damper system, whose equation is solved with an updated longitudinal force value in every loop as per the algorithm in Figure S1.
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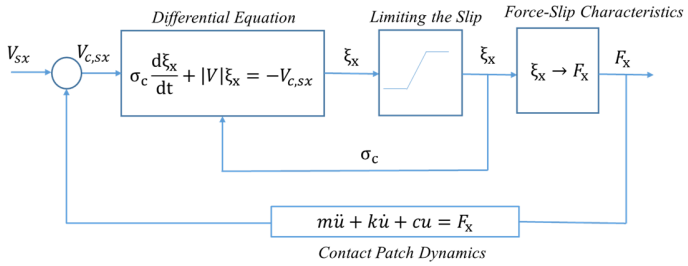


Figure S1. Pragmatic longitudinal slip calculation algorithm [20]

The model is validated against a fixed axle cleat test, where the tire is rolled over different cleat shapes at different velocities and axle loading. The rigid ring-based dynamic tire model response is then compared with test data. Figure S2 shows the

correlation of the quasi-static enveloping behavior of the tire model along with dynamic axle forces. With Pearson's correlation factor of 0.98, 0.84, and 0.86, the model is acceptable for further DOE study.

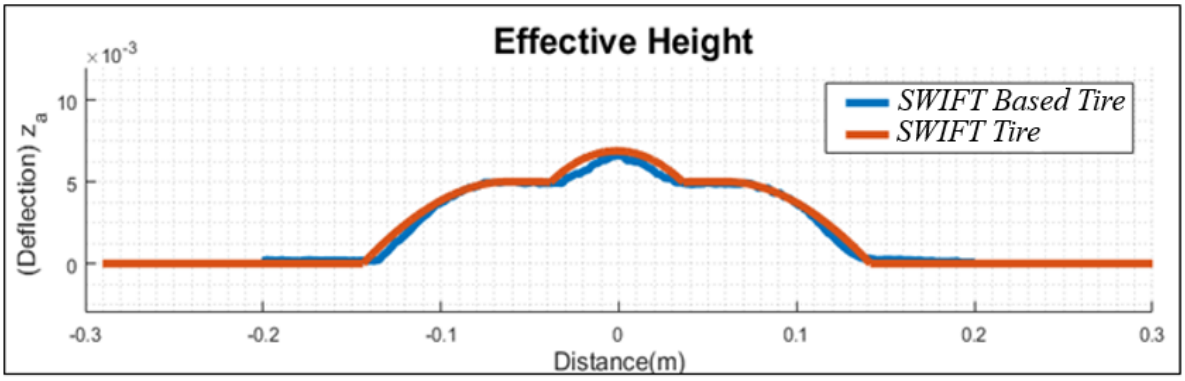


Figure S2. Comparison of simulation results of the enveloping model with SWIFT tire rolling with a constant vertical load of 4000 N over the trapezoidal cleat [20]

The effective height generated by the enveloping model, contact patch normal force, and axle forces in the x and z directions are compared with test results. Figure S3 compares the contact patch normal force in z direction with a constant vertical load of 4000 N over the trapezoidal cleat.

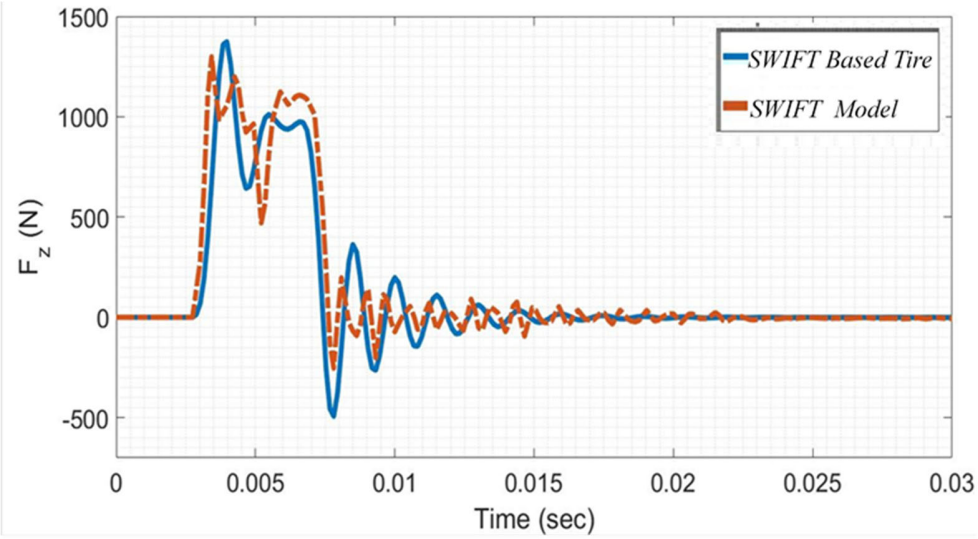


Figure S3. Comparison of simulation results of the Vertical Force at Axle (F_z) with SWIFT model rolling with a constant vertical load of 4000 N over the trapezoidal cleat.

Figure S4 compares the contact patch normal force in the x direction with a constant vertical load of 4000 N over the trapezoidal cleat

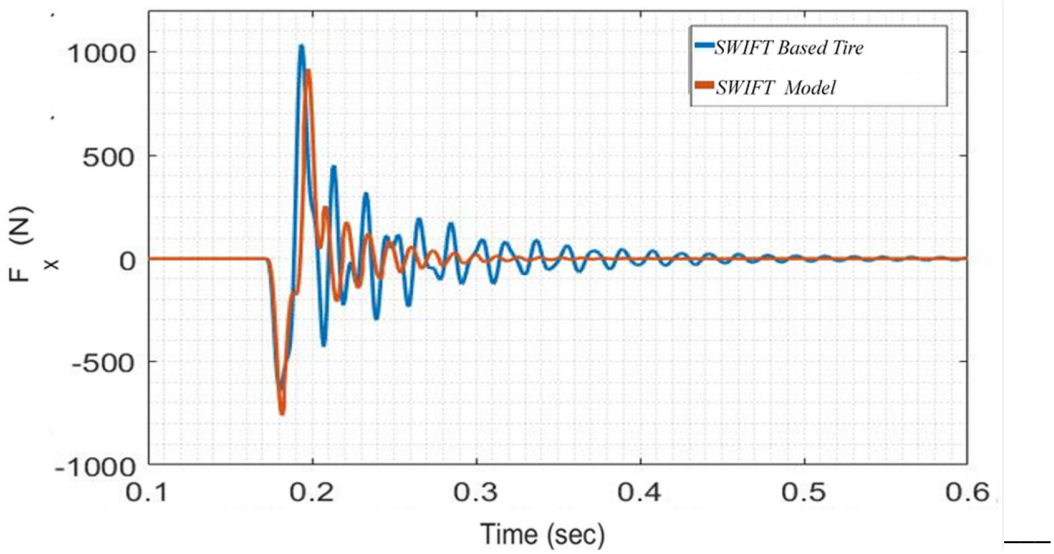


Figure S4. Comparison of simulation results of the Longitudinal Force at Axle (F_x) with SWIFT model rolling with a constant vertical load of 4000 N over the trapezoidal cleat

References

4. Schmeitz, A.J.C.; Besselink, I.J.M.; Jansen, S.T.H. TNO MF-SWIFT. *Vehicle System Dynamics* **2007**, *45*, 121-137, doi:10.1080/00423110701725208.
20. Birajdar, V.; Baqersad, J.; Bastiaan, J.; Behroozi, M. *Investigation and Development of a Slip Model for a Basic Rigid Ring Ride Model*; 0148-7191; SAE Technical Paper: 2018.
22. Zegelaar, P.W.A. The dynamic response of tyre to brake torque variations and road unevennesses. Ph-D Thesis, Delft University of Technology, Netherlands, 1998.