Proof of Lemma 1:

Commencing with the second-order condition for the shipping company's profit function, we have $\frac{\partial^2 \pi_s{}^{NBCT}}{\partial p^2} = -2 < 0$, which confirms that π_s^{NBCT} is a concave function with respect to the reefer freight rate p. Consequently, there exists an optimal reefer freight rate p that maximizes π_s^{NBCT}. For the port's profit function, the second-order condition yields $\frac{\partial^2 \pi_p{}^{NBCT}}{\partial w^2} = -1 < 0$, indicating that π_p^{NBCT} is concave in terms of the port service price w.There also exists an optimal w that maximizes π_p^{NBCT}.We obtain the equilibrium results as follows:

$$f^{NBCT*} = \frac{3a + bkr}{4b}$$

$$w^{NBCT*} = \frac{a - bkr}{2b}$$

$$q^{NBCT*} = \frac{1}{4}(a - bkr)$$

$$\pi_p^{NBCT*} = \frac{(a - bkr)^2}{8b}$$

$$\pi_s^{NBCT*} = \frac{(a - bkr)^2}{16b}$$

Proof of Lemma 2:

Firstly, the second-order partial derivative with respect to reefer freight rate p for the shipping company's profit function, $\frac{\partial^2 \pi_s{}^{(2)}}{\partial p^2} = -2 < 0$, demonstrates that π_s^{PBCT} is concave in p. Hence, there exists an optimal reefer freight rate p that maximizes π_s^{PBCT}.

Subsequently, by determining the Hessian matrix of π_p^{PBCT} with respect to w and e, as presented below, we observe that the first order of leading principal minor of the Hessian matrix is -1. Thus, when $\beta_p > \frac{(bkr+\lambda)^2}{4b}$, the Hessian matrix is negative definite, and π_p^{PBCT} is a jointly concave function in w and e. To ensure that $e < 1$, we obtain $\beta_p > \frac{(\lambda+a)(\lambda+bkr)}{4b}$

$$\begin{vmatrix} \frac{\partial^2 \pi_p{}^{PBCT}}{\partial w^2} & \frac{\partial^2 \pi_p{}^{PBCT}}{\partial w \partial e} \\ \frac{\partial^2 \pi_p{}^{PBCT}}{\partial e \partial w} & \frac{\partial^2 \pi_p{}^{PBCT}}{\partial e^2} \end{vmatrix} = \begin{vmatrix} -b & \frac{1}{2}(bkr + \lambda) \\ \frac{1}{2}(bkr + \lambda) & -\beta_p \end{vmatrix}$$

We obtain the equilibrium results as follows:

$$f^{PBCT*} = \frac{3a\beta_p - akr(bkr + \lambda) - kr(-b\beta_p + bkr\lambda + \lambda^2)}{4\beta_p b - (\lambda + bkr)^2}$$

$$w^{PBCT*} = \frac{2(a - bkr)\beta_p}{4\beta_p b - (\lambda + bkr)^2}$$

$$e^{PBCT*} = \frac{(a - bkr)(bkr + \lambda)}{4\beta_p b - (\lambda + bkr)^2}$$

$$q^{PBCT*} = \frac{b(a-bkr)\beta_p}{4\beta_p b-(\lambda+bkr)^2}$$

$$\pi_p^{PBCT*} = \frac{(a-bkr)^2\beta_p}{2(4\beta_p b-(\lambda+bkr)^2)}$$

$$\pi_s^{PBCT*} = \frac{b(a-bkr)^2{\beta_p}^2}{\left(4\beta_p b-(\lambda+bkr)^2\right)^2}$$

Proof of Lemma 3:

Similar to Proof of Proposition 3. The Hessian matrix is:

$$\begin{vmatrix} \frac{\partial^2 {\pi_s}^{SBCT}}{\partial p^2} & \frac{\partial^2 {\pi_s}^{SBCT}}{\partial p\partial e} \\ \frac{\partial^2 {\pi_s}^{SBCT}}{\partial e\partial p} & \frac{\partial^2 {\pi_s}^{SBCT}}{\partial e^2} \end{vmatrix} = \begin{vmatrix} -2\mathrm{b} & -\mathrm{b}kr+\lambda \\ -\mathrm{b}kr+\lambda & -\beta_s+2kr\lambda \end{vmatrix}$$

The first order of leading principal minor of the Hessian matrix is $-2b$. Thus, when $\beta_s > \frac{(bkr+\lambda)^2}{2b}$, the Hessian matrix is negative definite. To ensure that $e < 1$, we obtain $\beta_s > \frac{(a+2\lambda+\mathrm{b}kr)(\mathrm{b}kr+\lambda)}{4b}$.

We obtain the equilibrium results as follows:

$$f^{SBCT*} = \frac{\mathrm{b}kr(\mathrm{b}\beta-\mathrm{b}kr\lambda-\lambda^2)+a(3\mathrm{b}\beta_s-2\mathrm{b}^2k^2r^2-3\mathrm{b}kr\lambda-\lambda^2)}{2\mathrm{b}(2\beta_s\mathrm{b}-(\lambda+bkr)^2)}$$

$$w^{\mathrm{SBCT}*} = \frac{a-\mathrm{b}kr}{2\mathrm{b}}$$

$$e^{SBCT*} = \frac{(a-\mathrm{b}kr)(\mathrm{b}kr+\lambda)}{2(2\beta_s\mathrm{b}-(\lambda+bkr)^2)}$$

$$q^{\mathrm{SBCT}*} = \frac{\mathrm{b}(a-\mathrm{b}kr)\beta_s}{2(2\beta_s\mathrm{b}-(\lambda+bkr)^2)}$$

$$\pi_p^{\mathrm{SBCT}*} = \frac{(a-\mathrm{b}kr)^2\beta_s}{4(2\beta_s\mathrm{b}-(\lambda+bkr)^2)}$$

$$\pi_s^{SBCT*} = \frac{(a-\mathrm{b}kr)^2\beta_s}{8(2\beta_s\mathrm{b}-(\lambda+bkr)^2)}$$

Proof of Proposition 1:

Since $q^{PBCT*}-q^{NBCT*} = -\frac{(a-\mathrm{b}kr)(\mathrm{b}kr+\lambda)^2}{4(\mathrm{b}^2k^2r^2-4\mathrm{b}\beta+2\mathrm{b}kr\lambda+\lambda^2)} > 0$, $q^{SBCT*}-q^{NBCT*} = -\frac{(a-\mathrm{b}kr)(\mathrm{b}kr+\lambda)^2}{4(\mathrm{b}^2k^2r^2-2\mathrm{b}\beta+2\mathrm{b}kr\lambda+\lambda^2)} > 0$.We have $q^{NBCT*} < min\{q^{PBCT*}, q^{\mathrm{SBCT}*}\}$. Proposition 1(a) is proven.

Since $\pi_p^{PBCT*}-\pi_p^{\mathrm{NBCT}*} = -\frac{(a-\mathrm{b}kr)^2(\mathrm{b}kr+\lambda)^2}{8\mathrm{b}\left(\mathrm{b}^2k^2r^2-4\mathrm{b}\beta_p+2\mathrm{b}1kr\lambda+\lambda^2\right)} > 0$, $\pi_s^{PBCT*}-\pi_s^{NBCT*} = -\frac{(a-\mathrm{b}kr)^2(\mathrm{b}kr+\lambda)^2\left(\mathrm{b}^2k^2r^2-8\mathrm{b}\beta_p+2\mathrm{b}kr\lambda+\lambda^2\right)}{16\mathrm{b}\left(\mathrm{b}^2k^2r^2-4\mathrm{b}\beta_p+2\mathrm{b}kr\lambda+\lambda^2\right)^2} > 0$. We have $\pi_p^{NBCT*} < min\{\pi_p^{\mathrm{PBCT}*}, \pi_p^{\mathrm{SBCT}*}\}$.

Since $\pi_p^{SBCT*}-\pi_p^{\mathrm{NBCT}*} = -\frac{(a-\mathrm{b}kr)^2(\mathrm{b}kr+\lambda)^2}{8\mathrm{b}(\mathrm{b}^2k^2r^2-2\mathrm{b}\beta_s+2\mathrm{b}kr\lambda+\lambda^2)} > 0$, $\pi_s^{SBCT*}-\pi_s^{NBCT*} = -\frac{(a-\mathrm{b}kr)^2(\mathrm{b}kr+\lambda)^2}{16\mathrm{b}(\mathrm{b}^2k^2r^2-2\mathrm{b}\beta_s+2\mathrm{b}kr\lambda+\lambda^2)} > 0$. We have $\pi_s^{NBCT*} < min\{\pi_s^{\mathrm{PBCT}*}, \pi_s^{\mathrm{SBCT}*}$.Proposition 1(b) is proven.

Proof of Proposition 2:

We get the consumer surplus and social welfare under three models:

$$CS^{NBCT} = \frac{(a - \mathrm{b}kr)^2}{32\mathrm{b}}$$

$$CS^{PBCT} = \frac{\mathrm{b}(a - \mathrm{b}kr)^2{\beta_p}^2}{2(4\beta_p\mathrm{b} - (\lambda + bkr)^2)^2}$$

$$CS^{SBCT} = \frac{\mathrm{b}(a - \mathrm{b}kr)^2{\beta_s}^2}{8(2\beta_s b - (\lambda + bkr)^2)^2}$$

$$SW^{NBCT} = \frac{7(a - \mathrm{b}kr)^2}{32\mathrm{b}}$$

$$SW^{PBCT} = \frac{(a - \mathrm{b}kr)^2\beta_p(7\mathrm{b}\beta_p - (\lambda + bkr)^2)}{2(4\mathrm{b}\beta_p - (\lambda + bkr)^2)^2}$$

$$SW^{SBCT} = \frac{(a - bkr)^2\beta_s(7b\beta_s - 3(\lambda + bkr)^2)}{8(2b\beta_s - (\lambda + bkr)^2)^2}$$

Since $CS^{PBCT} - CS^{NBCT} = -\frac{(a-\mathrm{b}kr)^2(\mathrm{b}kr+\lambda)^2\left(\mathrm{b}^2k^2r^2-8\mathrm{b}\beta_p+2\mathrm{b}kr\lambda+\lambda^2\right)}{32\mathrm{b}\left(\mathrm{b}^2k^2r^2-4\mathrm{b}\beta_p+2\mathrm{b}kr\lambda+\lambda^2\right)^2} > 0$, $CS^{SBCT} - CS^{NBCT} = -\frac{(a-\mathrm{b}kr)^2(\mathrm{b}kr+\lambda)^2\left(\mathrm{b}^2k^2r^2-4\mathrm{b}\beta_s+2\mathrm{b}kr\lambda+\lambda^2\right)}{32\mathrm{b}(\mathrm{b}^2k^2r^2-2\mathrm{b}\beta_s+2\mathrm{b}kr\lambda+\lambda^2)^2} > 0$.We have $CS^{\mathrm{NBCT}} < min\{CS^{\mathrm{PBCT}}, CS^{SBCT}\}$. Proposition 2(a) is proven.

Since $SW^{\mathrm{PBCT}} - SW^{\mathrm{NBCT}} = -\frac{(a-\mathrm{b}kr)^2(\mathrm{b}kr+\lambda)^2\left(7\mathrm{b}^2k^2r^2-40\mathrm{b}\beta_p+14\mathrm{b}kr\lambda+7\lambda^2\right)}{32\mathrm{b}\left(\mathrm{b}^2k^2r^2-4\mathrm{b}\beta_p+2\mathrm{b}kr\lambda+\lambda^2\right)^2} > 0$, $SW^{\mathrm{SBCT}} - SW^{\mathrm{NBCT}} = -\frac{(a-\mathrm{b}kr)^2(\mathrm{b}kr+\lambda)^2\left(7\mathrm{b}^2k^2r^2+7\lambda^2+2\mathrm{b}(-8\beta_s+7kr\lambda)\right)}{32\mathrm{b}(\mathrm{b}^2k^2r^2-2\mathrm{b}\beta_s+2\mathrm{b}kr\lambda+\lambda^2)^2} > 0$. Proposition 2(b) is proven.

Proof of Proposition 3:

We have $\frac{q^{SBCT*}}{q^{PBCT*}} = \frac{((\mathrm{b}kr+\lambda)^2-4\mathrm{b}\beta_p)\beta_s}{2\beta_p((\mathrm{b}kr+\lambda)^2-2\mathrm{b}\beta_s)}$, if $\beta_p > \frac{\beta_s}{2}$, then $\frac{q^{SBCT*}}{q^{PBCT*}} > 1$, i.e $q^{SBCT*} > q^{\mathrm{PBCT}*}$; if $\frac{(\lambda+a)(\lambda+bkr)}{4b} < \beta_p < \frac{\beta_s}{2}$, then $\frac{q^{SBCT*}}{q^{PBCT*}} < 1$, i.e $q^{SBCT*} < q^{\mathrm{PBCT}*}$. Proposition 3 is proven.

Proof of Proposition 4:

We have $\frac{\pi_p^{\mathrm{SBCT}*}}{\pi_p^{\mathrm{PBCT}*}} = \frac{2\beta_p((\mathrm{b}kr+\lambda)^2-2\mathrm{b}\beta_s)}{((\mathrm{b}kr+\lambda)^2-4\mathrm{b}\beta_p)\beta_s}$, if $\beta_p > \frac{\beta_s}{2}$, then $\frac{\pi_p^{\mathrm{SBCT}*}}{\pi_p^{\mathrm{PBCT}*}} > 1$, i.e $\pi_s^{\mathrm{SBCT}*} > \pi_s^{\mathrm{PBCT}*}$; if $\frac{(\lambda+a)(\lambda+bkr)}{4b} < \beta_p < \frac{\beta_s}{2}$, then $\frac{\pi_p^{\mathrm{SBCT}*}}{\pi_p^{\mathrm{PBCT}*}} < 1$, i.e $\pi_s^{\mathrm{SBCT}*} < \pi_s^{\mathrm{PBCT}*}$. Proposition 4(a) is proven.

We have $\pi_s^{\mathrm{SBCT}*} - \pi_s^{\mathrm{PBCT}*} = -\frac{(-a+\mathrm{b}kr)^2(\mathrm{b}kr+\lambda)^2(8\mathrm{b}\beta_p^2+\mathrm{b}1^2k^2r^2\beta_s+2\mathrm{b}kr\lambda\beta_s+\lambda^2\beta_s-8\mathrm{b}\beta_p\beta_s)}{8(\mathrm{b}^2k^2r^2+2\mathrm{b}kr\lambda+\lambda^2-4\mathrm{b}1\beta_p)^2(\mathrm{b}^2k^2r^2+2\mathrm{b}kr\lambda+\lambda^2-2\mathrm{b}\beta_s)}$. If $\pi_s^{\mathrm{SBCT}*} - \pi_s^{\mathrm{PBCT}*} > 0$, we can obtain $\beta_p > \frac{\beta_s}{2} + \frac{\sqrt{2}\sqrt{-\mathrm{b}^3k^2r^2\beta_s-2\mathrm{b}^2kr\lambda\beta_s-\mathrm{b}\lambda^2\beta_s+2\mathrm{b}^2\beta_s^2}}{4\mathrm{b}}$, then we can simplify this to $\beta_p > M\beta_s(M > 1/2)$. If $\pi_s^{\mathrm{SBCT}*} - \pi_s^{\mathrm{PBCT}*} < 0$,we can obtain $\frac{(\lambda+a)(\lambda+bkr)}{4b} < \beta_p < M\beta_s$. Proposition 4(b) is proven.

Proof of Proposition 5:

We have $CS^{SBCT} - CS^{PBCT} = -\frac{(\mathrm{b}kr+\lambda)^2(2\beta_p-\beta_s)(2\mathrm{b}^2k^2r^2\beta_p+4\mathrm{b}kr\lambda\beta_p+2\lambda^2\beta_p+\mathrm{b}^2k^2r^2\beta_s+2\mathrm{b}kr\lambda\beta_s+\lambda^2\beta_s-8\mathrm{b}\beta_p\beta_s)}{(-\mathrm{b}^2k^2r^2-2\mathrm{b}kr\lambda-\lambda^2+4\mathrm{b}\beta_p)^2(\mathrm{b}^2k^2r^2+2\mathrm{b}kr\lambda+\lambda^2-2\mathrm{b}\beta_s)^2}$.

If $CS^{\mathrm{SBCT}} > CS^{PBCT}$, then $\beta_p > \frac{\beta_s}{2}$, if $CS^{\mathrm{SBCT}} < CS^{\mathrm{PBCT}}$,then $\frac{(\lambda+a)(\lambda+bkr)}{4b} < \beta_p < \frac{\beta_s}{2}$. Proposition 5 is proven.

Proof of Proposition 6:

We have $\frac{SW^{\mathrm{SBCT}}}{SW^{\mathrm{PBCT}}} = \frac{((\mathrm{b}kr+\lambda)^2-4\mathrm{b}\beta_p)^2\beta_s(3(\mathrm{b}kr+\lambda)^2-7\mathrm{b}\beta_s)}{4\beta_p((\mathrm{b}kr+\lambda)^2-7\mathrm{b}\beta_p)((\mathrm{b}kr+\lambda)^2-2\mathrm{b}\beta_s)^2}$, If $N_1\beta_s < \beta_p < N_2\beta_s$, then $SW^{\mathrm{SBCT}} < SW^{\mathrm{PBCT}}$, where

$$N_1 = \frac{((\mathrm{b}kr+\lambda)^4 + 2\mathrm{b}\beta_s((\mathrm{b}kr+\lambda)^2 - 5\mathrm{b}\beta_s) - \sqrt{(\mathrm{b}kr+\lambda)^8 + \mathrm{b}\beta_s(-17(\mathrm{b}kr+\lambda)^6 + \mathrm{b}\beta_s(81(\mathrm{b}kr+\lambda)^4 + 4\mathrm{b}\beta_s(-38(\mathrm{b}kr+\lambda)^2 + 25\mathrm{b}\beta_s))))}}{(2\mathrm{b}\beta_s(7(\mathrm{b}kr+\lambda)^2 - 16\mathrm{b}\beta_s))}$$

$$N_2 = \frac{((\mathrm{b}kr+\lambda)^4 + 2\mathrm{b}\beta_s((\mathrm{b}kr+\lambda)^2 - 5\mathrm{b}\beta_s) + \sqrt{(\mathrm{b}kr+\lambda)^8 + \mathrm{b}\beta_s(-17(\mathrm{b}kr+\lambda)^6 + \mathrm{b}\beta_s(81(\mathrm{b}kr+\lambda)^4 + 4\mathrm{b}\beta_s(-38(\mathrm{b}kr+\lambda)^2 + 25\mathrm{b}\beta_s))))}}{(2\mathrm{b}\beta_s(7(\mathrm{b}kr+\lambda)^2 - 16\mathrm{b}\beta_s))}$$

Proposition 6 is proven.

Proof of Proposition 7:

We have $\frac{\partial\pi_s^{SBCT*}}{\partial r} = \frac{\mathrm{b}k(a-\mathrm{b}kr)\beta_s(-2\mathrm{b}\beta_s+\mathrm{b}kr\lambda+\lambda^2+a(\mathrm{b}kr+\lambda))}{4(\mathrm{b}^2k^2r^2-2\mathrm{b}\beta_s+2\mathrm{b}kr\lambda+\lambda^2)^2}, \frac{\partial\pi_p^{SBCT*}}{\partial r} = \frac{\mathrm{b}k(a-\mathrm{b}kr)\beta_s(-2\mathrm{b}\beta_s+\mathrm{b}kr\lambda+\lambda^2+a(\mathrm{b}kr+\lambda))}{2(\mathrm{b}^2k^2r^2-2\mathrm{b}\beta_s+2\mathrm{b}kr\lambda+\lambda^2)^2}$.

When $\beta_2 < \beta_s < \beta_3$, then $\frac{\partial\pi_s^{SBCT*}}{\partial r}, \frac{\partial\pi_p^{SBCT*}}{\partial r} > 0, \mathrm{When}\beta_s > \beta_3, \mathrm{then}\ \frac{\partial\pi_s^{SBCT*}}{\partial r}, \frac{\partial\pi_p^{SBCT*}}{\partial r} < 0.$ Where $\beta_2 = \frac{(a+2\lambda+\mathrm{b}kr)(\mathrm{b}kr+\lambda)}{4b}, \beta_3 = \frac{(a+\lambda)(\mathrm{b}kr+\lambda)}{2\mathrm{b}}$.

We have $\frac{\partial q^{PBCT*}}{\partial r} = \frac{\mathrm{b}^2k\beta_p(\mathrm{b}kr(2a-\mathrm{b}kr)-4\beta_pb+2a\lambda+\lambda^2)}{(-4\beta_pb+(bkr+\lambda)^2)^2}, \frac{\partial q^{SBCT*}}{\partial r} = \frac{\mathrm{b}^2k\beta_s(\mathrm{b}kr(2a-\mathrm{b}kr)-2\mathrm{b}\beta_s+2a\lambda+\lambda^2)}{2(\mathrm{b}^2k^2r^2-2\mathrm{b}\beta_s+2\mathrm{b}kr\lambda+\lambda^2)^2}$.

When$\beta_2 < \beta_s < \beta_4, \mathrm{then}\ \frac{\partial q^{SBCT*}}{\partial r} > 0.$ Where $\beta_4 = \frac{(2a-\mathrm{b}kr+\lambda)(\mathrm{b}kr+\lambda)}{2\mathrm{b}}$, $\beta_3 < \beta_4$.