

# Supplementary material

## Time dependent models

Here we offer more detail as to how time dependence is included into an SDM/ENM. We consider a time dependent variable of interest,  $v(t)$ , defined by an epidemiological “who” ensemble, such as confirmed cases, deaths for age group  $> 60$ , mortality etc. The variable of interest may be ordinal, nominal or interval, and extensive or intensive. Its calculation requires a discretisation in time, so we may consider  $N_T$  time intervals, each of duration  $\delta t$ , to yield a total time period  $\Delta T = N_T * \delta t$ . We may consider a variable  $v(t, \delta t)$  specified over an interval of time  $\delta t$ , such as the number of confirmed cases of COVID-19 in the  $i$ th time interval,  $\delta t_i$ , or over the entire period,  $\Delta T$ . We may further consider  $v_\alpha(t, \delta t)$ , now calculated for a spatial cell  $\alpha$  on a grid partition of  $N_s$  spatial cells on a region of interest. For instance, the number of confirmed cases in a country,  $\alpha$ , in a month,  $\delta t$ , beginning at  $t$  being the first of June, 2021. Thus,  $(\alpha, \delta t_i)$  defines a spatio-temporal cell, to which we can assign a value of  $v(t)$ ,  $v(\alpha, \delta t_i)$ . A target class  $C$  can be formed via any criterion that specifies a subset  $\mathcal{S}_C \times \mathcal{T}_C \subset \mathcal{S} \times \mathcal{T}$  of cells. For instance, those spatial cells that have confirmed cases in the time interval  $\delta t_i$ , or the 10% of spatial cells that have the highest mortality, as calculated in the interval  $\delta t_i$ . Taking the discretisation  $(\alpha, \delta t_i)$  as the finest resolution, then  $v(\Delta S, \Delta T)$  can be calculated for any spatial region  $\Delta S$  and time interval  $\Delta T$ . In the case of an extensive variable, such as number of deaths, this corresponds to a simple aggregation.

Turning now to the habitat variables,  $\mathbf{X}$ ; in the same way, we can define a habitat variable  $X_i(\alpha, \delta t_i)$  on a spatial cell in the time interval  $\delta t_i$ . It may be that  $X_i(\alpha, \delta t_i) = X_i(\alpha)$ , i.e., it is independent of time over the interval of interest. For instance, over the lifetime of the pandemic we may take a climatic variable, such as average annual temperature, to be constant. On the other hand, if  $X_i(\alpha, \delta t_i)$  represents the number of confirmed cases in the interval  $\delta t_i$ , to be used as a niche variable, then certainly it will not be constant. The validity of an assumption of equilibrium depends on the rates of change and the magnitudes of change of  $C$  and  $\mathbf{X}$  over the time period of interest. If the changes are slow and small over this period, then equilibrium may be a good approximation. Clearly, many variables of interest for COVID-19 are changing rapidly, such as vaccination rates, social distancing measures, travel restrictions etc.

Considering first a purely spatial model, we calculate  $P(C(t)|\mathbf{X}(t))$  using the formalism of Method section. This will be calculated using data for a period  $\Delta T$ . A score for a niche variable  $X_i^m(t)$  is given by  $s(X_i^m(t)) = \ln\left(\frac{P(X_i^m(t)|C(t))}{P(X_i^m(t)|\bar{C}(t))}\right)$ , where  $P(X_i^m(t)|C(t)) = N(X_i^m(t)C(t))/N(C(t))$  and  $P(X_i^m(t)|\bar{C}(t)) = (N(X_i^m(t)) - N(X_i^m(t)C(t)))/(N - N(C(t)))$ .  $N(X_i^m(t)C(t))$  is the number of cells with a co-occurrence of  $X_i^m(t)$  and  $C(t)$ . For example,  $C(t)$  could represent the 10% of cells (Mexican municipalities) with the highest number of cases in month  $t$ , while  $X_i^m(t)$  could represent a dynamic habitat variable, such as the 10% of municipalities with the highest average mobility in that month, or a more static variable, such as the 10% of municipalities with the highest average income, or those municipalities that have pixels from an average annual temperature raster in the range  $20.6 - 22.3 \text{ deg } C$ .  $N(X_i^m(t)C(t))$  thus counts, for example, the number of municipalities that are in the top 10% with the highest number of cases in month  $t$  and that also are in the top 10% of municipalities with the highest intra-municipal daily labor flows. We can calculate  $s(X_i^m(t))$  in different time intervals,  $t_i$ , and compare  $s(X_i^m(t_i))$  with  $s(X_i^m(t_j))$ . This can be done for all  $X_i^m$  and the degree of correlation in time calculated.

Although  $X_i^m(t)$  and  $C(t)$  may change in time, this does not imply that the ENM  $P(C(t)|\mathbf{X}(t))$  itself changes in time. The difference between  $s(X_i^m(t_i))$  and  $s(X_i^m(t_j))$ , however, will inform us of any changes in the ENM itself. In the case that the niche is conserved, i.e.,  $s(X_i^m(t_i)) \sim s(X_i^m(t_j))$ , to predict the spatial distribution of the target class at  $t'$ , we use the scores,  $s(X_i^m(t))$ , from the ENM at time  $t$ . For a given spatial cell,  $\alpha$  at time  $t' > t$ , we associate the score contribution  $s(X_i^m(\alpha, t'))$ . Thus, for example, if  $X_i$  is divided into deciles,  $X_i^m, m = [1, 10]$ , a given spatial cell at time  $t$  may be associated with decile  $m$ , whereas at time  $t'$  it may be associated with a different decile  $m'$ , so that the cell  $\alpha$  has become more niche-like/anti-niche-like in the cases that  $s(X_i^m) < s(X_i^{m'})$  or  $> s(X_i^{m'})$  respectively.

Niche conservation implies that for a given habitat state,  $\mathbf{X}$ , then  $P(C(t)|\mathbf{X})$  will not depend on time, i.e., there is the same probability to find the target species at any time  $t$ . When the niche is not conserved, however, then  $P(C(t)|\mathbf{X}) \neq P(C(t')|\mathbf{X})$ , i.e., the probability to find the target species given a certain habitat changes. In this case, the ENM  $P(C(t)|\mathbf{X}(t))$  will have limited predictability for  $t' > t$ . To handle this situation we must only use  $P(C(t)|\mathbf{X}(t))$  to predict into the future with a time horizon  $\Delta_H = (t' - t)$  such that  $s(X_i^m(t_i)) \sim s(X_i^m(t_j))$ . We may then create a new model at  $t + \Delta_H$ ,  $P(C(t + \Delta_H)|\mathbf{X}(t + \Delta_H))$  and use it to predict at  $t + 2\Delta_H$  and so on. In terms of a SDM, the fact that the habitat is changing means that spatial cells where the species was present/absent at  $t$  may now exhibit absence/presence. A useful way to quantify this, instead of using as target where the species is present in a given time period, is to focus on the *changes* between one period and the next. For example, we can take as target  $\Delta_C(t, t')$  those spatial cells where the species was present/absent in period  $t$  but then is absent/present in period  $t'$ . An explicit example is to consider those spatial cells where there were no confirmed cases in period  $t - 1$  but there were cases in period  $t$ . The data for the pair of periods  $t - 1$  and  $t$  is taken as training data and the corresponding scores calculated. The target class,  $\Delta_{C_{01}}(t, t - 1)$ , now consists of those spatial cells where there was an absence,  $C(t - 1) = 0$ , in period  $t - 1$  and a presence,  $C(t) = 1$ , in period  $t$ .

Thus, for a habitat variable  $X_i^m$  one calculates the score  $s(X_i^m(t - 1)) = \ln(P(X_i^m(t - 1)|\Delta_{C_{01}}(t, t - 1))$

$1)) / (P(X_i^m(t-1) | \Delta_{\bar{C}_{01}}(t, t-1))$ , where  $P(X_i^m(t-1) | \Delta_{C_{01}}(t, t-1)) = N(\Delta_{C_{01}}(t, t-1) X_i^m(t-1)) / N(\Delta_{C_{01}}(t, t-1))$  and  $P(X_i^m(t-1) | \Delta_{\bar{C}_{01}}(t, t-1)) = N(\Delta_{\bar{C}_{01}}(t, t-1) X_i^m(t-1)) / N(\Delta_{\bar{C}_{01}}(t, t-1))$ , where  $\bar{C}_{01}(t, t-1)$  is the set complement of  $\Delta_{C_{01}}(t, t-1)$ . The scores  $s(X_i^m(t-1))$  now define the ENM which can now be applied to the habitat variables  $X_i^m(t)$  at time  $t$  to predict those cells that have an absence for confirmed cases at  $t$  but are predicted to have cases at  $t+1$ . Similarly, for the class variable being top 10% of municipalities with highest number of cases, we may consider  $\Delta_{C_{01}}(t, t-1)$  to represent those municipalities that were not in the top 10% at  $t-1$  but, due to a worsening epidemiological situation, passed into the top 10% at time  $t$ .

Finally, although we will not enter into detail here, we may also consider as habitat variables, changes in the habitat. For example, we may consider  $\Delta_{X_i^{m m'}}(t, t')$ , which represents those cells where there was a change in the habitat variable  $X_i$  from bin  $m'$  to bin  $m$  passing from period  $t'$  to period  $t$ .