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Supplementary File

Optimal Decisions in a Sea-Cargo Supply Chain with Two Competing Freight Forwarders Considering Altruistic Preference and Brand Investment

S1. Real Case (Including Key Parts in English)

Corporate Name: Intent Logistics Co., Ltd (Intent International Freight Forwarding Co., Ltd, Shenzhen, China.)

Web Site: http://www.szycil.com/show-15-18844-1.html (Accessed on 14 July 2023)

Webpage Snapshot:

Figure S1. Introduction to China COSCO Shipping Group.

Key Parts in English: The name of the shipping and airfreight forwarding company is China COSCO Shipping Air Freight Agency Co., Ltd, Tianjin, China. It was established on 11 July 1995, with its registered office located at Room 215, No. 1, Second Street, International Logistics Area, Tianjin Free Trade Zone. The company's scope of operations includes international air freight forwarding, cultural brokerage services, international road and sea freight forwarding, road freight agency, warehousing services, economic information consulting, import and export of goods, as well as the provision of application software, jewelry, handicrafts, machinery and equipment sales, non-vessel operating common carrier (NVOCC) services, packaging services, general cargo transportation, international express delivery, ship supplies, wholesale and retail of ship equipment and accessories. Additionally, the company offers ship technology consulting services and multimodal transport agency services.

China COSCO Shipping Air Freight Agency Co., Ltd. has invested in six external companies and has 15 branch offices. China COSCO Shipping Group company was formed on February 18, 2016, through the restructuring of China COSCO Shipping Corporation Limited, which was a merger between China COSCO Shipping Group and China Shipping Group. The establishment of China COSCO Shipping Group marked a new chapter in China's maritime history. Over the past five years, the company has focused on high-quality development and has become an important player in the international shipping industry, balancing the influence of the East and the West.

China COSCO Shipping Group operates in 11 countries and 12 ports, including the ports of Piraeus in Greece, Abu Dhabi, Zeebrugge in Belgium, NOATUM, New Port, Antwerp in Belgium, KUMPORT in Turkey, Suez Canal, EYROMAX in Rotterdam, Vado in Italy, Busan in South Korea, and Seattle in the United States.

S2. Proofs of Lemmas and Propositions

S2.1. Proof of Lemma 1

Simultaneously solving Equations (12) and (16) for p_1 and p_2 yields the unique optimal decision pair $(p_1^*(w_1,e), t_1^*(w_1,e))$ in reaction to the shipping prices. In fact, from Equation (12) we have

$$(-2+B)p_1+\mu p_2=b_1 \tag{S1}$$

and from Equation (16) we have

$$\mu p_1+(-2+B)p_2=b_2 \tag{S2}$$

According to Cramer's Rule, solving Equations (S1) and (S2) for p_1 and p_2 simultaneously yields the optimal decisions p_1^* in Equation (19) and p_2^* in Equation (21). Note that p_1^* and p_2^* are unique because the determinant of the coefficient matrix of the equation system composed of Equations (S1) and (S2) is nonzero based on Equation (5), i.e., $\begin{vmatrix} -2+B & \mu \\ \mu & -2+B \end{vmatrix} = (-2+B)^2-\mu^2 \neq 0$.

Substituting Equations (19) and (21) into Equations (13) and (17), respectively, we can derive t_1^* in Equation (20) and t_2^* in Equation (22). In fact, note that

$$\begin{aligned} p_1^*(w_1,e)-w_1 &= \frac{(-2+B)b_1-\mu b_2}{\left(-2+B\right)^2-\mu^2}-w_1=\frac{(-2+B)b_1-\mu b_2-\left(-2+B\right)^2 w_1+\mu^2 w_1}{\left(-2+B\right)^2-\mu^2} \\ &= \frac{(-2+B)[-k-\lambda e+(B-1)w_1-\left(-2+B\right)w_1]-\mu b_2+\mu^2 w_1}{\left(-2+B\right)^2-\mu^2} \\ &= \frac{(-2+B)[-k-\lambda e+w_1]-\mu b_2+\mu^2 w_1}{\left(-2+B\right)^2-\mu^2} \end{aligned} \tag{S3}$$

Thus, from Equations (13) and (S3), Formula (20) holds, and similarly we can obtain Formula (22). It is clear that $t_1^*(w_1,e)$ and $t_2^*(w_2,e)$ are also unique according to the uniqueness of $p_1^*(w_1,e)$ and $p_2^*(w_2,e)$. So, Lemma 1 is proved. □

S2.2. Proof of Lemma 2

From Equation (23), it can be deduced that

$$\begin{aligned} d &= (2-B+\mu)k+c[-2+B+\mu^2+\mu(1-B)]=(2-B+\mu)k+c[-2+B+\mu^2+\mu-\mu B] \\ &= (2-B+\mu)k-c[2-B-\mu^2-\mu+\mu B]=(2-B+\mu)k-c[2-B+\mu-2\mu-\mu^2+\mu B] \\ &= (2-B+\mu)k-c[2-B+\mu-\mu(2-B+\mu)] \\ &= (2-B+\mu)k-c(2-B+\mu)(1-\mu)=(2-B+\mu)[k-c(1-\mu)] \end{aligned} \tag{S4}$$

According to Inequality (4) and Assumption 3, $d>0$. □

S2.3. Proof of Lemma 3

From Equation (24), it follows that

$$\begin{aligned} C+D &= -2+B+\mu^2+\mu(1-B)=-2+B-\mu+2\mu+\mu^2-\mu B \\ &= -(2-B+\mu)+\mu(2-B+\mu)=-(2-B+\mu)(1-\mu)<0 \end{aligned} \tag{S5}$$

$$\begin{aligned} C-D &= -2+B+\mu^2-\mu(1-B)=-2+B+\mu-2\mu+\mu B+\mu^2 \\ &= -(2-B-\mu)-\mu(2-B-\mu)=-(2-B-\mu)(1+\mu)<0 \end{aligned} \tag{S6}$$

The proof is completed. □

S2.4. Proof of Proposition 1

First, substituting Equations (14) and (18) into (19) yields

$$\begin{aligned} p_1^*(w_1,e) &= \frac{(-2+B)[-k-\lambda e-(1-B)w_1]-\mu[-k-\lambda e-(1-B)w_2]}{\left(-2+B\right)^2-\mu^2} \\ &= \frac{(2-B+\mu)(k+\lambda e)+(2-B)(1-B)w_1+\mu(1-B)w_2}{\left(-2+B\right)^2-\mu^2} \\ &= \frac{(2-B+\mu)(k+\lambda e)+(2-B)(1-B)(w_1-c)+\mu(1-B)(w_2-c)+c(1-B)[2-B+\mu]}{\left(-2+B\right)^2-\mu^2} \end{aligned} \tag{S7}$$

Similarly, we have

$$p_2^*(w_2,e)=\frac{(2-B+\mu)(k+\lambda e)+\mu(1-B)(w_1-c)+(2-B)(1-B)(w_2-c)+c(1-B)[2-B+\mu]}{\left(-2+B\right)^2-\mu^2} \tag{S8}$$

Second, substituting Equation (18) into (20), we derive the following expression for $t_1^*(w_1,e)$.

$$t_1^*(w_1,e)=B_0\cdot\frac{(-2+B)[-k-\lambda e+w_1]-\mu[-k-\lambda e-(1-B)w_2]+\mu^2 w_1}{(-2+B)^2-\mu^2} \tag{S9}$$

According to Equation (23), we obtain

$$\begin{aligned}&(-2+B)[-k-\lambda e+w_1]-\mu[-k-\lambda e-(1-B)w_2]+\mu^2 w_1\\&=(2-B+\mu)(k+\lambda e)+(-2+B)w_1+\mu(1-B)w_2+\mu^2 w_1\\&=(2-B+\mu)\lambda e+[-2+B+\mu^2](w_1-c)+\mu(1-B)(w_2-c)\\&\quad+(2-B+\mu)k+c[-2+B+\mu^2+\mu(1-B)]\\&=(2-B+\mu)\lambda e+(-2+B+\mu^2)(w_1-c)+\mu(1-B)(w_2-c)+d\end{aligned} \tag{S10}$$

Thus,

$$t_1^*(w_1,e)=B_0\cdot\frac{(2-B+\mu)\lambda e+(-2+B+\mu^2)(w_1-c)+\mu(1-B)(w_2-c)+d}{(-2+B)^2-\mu^2} \tag{S11}$$

Similarly, we can obtain

$$t_2^*(w_2,e)=B_0\cdot\frac{(2-B+\mu)\lambda e+\mu(1-B)(w_1-c)+(-2+B+\mu^2)(w_2-c)+d}{(-2+B)^2-\mu^2} \tag{S12}$$

Third, substituting Equations (S7), (S8) and (S11) into the demand function (1), we get

$$\begin{aligned}&Q_1=Q_1(p_1,p_2,e,t_1)=k-p_1+\mu p_2+\lambda e+\eta t_1\\&=k+\lambda e-\frac{(2-B+\mu)(k+\lambda e)+(2-B)(1-B)(w_1-c)+\mu(1-B)(w_2-c)+c(1-B)(2-B+\mu)}{(-2+B)^2-\mu^2}\\&\quad+\mu\frac{(2-B+\mu)(k+\lambda e)+\mu(1-B)(w_1-c)+(2-B)(1-B)(w_2-c)+c(1-B)(2-B+\mu)}{(-2+B)^2-\mu^2}\\&\quad+\eta B_0\cdot\frac{(2-B+\mu)\lambda e+(-2+B+\mu^2)(w_1-c)+\mu(1-B)(w_2-c)+d}{(-2+B)^2-\mu^2}\\&=\frac{\Delta_1(w_1-c)}{(-2+B)^2-\mu^2}+\frac{\Delta_2(w_2-c)}{(-2+B)^2-\mu^2}+\frac{\Delta_3\lambda e}{(-2+B)^2-\mu^2}+\frac{\Delta_4}{(-2+B)^2-\mu^2}\end{aligned} \tag{S13}$$

where

$$\begin{aligned}\Delta_1 &= -(2-B)(1-B)+\mu^2(1-B)+B(-2+B+\mu^2) = -2+B+\mu^2 \\ \Delta_2 &= -\mu(1-B)+\mu(2-B)(1-B)+B\mu(1-B) = \mu(1-B) \\ \Delta_3 &= (2-B+\mu)(-1+\mu+B)\lambda e+[(-2+B)^2-\mu^2] \\ &= (2-B+\mu)(-1+\mu+B+2-B-\mu) \\ &= 2-B+\mu, \\ \Delta_4 &= (2-B+\mu)(-1+\mu)k-c(1-B)(2-B+\mu)+\mu c(1-B)(2-B+\mu) \\ &\quad +Bd+[(-2+B)^2-\mu^2]k \\ &= (2-B+\mu)\{k-c+cB+\mu c-\mu cB-Bk+B[k-c(1-\mu)]\} \\ &= (2-B+\mu)[k-c(1-\mu)] \\ &= d\end{aligned} \tag{S14}$$

Thus, the demand function (1) can be expressed as

$$\begin{aligned}Q_1 = &\frac{-2+B+\mu^2}{(-2+B)^2-\mu^2}(w_1-c)+\frac{\mu(1-B)}{(-2+B)^2-\mu^2}(w_2-c) \\ &+\frac{\lambda(2-B+\mu)}{(-2+B)^2-\mu^2}e+\frac{d}{(-2+B)^2-\mu^2}\end{aligned} \tag{S15}$$

Similar to the above derivation, demand function (2) can be represented as

$$\begin{aligned}Q_2 = &\frac{\mu(1-B)}{(-2+B)^2-\mu^2}(w_1-c)+\frac{-2+B+\mu^2}{(-2+B)^2-\mu^2}(w_2-c) \\ &+\frac{\lambda(2-B+\mu)}{(-2+B)^2-\mu^2}e+\frac{d}{(-2+B)^2-\mu^2}\end{aligned} \tag{S16}$$

By taking partial derivatives of Equation (6) with respect to w_1, w_2, and e, respectively, one can derive that

$$\begin{aligned}\frac{\partial \pi_s}{\partial w_1} &= Q_1+(w_1-c)\frac{\partial Q_1}{\partial w_1}+(w_2-c)\frac{\partial Q_2}{\partial w_1} \\ \frac{\partial \pi_s}{\partial w_2} &= (w_1-c)\frac{\partial Q_1}{\partial w_2}+Q_1+(w_2-c)\frac{\partial Q_2}{\partial w_2} \\ \frac{\partial \pi_s}{\partial e} &= (w_1-c)\frac{\partial Q_1}{\partial e}+(w_2-c)\frac{\partial Q_2}{\partial e}-2\alpha e\end{aligned} \tag{S17}$$

By taking partial derivatives of Equations (S15) and (S16) with respect to w_1, w_2, and e, respectively, and substituting Equation (S17), we obtain that

$$\begin{aligned}\frac{\partial \pi_s}{\partial w_1} &= \frac{2C(w_1-c)+2D(w_2-c)+\lambda(2-B+\mu)e+d}{(-2+B)^2-\mu^2} \\ \frac{\partial \pi_s}{\partial w_2} &= \frac{2D(w_1-c)+2C(w_2-c)+\lambda(2-B+\mu)e+d}{(-2+B)^2-\mu^2} \\ \frac{\partial \pi_s}{\partial e} &= \frac{\lambda(2-B+\mu)}{(-2+B)^2-\mu^2}(w_1-c)+\frac{\lambda(2-B+\mu)}{(-2+B)^2-\mu^2}(w_2-c)-2\alpha e\end{aligned} \tag{S18}$$

where C and D are defined in Equation (24).

By setting the first-order partial derivatives to zero, we can obtain the following system of linear equations:

$$\begin{gathered} 2C(w_1-c)+2D(w_2-c)+\lambda(2-B+\mu)e+d=0 \\ 2D(w_1-c)+2C(w_2-c)+\lambda(2-B+\mu)e+d=0 \\ \lambda(w_1-c)+\lambda(w_2-c)-2\alpha(2-B-\mu)e=0 \end{gathered} \tag{S19}$$

For simplicity, we denote

$$\Delta=(-2+B)^2-\mu^2 \tag{S20}$$

Hence, the Hessian matrix is

$$H_1=\begin{bmatrix} 2C & 2D & \lambda(2-B+\mu) \\ 2D & 2C & \lambda(2-B+\mu) \\ \lambda & \lambda & -2\alpha(2-B-\mu) \end{bmatrix} \tag{S21}$$

The first leading principal minor of H_1 is $D_1=2C=2[-2+B+\mu^2]$. From Assumption 2, we have $D_1<2[-2+2-\mu+\mu^2]=2[-\mu(1-\mu)]<0$.

The second leading principal minor of H_1 is

$$D_2=4(C^2-D^2)=4(C-D)(C+D) \tag{S22}$$

Based on Lemma 3, it is clear that $D_2>0$.

The third leading principal minor of H_1 is

$$D_3=-4(C-D)[2\alpha(C+D)(2-B-\mu)+\lambda^2(2-B+\mu)] \tag{S23}$$

According to $C-D<0$ and the condition of the proposition, we have $D_3<0$.

Therefore, the matrix H_1 is negative definite, meaning that π_s is a strictly concave function of w_1, w_2, and e, and it has a unique solution to maximize π_s.

Finally, solving the system of linear equations (S18) for w_1, w_2, and e yields the optimal pricing decisions w_1^* and w_2^* in Equation (29) and brand value e^* in Equation (30).

From Inequality (4) and Condition (28) in this proposition, we have $\lambda^2(2-B+\mu)+2\alpha(C+D)(2-B-\mu)<0$. Based on this, Assumption 2 and Formula (25), it is clear that w_1^*, w_2^*, and e^* are positive and $w_1^*=w_2^*>c$.

Furthermore, substituting w_1^*, w_2^*, and e^* into Equation (S7) and noting that $w_1^*=w_2^*$, we can obtain the following

$$p_1^*=\frac{(2-B+\mu)(k+\lambda e^*)+(1-B)[(2-B)+\mu](w_1^*-c)+c(1-B)[2-B+\mu]}{\left(-2+B\right)^2-\mu^2} \tag{S24}$$

$$
\begin{aligned}
&=\frac{(k+\lambda e^{*})+(1-B)w_{1}^{*}+c(1-B)}{2-B-\mu}\\
&=\frac{k+(1-B)c}{(2-B-\mu)}+\frac{-d\lambda^{2}-d\alpha(1-B)(2-B-\mu)}{(2-B-\mu)[\lambda^{2}(2-B+\mu)+2\alpha(C+D)(2-B-\mu)]}\\
&=\frac{k+(1-B)c}{(2-B-\mu)}-c+\frac{-d\lambda^{2}-d\alpha(1-B)(2-B-\mu)}{(2-B-\mu)[\lambda^{2}(2-B+\mu)+2\alpha(C+D)(2-B-\mu)]}+c\\
&=\frac{k-(1-\mu)c}{(2-B-\mu)}+\frac{-d\lambda^{2}-d\alpha(1-B)(2-B-\mu)}{(2-B-\mu)[\lambda^{2}(2-B+\mu)+2\alpha(C+D)(2-B-\mu)]}+c.
\end{aligned}
$$

From Formula (25), we have

$$
\begin{aligned}
p_{1}^{*}&=\frac{[k-(1-\mu)c]\cdot 2\alpha(C+D)(2-B-\mu)-d\alpha(1-B)(2-B-\mu)}{(2-B-\mu)[\lambda^{2}(2-B+\mu)+2\alpha(C+D)(2-B-\mu)]}-(w_{1}^{*}-c)+w_{1}^{*}\\
&=\frac{[k-(1-\mu)c]\cdot 2\alpha(C+D)-d\alpha[(1-B)-(2-B-\mu)]}{\lambda^{2}(2-B+\mu)+2\alpha(C+D)(2-B-\mu)}+w_{1}^{*}\\
&=\frac{[k-(1-\mu)c]\cdot 2\alpha(C+D)+d\alpha(1-\mu)}{\lambda^{2}(2-B+\mu)+2\alpha(C+D)(2-B-\mu)}+w_{1}^{*}\\
&=\frac{\alpha[k-(1-\mu)c]\cdot[2(C+D)+(2-B+\mu)(1-\mu)]}{\lambda^{2}(2-B+\mu)+2\alpha(C+D)(2-B-\mu)}+w_{1}^{*}.
\end{aligned}
\tag{S25}
$$

Thus, from Lemma 3, we have

$$
\begin{aligned}
p_{1}^{*}&=\frac{\alpha[k-(1-\mu)c]\cdot[2(C+D)-(C+D)]}{\lambda^{2}(2-B+\mu)+2\alpha(C+D)(2-B-\mu)}+w_{1}^{*}\\
&=\frac{\alpha[k-(1-\mu)c]\cdot(C+D)}{\lambda^{2}(2-B+\mu)+2\alpha(C+D)(2-B-\mu)}+w_{1}^{*}\\
&=\frac{\alpha[k-(1-\mu)c]\cdot(C+D)}{\lambda^{2}(2-B+\mu)+2\alpha(C+D)(2-B-\mu)}+c+\frac{-d\alpha(2-B-\mu)}{\lambda^{2}(2-B+\mu)+2\alpha(C+D)(2-B-\mu)}\\
&=\frac{\alpha[k-c(1-\mu)]\cdot(C+D)-d\alpha(2-B-\mu)}{\lambda^{2}(2-B+\mu)+2\alpha(C+D)(2-B-\mu)}+c
\end{aligned}
\tag{S26}
$$

This leads to p_1^* in Equation (31). According to Assumption 3, Formula (26), and under the condition that $\lambda^2(2-B+\mu)+2\alpha(C+D)(2-B-\mu)<0$ (i.e., Condition (28)), it is clear that $p_1^* > w_1^*$.

Substituting w_1^*, w_2^*, and e^* into Equation (S8) and following a similar proof as above, we can obtain $p_2^* = p_1^* > w_1^* = w_2^*$.

Replacing w_1, w_2, and e with w_1^*, w_2^*, and e^* in (S11), respectively, and noting that $w_1^* = w_2^*$, we can obtain

$$
\begin{aligned}
t_1^* &= B_0 \cdot \frac{(2-B+\mu)\lambda e^* + (-2+B+\mu^2)(w_1^*-c) + \mu(1-B)(w_2^*-c) + d}{(-2+B)^2-\mu^2} \\
&= B_0 \cdot \frac{(2-B+\mu)\lambda e^* + [-2+B+\mu^2+\mu(1-B)](w_1^*-c) + d}{(-2+B)^2-\mu^2} \\
&= B_0 \cdot \frac{d + \lambda(2-B+\mu)e^* - [2-B-\mu+B\mu-\mu^2](w_1^*-c)}{(-2+B)^2-\mu^2}
\end{aligned} \tag{S27}
$$

Using Formula (S5) and substituting w_1^* and e^* into Equation (S27), it follows that

$$
\begin{aligned}
t_1^* &= B_0 \cdot \frac{d + (2-B+\mu)\lambda e^* + (C+D)(w_1^*-c)}{(-2+B)^2-\mu^2} \\
&= B_0 \cdot \frac{(2-B+\mu)[k-c(1-\mu)]}{(-2+B)^2-\mu^2} \cdot \frac{\alpha(C+D)(2-B-\mu)}{\lambda^2(2-B+\mu)+2\alpha(C+D)(2-B-\mu)} \\
&= \frac{B_0 \cdot \alpha(C+D)[k-c(1-\mu)]}{\lambda^2(2-B+\mu)+2\alpha(C+D)(2-B-\mu)}
\end{aligned} \tag{S28}
$$

Thus, we obtain t_1^* in Equation (32). According to Assumption 3, Formula (26), and the condition that $\lambda^2(2-B+\mu)+2\alpha(C+D)(2-B-\mu)<0$ (i.e., Condition (28)), it is clear that $t_1^*>0$.

Similarly, substituting w_1^*, w_2^*, and e^* into Equation (S12), it follows that $t_2^*=t_1^*>0$.

Substituting w_1^*, w_2^*, and e^* into Equation (S15), one can derive that

$$
\begin{aligned}
Q_1^* &= \frac{-2+B+\mu^2}{(-2+B)^2-\mu^2}(w_1^*-c) + \frac{\mu(1-B)}{(-2+B)^2-\mu^2}(w_2^*-c) \\
&\quad + \frac{\lambda(2-B+\mu)}{(-2+B)^2-\mu^2}e^* + \frac{d}{(-2+B)^2-\mu^2} \\
&= \frac{-2+B+\mu^2+\mu(1-B)}{(-2+B)^2-\mu^2}(w_1^*-c) + \frac{\lambda(2-B+\mu)}{(-2+B)^2-\mu^2}e^* + \frac{d}{(-2+B)^2-\mu^2} \\
&= \frac{-d[\alpha(2-B-\mu)(C+D)+\lambda^2(2-B+\mu)]}{[(-2+B)^2-\mu^2][\lambda^2(2-B+\mu)+2\alpha(C+D)(2-B-\mu)]} + \frac{d}{(-2+B)^2-\mu^2} \\
&= \frac{-d}{(-2+B)^2-\mu^2}\left[\frac{\alpha(2-B-\mu)(C+D)+\lambda^2(2-B+\mu)}{\lambda^2(2-B+\mu)+2\alpha(C+D)(2-B-\mu)} - 1\right] \\
&= \frac{\alpha(C+D)d}{(2-B+\mu)[\lambda^2(2-B+\mu)+2\alpha(C+D)(2-B-\mu)]} \\
&= \frac{\alpha(C+D)(2-B+\mu)[k-c(1-\mu)]}{(2-B+\mu)[\lambda^2(2-B+\mu)+2\alpha(C+D)(2-B-\mu)]} \\
&= \frac{\alpha(C+D)[k-c(1-\mu)]}{\lambda^2(2-B+\mu)+2\alpha(C+D)(2-B-\mu)}
\end{aligned} \tag{S29}
$$

Therefore, we get Q_1^* in Equation (33). According to Assumption 3, Formula (26), and the condition that $\lambda^2(2-B+\mu)+2\alpha(C+D)(2-B-\mu)<0$ (i.e., Condition (28)), it is clear that $Q_1^*>0$.

Substituting w_1^*, w_2^*, and e^* into Equation (S16), similar to the proof of Q_1^*, we obtain $Q_2^*=Q_1^*>0$.

Substituting w_1^*, w_2^*, e^*, Q_1^*, and Q_2^* into Equation (6) yields the maximal profit shown as follows:

$$\begin{aligned}\pi_s^* &= \left(w_1^*-c\right)Q_1^*+\left(w_2^*-c\right)Q_2^*-\alpha e^{*2}=(Q_1^*+Q_2^*)(w_1^*-c)-\alpha e^{*2}\\ &=\frac{\alpha d[k-c(1-\mu)][-2\alpha(C+D)(2-B-\mu)-\lambda^2(2-B+\mu)]}{[\lambda^2(2-B+\mu)+2\alpha(C+D)(2-B-\mu)]^2}\\ &=\frac{-\alpha d[k-c(1-\mu)]}{\lambda^2(2-B+\mu)+2\alpha(C+D)(2-B-\mu)}\end{aligned} \tag{S30}$$

Noting that $p_1-w_1=\dfrac{t_1}{B_0}$ from Equation (13), one can derive that

$$\begin{aligned}p_1^*-w_1^* &= \frac{\alpha[k-c(1-\mu)]\cdot(C+D)-d\alpha(2-B-\mu)}{\lambda^2(2-B+\mu)+2\alpha(C+D)(2-B-\mu)}+c-w_1\\ &=\frac{\alpha[k-c(1-\mu)]\cdot(C+D)}{\lambda^2(2-B+\mu)+2\alpha(C+D)(2-B-\mu)}\end{aligned} \tag{S31}$$

Substituting Equation (S31), Q_1^*, and t_1^* into Equation (7) yields the maximal profits of the two forwarders:

$$\begin{aligned}\pi_{f_1}^* &= \pi_{f_2}^*=\left(p_1^*-w_1^*\right)Q_1^*-\beta t_1^{*2}\\ &=\frac{(1-\beta B_0{}^2)[\alpha(C+D)[k-c(1-\mu)]^2}{[\lambda^2(2-B+\mu)+2\alpha(C+D)(2-B-\mu)]^2}\\ &=\frac{(1-\beta B_0{}^2)[\alpha(2-B+\mu)(1-\mu)]^2[k-c(1-\mu)]^2}{[\lambda^2(2-B+\mu)-2\alpha(2-B+\mu)(1-\mu)(2-B-\mu)]^2}\\ &=\frac{(1-\beta B_0{}^2)[\alpha(1-\mu)]^2[k-c(1-\mu)]^2}{[\lambda^2-2\alpha(1-\mu)(2-B-\mu)]^2}\end{aligned} \tag{S32}$$

From Assumption 1, we have $1-\beta B_0{}^2=1-\beta\left(\dfrac{\eta}{2\beta}\right)^2=1-\dfrac{\eta^2}{4\beta}>0$. Thus, $\pi_{f_1}^*=\pi_{f_2}^*>0$. □

S2.5. Proof of Lemma 4

Similar to the proof of Lemma 1, solving Equations (41) and (46) for p_1 and p_2 simultaneously yields the unique optimal decision pair (p_1^{**},p_2^{**}) in reaction to the shipping prices.

Substituting Equations (49) and (51) into Equations (42) and (47), respectively, we can obtain $t_1^{**}(w_1,e)$ in Equation (50) and $t_2^{**}(w_2,e)$ in Equation (52). In fact, note that

$$\begin{aligned}
&p_1^{**}(w_1,e)-\overline{w}_1=\frac{(-2+B)b_3-\mu b_4}{\left(-2+B\right)^2-\mu^2}-\overline{w}_1\\
&=\frac{(-2+B)[-k-\lambda e+(B-1)\overline{w}_1-\varepsilon\mu\left(w_2-c\right)-\left(-2+B\right)\overline{w}_1]-\mu b_4+\mu^2\overline{w}_1}{\left(-2+B\right)^2-\mu^2}\\
&=\frac{(-2+B)[-k-\lambda e+\overline{w}_1-\varepsilon\mu\left(w_2-c\right)]-\mu b_4+\mu^2\overline{w}_1}{\left(-2+B\right)^2-\mu^2}
\end{aligned} \tag{S33}$$

From Equations (42) and (S33), Formula (50) holds. Following a similar proof as above, we can obtain Equation (52). It is clear that $t_1^{**}(w_1,e)$ and $t_2^{**}(w_2,e)$ are also unique, given the uniqueness of $p_1^{**}(w_1,e)$ and $p_2^{**}(w_2,e)$. □

S2.6. Proof of Lemma 5

Directly from Equation (53), we have

$$\begin{aligned}
M+N&=[2-B+\mu(B-1)(3-B)+\mu^3]\varepsilon-[2-B-\mu^2+\mu(B-1)]\\
&=s_1\varepsilon+s_2
\end{aligned} \tag{S34}$$

where

$$\begin{aligned}
s_1&=2-B+\mu(B-1)(3-B)+\mu^3\\
&=2-B+\mu-\mu(B^2-4B+4)+\mu^3\\
&=(2-B+\mu)[1-\mu(2-B-\mu)]\\
&=(2-B+\mu)[(1-\mu)^2+\mu B]\\
&>0
\end{aligned} \tag{S35}$$

and

$$s_2=-[2-B-\mu^2+\mu(B-1)]=C+D=-(2-B+\mu)(1-\mu)<0 \tag{S36}$$

based on Lemma 3. Thus, when $\varepsilon<\frac{-s_2}{s_1}$, $M+N<0$.

From Equation (53), we also have

$$\begin{aligned}
M-N&=(-2+B)(1-\varepsilon)+\mu^2-[\mu(B-1)(3\varepsilon-\varepsilon B-1)+\mu^3\varepsilon]\\
&=[(2-B)-\mu(B-1)(3-B)-\mu^3]\varepsilon+(-2+B)+\mu^2+\mu(B-1)\\
&=s_3\varepsilon+s_4
\end{aligned} \tag{S37}$$

where

$$\begin{aligned}
s_3&=(2-B)-\mu(B-1)(3-B)-\mu^3\\
&=\mu B^2-(1+4\mu)B+2+3\mu-\mu^3\\
&=\mu B^2-(1+\mu)^2B-(2-\mu)\mu B+(2-\mu)(1+\mu)^2\\
&=\mu B(B-2+\mu)-(1+\mu)^2(B-2+\mu)\\
&=(B-2+\mu)[\mu B-(1+\mu)^2]=(2-B-\mu)[(1+\mu)^2-\mu B]
\end{aligned} \tag{S38}$$

and

$$\begin{aligned} s_4 &= (-2+B)+\mu^2+\mu(B-1) \\ &= -[2-B-\mu+\mu(2-\mu-B)] \\ &= -(2-B-\mu)(1+\mu) \end{aligned} \tag{S39}$$

Because $-B > -2+\mu$ under Assumption 2, we have

$$\begin{aligned} &(1+\mu)^2-\mu B > (1+\mu)^2+\mu(-2+\mu) = (1+2\mu+\mu^2)-2\mu+\mu^2 \\ &= 2\mu^2+1>0 \end{aligned} \tag{S40}$$

Thus, $s_3 > 0$. From Assumption 2, it is easy to get $s_4 < 0$.

Finally, we conclude that $M - N < 0$ when $\varepsilon < \frac{-s_4}{s_3}$. □

S2.7. Proof of Lemma 6

First, note that we can rewrite Equation (56) as

$$\begin{aligned} s_5 &= (2-B+\mu)[2(1-\mu)^2+2\mu B-1+(2-B-\mu)] \\ &= (2-B+\mu)[3-5\mu+2\mu^2+2\mu B-B] \\ &= (2-B+\mu)[2\mu^2-(5-2B)\mu+3-B] \end{aligned} \tag{S41}$$

We claim that the term $2\mu^2-(5-2B)\mu+3-B$ in Equation (S41) is positive, and this can be proven as follows. If $2\mu^2-(5-2B)\mu+3-B\le 0$, then $(5-2B)^2-8(3-B) = 1-12B+4B^2 \ge 0$ and $\mu_1<\mu<\mu_2$, where $\mu_{1,2}=\frac{(5-2B)\pm\sqrt{1-12B+4B^2}}{4}$. Combined with Assumption 2, we have $B<\frac{3-\sqrt{8}}{2}$. It is clear that $1-12B+4B^2<1-4B+4B^2=(1-2B)^2$. Thus, $\sqrt{1-12B+4B^2}<1-2B$, and therefore $(5-2B)-\sqrt{1-12B+4B^2}>4$, which leads to $\mu_1>1$. This contradicts with $\mu<1$. When combined with Formulas (4) and (S11), this results in $s_5>0$. □

S2.8. Proof of Proposition 2

We first need to show that ε is well-defined. It is clear that $\mu^2<\mu$ because $0<\mu<1$. Thus, $2-B-\mu^2>2-B-\mu>0$ based on Assumption 2. Combining with Lemmas 5 and 6, it can be concluded that ε is well-defined.

(a) We express p_i, t_i, and Q_i as a linear combination of w_i-c_i ($i=1,2$) and e.

Substituting Equations (43) and (48) into Equation (49), we get

$$
\begin{aligned}
p_1^{**}(w_1,e) &= \frac{(-2+B)b_3-\mu b_4}{\left(-2+B\right)^2-\mu^2} \\
&= \frac{(-2+B)[-k-\lambda e-(1-B)\overline{w}_1-\varepsilon\mu(w_2-c)]-\mu[-k-\lambda e-(1-B)\overline{w}_2-\varepsilon\mu(w_1-c)]}{\left(-2+B\right)^2-\mu^2} \\
&= \frac{(-2+B-\mu)(-k-\lambda e)-(-2+B)(1-B)\overline{w}_1+\varepsilon\mu^2(w_1-c)-(-2+B)\varepsilon\mu(w_2-c)+\mu(1-B)\overline{w}_2}{\left(-2+B\right)^2-\mu^2}
\end{aligned}
\tag{S42}
$$

Based on Equations (40) and (45), the last four terms in the numerator in Equation (S42) can be rewritten as

$$
\begin{aligned}
&-(-2+B)(1-B)\overline{w}_1+\varepsilon\mu^2(w_1-c)-(-2+B)\varepsilon\mu(w_2-c)+\mu(1-B)\overline{w}_2 \\
&=[(2-B)(1-B)(1-\varepsilon)+\varepsilon\mu^2](w_1-c)+[(2-B)\varepsilon\mu+\mu(1-B)(1-\varepsilon)](w_2-c) \\
&\quad +c(1-B)(2-B+\mu)
\end{aligned}
\tag{S43}
$$

Thus, we obtain the following expression for $p_1^{**}(w_1,e)$:

$$
\begin{aligned}
p_1^{**}(w_1,e) &= \frac{[(2-B)(1-B)(1-\varepsilon)+\varepsilon\mu^2](w_1-c)+[(2-B)\varepsilon\mu+\mu(1-B)(1-\varepsilon)](w_2-c)}{\left(-2+B\right)^2-\mu^2} \\
&\quad +\frac{(2-B+\mu)(k+\lambda e)+c(1-B)[2-B+\mu]}{(-2+B)^2-\mu^2}
\end{aligned}
\tag{S44}
$$

Since $(2-B)\varepsilon\mu+\mu(1-B)(1-\varepsilon)=\mu(1-B)+\varepsilon\mu$, and considering Equation (S44), we get

$$
\begin{aligned}
p_1^{**}(w_1,e) &= \frac{[(2-B)(1-B)(1-\varepsilon)+\varepsilon\mu^2](w_1-c)+[\mu(1-B)+\varepsilon\mu](w_2-c)}{\left(-2+B\right)^2-\mu^2} \\
&\quad +\frac{(2-B+\mu)(k+\lambda e)+c(1-B)[2-B+\mu]}{(-2+B)^2-\mu^2}
\end{aligned}
\tag{S45}
$$

Similarly, we have

$$
\begin{aligned}
p_2^{**}(w_2,e) &= \frac{[\mu(1-B)+\varepsilon\mu](w_1-c)+[(2-B)(1-B)(1-\varepsilon)+\varepsilon\mu^2](w_2-c)}{\left(-2+B\right)^2-\mu^2} \\
&\quad +\frac{(2-B+\mu)(k+\lambda e)+c(1-B)[2-B+\mu)]}{(-2+B)^2-\mu^2}
\end{aligned}
\tag{S46}
$$

Substituting Equation (48) into Equation (50) yields the following expression for $t_1^{**}(w_1,e)$:

$$t_1^{**}(w_1,e)=B_0\cdot\frac{(-2+B)[-k-\lambda e+\overline{w}_1-\varepsilon\mu(w_2-c)]-\mu b_4+\mu^2\overline{w}_1}{\left(-2+B\right)^2-\mu^2}$$
$$=B_0\cdot\frac{(-2+B)[-k-\lambda e+\overline{w}_1-\varepsilon\mu(w_2-c)]-\mu[-k-\lambda e-(1-B)\overline{w}_2-\varepsilon\mu(w_1-c)]+\mu^2\overline{w}_1}{\left(-2+B\right)^2-\mu^2} \tag{S47}$$
$$=B_0\cdot\frac{(2-B+\mu)(k+\lambda e)+(-2+B)\overline{w}_1+(2-B)\varepsilon\mu(w_2-c)+\mu(1-B)\overline{w}_2+\varepsilon\mu^2(w_1-c)+\mu^2\overline{w}_1}{\left(-2+B\right)^2-\mu^2}$$

where the last four terms in the numerator of $t_1^{**}(w_1,e)$ can be rewritten as

$$\begin{aligned}&(-2+B)\overline{w}_1+(2-B)\varepsilon\mu(w_2-c)+\mu(1-B)\overline{w}_2+\varepsilon\mu^2(w_1-c)+\mu^2\overline{w}_1\\&=[(-2+B)+\mu^2]\overline{w}_1+\varepsilon\mu^2(w_1-c)+(2-B)\varepsilon\mu(w_2-c)+\mu(1-B)\overline{w}_2\\&=[(-2+B)(1-\varepsilon)+\mu^2](w_1-c)+[\mu(1-B)+\varepsilon\mu](w_2-c)\\&\quad+c[-2+B+\mu^2+\mu(1-B)]\end{aligned} \tag{S48}$$

From Equation (S48) and by combining with Equation (23), we get

$$t_1^{**}(w_1,e)=B_0\cdot\frac{d+\lambda(2-B+\mu)e+[(-2+B)(1-\varepsilon)+\mu^2](w_1-c)+[\mu(1-B)+\varepsilon\mu](w_2-c)}{\left(-2+B\right)^2-\mu^2} \tag{S49}$$

Similarly, we obtain

$$t_2^{**}(w_2,e)=B_0\cdot\frac{d+\lambda(2-B+\mu)e+[\mu(1-B)+\varepsilon\mu](w_1-c)+[(-2+B)(1-\varepsilon)+\mu^2](w_2-c)}{\left(-2+B\right)^2-\mu^2} \tag{S50}$$

Substituting Equations (S45), (S46), and (S49) into the demand function (1), we can obtain

$$\begin{aligned}Q_1=&\frac{(-2+B)(1-\varepsilon)+\mu^2}{(-2+B)^2-\mu^2}(w_1-c)+\frac{\mu(B-1)(3\varepsilon-\varepsilon B-1)+\mu^3\varepsilon}{(-2+B)^2-\mu^2}(w_2-c)\\&+\frac{\lambda(2-B+\mu)}{(-2+B)^2-\mu^2}e+\frac{d}{(-2+B)^2-\mu^2}\end{aligned} \tag{S51}$$

Substituting Equations (S45), (S46), and (S50) into the demand function (2), we can obtain

$$\begin{aligned}Q_2=&\frac{\mu(B-1)(3\varepsilon-\varepsilon B-1)+\mu^3\varepsilon}{(-2+B)^2-\mu^2}(w_1-c)+\frac{(-2+B)(1-\varepsilon)+\mu^2}{(-2+B)^2-\mu^2}(w_2-c)\\&+\frac{\lambda(2-B+\mu)}{(-2+B)^2-\mu^2}e+\frac{d}{(-2+B)^2-\mu^2}\end{aligned} \tag{S52}$$

(b) Next, to obtain optimal decisions we solve the first-order conditions. For this, differentiating Q_1 and Q_2 with respect to w_1 and w_2 yields

$$\begin{aligned}\frac{\partial Q_1}{\partial w_1}&=\frac{(-2+B)(1-\varepsilon)+\mu^2}{(-2+B)^2-\mu^2}\\\frac{\partial Q_2}{\partial w_1}&=\frac{\mu(B-1)(3\varepsilon-\varepsilon B-1)+\mu^3\varepsilon}{(-2+B)^2-\mu^2}\end{aligned} \tag{S53}$$

$$\frac{\partial Q_1}{\partial w_2}=\frac{\partial Q_2}{\partial w_1}=\frac{\mu(B-1)(3\varepsilon-\varepsilon B-1)+\mu^3\varepsilon}{(-2+B)^2-\mu^2}$$
$$\frac{\partial Q_2}{\partial w_2}=\frac{\partial Q_1}{\partial w_1}=\frac{(-2+B)(1-\varepsilon)+\mu^2}{(-2+B)^2-\mu^2} \tag{S54}$$

and note that

$$\frac{\partial \pi_s}{\partial w_1}=Q_1+(w_1-c)\frac{\partial Q_1}{\partial w_1}+(w_2-c)\frac{\partial Q_2}{\partial w_1}$$
$$\frac{\partial \pi_s}{\partial w_2}=(w_1-c)\frac{\partial Q_1}{\partial w_2}+Q_2+(w_2-c)\frac{\partial Q_2}{\partial w_2} \tag{S55}$$
$$\frac{\partial \pi_s}{\partial e}=(w_1-c)\frac{\partial Q_1}{\partial e}+(w_2-c)\frac{\partial Q_2}{\partial e}-2\alpha e$$

Substituting Equations (S53) and (S54) into (S55) yields

$$\frac{\partial \pi_s}{\partial w_1}=\frac{1}{(-2+B)^2-\mu^2}[2M(w_1-c)+2N(w_2-c)+\lambda(2-B+\mu)e+d]$$
$$\frac{\partial \pi_s}{\partial w_2}=\frac{1}{(-2+B)^2-\mu^2}[2N(w_1-c)+2M(w_2-c)+\lambda(2-B+\mu)e+d] \tag{S56}$$
$$\frac{\partial \pi_s}{\partial e}=\frac{\lambda(2-B+\mu)}{(-2+B)^2-\mu^2}(w_1-c)+\frac{\lambda(2-B+\mu)}{(-2+B)^2-\mu^2}(w_2-c)-2\alpha e$$

where M and N are defined in Equation (53).

Setting the above first-order derivatives equal to 0, we can obtain

$$2M(w_1-c)+2N(w_2-c)+\lambda(2-B+\mu)e+d=0$$
$$2N(w_1-c)+2M(w_2-c)+\lambda(2-B+\mu)e+d=0 \tag{S57}$$
$$\lambda(w_1-c)+\lambda(w_2-c)-2\alpha(2-B-\mu)e=0$$

The Hessian matrix is

$$\bar{H}_1=\begin{bmatrix} 2M & 2N & \lambda(2-B+\mu) \\ 2N & 2M & \lambda(2-B+\mu) \\ \lambda & \lambda & -2\alpha(2-B-\mu) \end{bmatrix} \tag{S58}$$

The first leading principal minor of H_1 is

$$\bar{D}_1=2M=2[(-2+B)(1-\varepsilon)+\mu^2]=2[(-2+B)+\mu^2+(2-B)\varepsilon] \tag{S59}$$

Since $\varepsilon<\dfrac{2-B-\mu^2}{2-B}$, we have $\bar{D}_1<0$.

The second leading principal minor of H_1 is equal to

$$\bar{D}_2=4(M^2-N^2)=4(M-N)(M+N) \tag{S60}$$

According to Lemma 5, $\bar{D}_2>0$.

The third leading principal minor of H_1 is as follows:

$$\bar{D}_3 = -4(M-N)[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)] \tag{S61}$$

Based on Condition (58) and Inequality (4), it is clear that

$$\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)<0 \tag{S62}$$

Based on Lemma 5 and Inequality (S62), we have $\bar{D}_3<0$.

Therefore, the matrix H_1 is negative definite, indicating that π_s is a strictly concave function of w_1, w_1, and e, and has a unique optimal solution.

Finally, solving the linear system of Equation (S56) yields the equilibrium pricing decisions w_1^{**} and w_2^{**} in Equation (59) and e^{**} in Equation (60).

From Assumption 2 and Inequalities (25) and (S62), we have w_1^{**}, w_2^{**}, and e^{**} as positive and $w_1^{**}=w_2^{**}>c$.

(c) Finally, we can derive the freight service prices, brand extension efforts, market demands, and profits under the optimal decisions.

By substituting Equations (59) and (60) back into Equations (S45) and (S46), the optimal freight service prices can be determined. In fact, considering Equation (S45) and given that $w_1^{**}=w_2^{**}$, we can deduce that

$$\begin{aligned}
p_1^{**} &= \frac{(k+\lambda e^{**})+c(1-B)}{2-B-\mu}+\frac{[(1-B)(1-\varepsilon)+\varepsilon\mu](w_1^{**}-c)}{2-B-\mu} \\
&= \frac{k+(1-B)c}{(2-B-\mu)}+\frac{-d\lambda^2-d\alpha[(1-B)(1-\varepsilon)+\varepsilon\mu](2-B-\mu)}{(2-B-\mu)[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]} \\
&= \frac{k+(1-B)c}{(2-B-\mu)}-c+\frac{-d\lambda^2-d\alpha[(1-B)(1-\varepsilon)+\varepsilon\mu](2-B-\mu)}{(2-B-\mu)[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]}+c \\
&= \frac{k-(1-\mu)c}{(2-B-\mu)}+\frac{-d\lambda^2-d\alpha[(1-B)(1-\varepsilon)+\varepsilon\mu](2-B-\mu)}{(2-B-\mu)[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]}+c
\end{aligned} \tag{S63}$$

Let $\bar{w}_1^{**}=c+\dfrac{-d\alpha(2-B-\mu)(1-\varepsilon)}{\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)}$. Thus, from Lemma 2, Equation (S63) can be rewritten as

$$\begin{aligned}
&p_1^{**} \\
&= \frac{[k-(1-\mu)c]\cdot 2\alpha(M+N)(2-B-\mu)-d\alpha[(1-B)(1-\varepsilon)+\varepsilon\mu](2-B-\mu)}{(2-B-\mu)[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]}-\bar{w}_1^{**}+\bar{w}_1^{**}+c \\
&= \frac{[k-(1-\mu)c]\cdot 2\alpha(M+N)-d\alpha[(1-B)(1-\varepsilon)+\varepsilon\mu-(2-B-\mu)(1-\varepsilon)]}{\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)}+\bar{w}_1^{**} \\
&= \frac{[k-(1-\mu)c]\cdot 2\alpha(M+N)+d\alpha[(1-\mu)(1-\varepsilon)-\varepsilon\mu]}{\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)}+\bar{w}_1^{**} \\
&= \frac{\alpha[k-(1-\mu)c]\cdot\{2(M+N)+(2-B+\mu)(1-\mu)-(2-B+\mu)[(1-\mu)\varepsilon+\varepsilon\mu]}{\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)}+\bar{w}_1^{**}
\end{aligned} \tag{S64}$$

By noting that $M+N=s_1\varepsilon+s_2$ and $s_2=-(2-B+\mu)(1-\mu)$ based on Lemma 5, Equation (S64) can be rewritten as

$$\begin{aligned}
& p_1^{**} \\
&= \frac{\alpha[k-(1-\mu)c]\cdot\{2(s_1\varepsilon+s_2)-s_2-(2-B+\mu)\varepsilon]}{\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)}+\overline{w}_1^{**} \\
&= \frac{\alpha[k-(1-\mu)c]\cdot\{2s_1\varepsilon+s_2-(2-B+\mu)\varepsilon]}{\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)}+\overline{w}_1^{**} \\
&= \frac{\alpha[k-(1-\mu)c]\cdot[2s_1\varepsilon+s_2-(2-B+\mu)\varepsilon]-d\alpha(2-B-\mu)(1-\varepsilon)}{\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)}+c
\end{aligned} \tag{S65}$$

Thus, Equation (61) holds.

In the subsequent analysis, we will prove that $p_1^{**} > w_1^{**}$. In fact, from Equations (S65) and (59), we can obtain

$$\begin{aligned}
p_1^{**} &= \frac{\alpha[k-(1-\mu)c]\cdot[2s_1\varepsilon+s_2-(2-B+\mu)\varepsilon]}{\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)}+\frac{-d\alpha(2-B-\mu)(-\varepsilon)}{\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)}+w_1^{**} \\
&= \frac{\alpha[k-(1-\mu)c]\cdot[2s_1\varepsilon+s_2-(2-B+\mu)\varepsilon+(2-B+\mu)(2-B-\mu)\varepsilon]}{\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)}+w_1^{**}
\end{aligned} \tag{S66}$$

Because $\varepsilon < \frac{-s_2}{s_5} = \frac{-s_2}{2s_1-(2-B+\mu)+(2-B+\mu)(2-B-\mu)}$, it follows that $2s_1\varepsilon+s_2-(2-B+\mu)\varepsilon+(2-B+\mu)(2-B-\mu)\varepsilon<0$. Therefore, based on Assumption 3 and Inequality (S62), we can immediately get $p_1^{**} > w_1^{**} > 0$.

Following a similar proof as above, we can obtain the expression for p_2^{**} and prove that $p_2^{**} = p_1^{**} > w_2^{**}$.

Replacing w_1, w_2, and e with w_1^{**}, w_2^{**}, and e^{**} in $t_1^{**}(w_1,e)$ in Equation (S49) and $t_2^{**}(w_1,e)$ in Equation (S50), respectively, we can obtain the equilibrium brand extension efforts in Equation (62). Indeed, as $w_1^{**}=w_2^{**}$, $s_2=-2+B+\mu^2-\mu(B-1)$ $=C+D$ in Equation (54) and $M+N=s_1\varepsilon+s_2$ from Lemma 5, it is apparent that

$$\begin{aligned}
t_1^{**} &= B_0\cdot\frac{d+\lambda(2-B+\mu)e^{**}+[(-2+B)(1-\varepsilon)+\mu^2+\mu(1-B)+\varepsilon\mu](w_1^{**}-c)}{(-2+B)^2-\mu^2} \\
&= B_0\cdot\frac{d+\lambda(2-B+\mu)e^{**}+(C+D)(w_1-c)+\varepsilon[2-B+\mu](w_1^{**}-c)}{(-2+B)^2-\mu^2} \\
&= \frac{B_0}{(-2+B)^2-\mu^2}\cdot\frac{d\alpha(2s_1\varepsilon+s_2)(2-B-\mu)-\varepsilon d\alpha(2-B-\mu)(2-B+\mu)}{\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)} \\
&= \frac{B_0\cdot d\alpha[2s_1\varepsilon+s_2-\varepsilon(2-B+\mu)]}{(2-B+\mu)[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]} \\
&= \frac{\alpha B_0\cdot[k-(1-\mu)c]\cdot[2s_1\varepsilon+s_2-\varepsilon(2-B+\mu)]}{\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)}
\end{aligned} \tag{S67}$$

Based on Lemma 5, we have $2s_1\varepsilon+s_2-\varepsilon(2-B+\mu)=M+N+s_1\varepsilon-\varepsilon(2-B+\mu)$, and since

$$s_1\varepsilon-\varepsilon(2-B+\mu)=(2-B+\mu)[(1-\mu)^2+\mu B]\varepsilon-\varepsilon(2-B+\mu)$$
$$=(2-B+\mu)[(1-\mu)^2+\mu B-1]\varepsilon=(2-B+\mu)[-2\mu+\mu^2+\mu B] \quad \text{(S68)}$$
$$=-\mu\varepsilon(2-B+\mu)[2-B-\mu]<0$$

and $M+N<0$ from Lemma 5, it is clear that t_1^{**} is positive as a result of Assumption 3 and Inequality (S62).

Similar to the above derivation, we can obtain $t_2^{**}=t_1^{**}$.

Replacing w_1, , w_2 and e with w_1^{**}, w_2^{**}, and e^{**} in Q_1 in Equation (S51) and Q_2 in Equation (S52), respectively, we can derive the market demands under the optimal decision. Specifically, since $w_1^{**}=w_2^{**}$, it can be concluded that

$$Q_1^{**}=\frac{M+N}{(-2+B)^2-\mu^2}(w_1^{**}-c)+\frac{\lambda(2-B+\mu)}{(-2+B)^2-\mu^2}e^{**}+\frac{d}{(-2+B)^2-\mu^2}$$
$$=\frac{-d[\alpha(2-B-\mu)(M+N)+\lambda^2(2-B+\mu)]}{[(-2+B)^2-\mu^2][\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]}+\frac{d}{(-2+B)^2-\mu^2}$$
$$=\frac{-d}{(-2+B)^2-\mu^2}\left[\frac{\alpha(2-B-\mu)(M+N)+\lambda^2(2-B+\mu)}{\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)}-1\right] \quad \text{(S69)}$$
$$=\frac{\alpha(M+N)(2-B+\mu)[k-c(1-\mu)]}{(2-B+\mu)[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]}$$
$$=\frac{\alpha(M+N)[k-c(1-\mu)]}{\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)}$$

Owing to Lemma 5, Assumption 3, and Inequality (S62), it is clear that Q_1^{**} is positive.

Similarly, we can obtain $Q_2^{**}=Q_1^{**}$.

We substitute w_1^{**}, w_2^{**} Q_1^{**}, Q_2^{**}, and e^{**} into Equation (6) and then get the equilibrium profits π_s^{**} in Equation (64). In fact, by taking into account $w_1^{**}=w_2^{**}$, we can deduce that

$$\pi_s^{**}=\left(w_1^{**}-c\right)Q_1^{**}+\left(w_2^{**}-c\right)Q_2^{**}-\alpha e^{**2}=(Q_1^{**}+Q_2^{**})(w_1^{**}-c)-\alpha e^{**2}$$
$$=\frac{\alpha d[k-c(1-\mu)]\cdot[-2\alpha(M+N)(2-B-\mu)-\lambda^2(2-B+\mu)]}{[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]^2} \quad \text{(S70)}$$
$$=\frac{-\alpha d[k-c(1-\mu)]}{\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)}$$

Resulting from Lemma 2, Assumption 3, and Inequality (S62), it is evident that π_s^{**} is positive.

Subsequently, the maximal profits $\pi_{f_1}^{**}$ and $\pi_{f_2}^{**}$ can be obtained by substituting $w_1^{**}=w_2^{**}$ in Equation (59), $p_1^{**}=p_2^{**}$ in Equation (61), $t_1^{**}=t_2^{**}$ in (62), and $Q_2^{**}=Q_1^{**}$ in (63) into the π_{f_1} in (7) and π_{f_2} in (8), respectively. In fact, we have

$$\begin{aligned}
\pi_{f_1}^{**} &= \pi_{f_2}^{**} = \left(p_1^{**} - w_1^{**}\right)Q_1^{**} - \beta t_1^{**2} \\
&= \frac{\alpha^2 \cdot [k-(1-\mu)c]^2 \cdot [2s_1\varepsilon + s_2 - (2-B+\mu)\varepsilon + (2-B+\mu)(2-B-\mu)\varepsilon](M+N)}{[\lambda^2(2-B+\mu) + 2\alpha(M+N)(2-B-\mu)]^2} \\
&\quad - \frac{\alpha^2\beta \cdot {B_0}^2 \cdot [k-(1-\mu)c]^2 \cdot [2s_1\varepsilon + s_2 - \varepsilon(2-B+\mu)]^2}{[\lambda^2(2-B+\mu) + 2\alpha(M+N)(2-B-\mu)]^2} \\
&= \frac{\alpha^2 \cdot [k-(1-\mu)c]^2 \cdot L}{[\lambda^2(2-B+\mu) + 2\alpha(M+N)(2-B-\mu)]^2}
\end{aligned} \tag{S71}$$

where

$$\begin{aligned}
L &= [2s_1\varepsilon + s_2 - (2-B+\mu)\varepsilon + (2-B+\mu)(2-B-\mu)\varepsilon](M+N) - \beta \cdot {B_0}^2[2s_1\varepsilon + s_2 - \varepsilon(2-B\cdot \\
&= [M+N+s_1\varepsilon - (2-B+\mu)\varepsilon + (2-B+\mu)(2-B-\mu)\varepsilon](M+N) \\
&\quad - \beta \cdot {B_0}^2[M+N]^2 - 2\beta \cdot {B_0}^2[M+N][s_1\varepsilon - \varepsilon(2-B+\mu)] - \beta \cdot {B_0}^2[s_1\varepsilon - \varepsilon(2-B+\mu)]^2 \\
&= (1-\beta \cdot {B_0}^2)(M+N)^2 + \varepsilon E(M+N) - \beta \cdot {B_0}^2[s_1\varepsilon - \varepsilon(2-B+\mu)]^2
\end{aligned} \tag{S72}$$

in which

$$\begin{aligned}
E &= s_1 - (2-B+\mu) + (2-B+\mu)(2-B-\mu) - B[s_1 - (2-B+\mu)] \\
&= (1-B)[s_1 - (2-B+\mu)] + (2-B+\mu)(2-B-\mu) \\
&= (1-B)s_1 - (2-B+\mu)[-1+\mu] \\
&= (1-B)s_1 - s_2
\end{aligned} \tag{S73}$$

Note that in the above derivation, we used the relation $B = \eta B_0 = \eta\dfrac{\eta}{2\beta} = 2\beta\dfrac{\eta^2}{4\beta^2} = 2\beta {B_0}^2$ from Equation (3). Therefore, Proposition 2 is proved.

S2.9. Proof of Corollary 1

Proof. In order to prove that $\pi_{f_1} > 0$, it suffices to show that $L > 0$. Since $\varepsilon < \dfrac{-s_2(2-B)}{(2-B)s_1 + E + \sqrt{E^2 + (2-B)B[s_1 - (2-B+\mu)]^2}}$, we have

$$(2-B)(s_1\varepsilon + s_2) < -\varepsilon E - \varepsilon\sqrt{E^2 + (2-B)B[s_1 - (2-B+\mu)]^2} \tag{S74}$$

We note that $s_1\varepsilon + s_2 = M + N$ from Lemma 5 and $B = \eta B_0 = \eta\dfrac{\eta}{2\beta} = 2\beta\dfrac{\eta^2}{4\beta^2} = 2\beta {B_0}^2$ from Equation (3). Therefore,

$$\begin{aligned}
M + N &< \frac{-\varepsilon E - \varepsilon\sqrt{E^2 + (2-B)B[s_1 - (2-B+\mu)]^2}}{2-B} \\
&= \frac{-\varepsilon E - \varepsilon\sqrt{E^2 + 4(1-\beta \cdot {B_0}^2)\beta \cdot {B_0}^2[s_1 - (2-B+\mu)]^2}}{2(1-\beta \cdot {B_0}^2)}
\end{aligned} \tag{S75}$$

Let

$$x_{1,2} = \frac{-\varepsilon E \ \mathrm{m}\varepsilon \sqrt{E^2 + 4(1-\beta \cdot B_0{}^2)\beta \cdot B_0{}^2[s_1 - (2-B+\mu)]^2}}{2(1-\beta \cdot B_0{}^2)} \quad \text{(S76)}$$

and note that $\beta \cdot B_0{}^2 = \beta \frac{\eta^2}{4\beta^2} = \frac{\eta^2}{4\beta} < 1$ from Assumption 1. Therefore,

$$(1-\beta \cdot B_0{}^2)(M+N-x_1)(M+N-x_2) > 0 \quad \text{(S77)}$$

By substituting the values of x_1 and x_2 into the above inequality, it can be rewritten as

$$(1-\beta \cdot B_0{}^2)(M+N)^2 + \varepsilon E(M+N) - \beta \cdot B_0{}^2[s_1 - (2-B+\mu)]^2 > 0 \quad \text{(S78)}$$

As a results, $L > 0$, and the proof is established.

S2.10. Proof of Proposition 3

First, the effects of the parameters λ and ε on w_i^{**} and e^{**} are explored. By taking the partial derivatives of w_i^{**} with respect to λ and ε, respectively, we obtain

$$\begin{aligned} \frac{\partial w_i^{**}}{\partial \lambda} &= \frac{2\lambda d\alpha(2-B-\mu)(2-B+\mu)}{[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]^2} > 0 \\ \frac{\partial w_i^{**}}{\partial \varepsilon} &= \frac{2\alpha^2(2-B-\mu)^2 ds_1}{[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]^2} > 0 \end{aligned} \quad \text{(S79)}$$

This indicates that w_i^{**} increases with λ and ε.

Similarly, by taking the partial derivative of e^{**} with respect to λ and ε, respectively, we obtain

$$\begin{aligned} \frac{\partial e^{**}}{\partial \lambda} &= \frac{2d\lambda^2(2-B+\mu)}{[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]^2} > 0 \\ \frac{\partial e^{**}}{\partial \varepsilon} &= \frac{2\alpha d\lambda(2-B-\mu)s_1}{[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]^2} > 0 \end{aligned} \quad \text{(S80)}$$

This reveals that e^{**} increases with λ and ε.

Second, we analyze the effects of parameters λ and ε on p_i^{**}.

Given that $d = [k-(1-\mu)c] \cdot (2-B+\mu)$, Equation (56), and Condition (57), it follows that

$$\begin{aligned} &\alpha[k-(1-\mu)c] \cdot [2s_1\varepsilon + s_2 - (2-B+\mu)\varepsilon] - d\alpha(2-B-\mu)(1-\varepsilon) \\ &= \alpha[k-(1-\mu)c] \cdot [s_5\varepsilon + s_2 - (2-B+\mu)(2-B-\mu)] \\ &< 0 \end{aligned} \quad \text{(S81)}$$

Thus, differentiating p_i^{**} with respect to λ yields

$$\frac{\partial p_i^{**}}{\partial \lambda} = \alpha[k-(1-\mu)c] \cdot \frac{-[s_5\varepsilon + s_2 - (2-B+\mu)(2-B-\mu)] \cdot 2\lambda(2-B+\mu)}{[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]^2} > 0 \tag{S82}$$

This indicates that p_i^{**} increases with λ.

Differentiating p_i^{**} with respect to ε results in

$$\frac{\partial p_i^{**}}{\partial \varepsilon} = \alpha[k-(1-\mu)c] \cdot \frac{s_5 \cdot F - [s_5\varepsilon + s_2 - (2-B+\mu)(2-B-\mu)]2\alpha s_1(2-B-\mu)}{[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]^2} \tag{S83}$$

From Assumption 3, $k-(1-\mu)c>0$. To prove that $\frac{\partial p_i^{**}}{\partial \varepsilon} > 0$, it is only necessary to demonstrate that $s_5 \cdot F - [s_5\varepsilon + s_2 - (2-B+\mu)(2-B-\mu)]2\alpha s_1(2-B-\mu) > 0$. It is clear that $\mu - 1 < B + \mu - 1$. By multiplying both sides of the above inequality by $\mu(2-B-\mu)$, we get $\mu[2-B-\mu](\mu-1) < \mu(2-B-\mu)\cdot(B+\mu-1)$. That is, $(1-\mu)[-2\mu+\mu B+\mu^2] < (2-B-\mu)\cdot[\mu B+\mu^2-\mu]$, which is equivalent to

$$(1-\mu)[\mu B+(1-\mu)^2-1] < (2-B-\mu)\cdot[\mu B+(1-\mu)^2+\mu-1] \tag{S84}$$

By multiplying $(2-B+\mu)$ on both sides of Equation (S84), it follows from Formula (54) that

$$-s_2\cdot[\mu B+(1-\mu)^2-1] < (2-B-\mu)\cdot[s_1+s_2] \tag{S85}$$

Multiplying both sides of Equation (S85) by $(2-B+\mu)$ leads to

$$-s_2\cdot s_1 + (2-B+\mu)s_2 < (2-B+\mu)(2-B-\mu)\cdot[s_1+s_2] \tag{S86}$$

This is equivalent to

$$-s_2\cdot s_1 - (2-B+\mu)(2-B-\mu)\cdot s_1 < -(2-B+\mu)s_2 + (2-B+\mu)(2-B-\mu)s_2 \tag{S87}$$

Adding $2s_1s_2$ to both sides of Equation (S87) gives

$$[s_2 - (2-B+\mu)(2-B-\mu)]\cdot s_1 < [2s_1 - (2-B+\mu) + (2-B+\mu)(2-B-\mu)]s_2 \tag{S88}$$

By adding $s_5s_1\varepsilon$ to both sides of Equation (S88) and utilizing Equation (56), we obtain

$$[s_5\varepsilon + s_2 - (2-B+\mu)(2-B-\mu)]\cdot s_1 < s_5(s_1\varepsilon + s_2) \tag{S89}$$

Multiplying both sides of Equation (S89) with $2\alpha(2-B-\mu)$ and employing Lemma 5 yields

$$[s_5\varepsilon + s_2 - (2-B+\mu)(2-B-\mu)]\cdot 2\alpha s_1(2-B-\mu) < s_5 2\alpha(M+N)(2-B-\mu) \tag{S90}$$

Additionally, it is clear that

$$[s_5\varepsilon + s_2 - (2-B+\mu)(2-B-\mu)]\cdot 2\alpha s_1(2-B-\mu) < s_5\cdot\lambda^2(2-B+\mu) + s_5\cdot 2\alpha(M+N)(2-B-\mu) \tag{S91}$$

This is equivalent to $s_5 \cdot F - [s_5\varepsilon + s_2 - (2-B+\mu)(2-B-\mu)]\cdot 2\alpha s_1(2-B-\mu) > 0$.

Therefore, $\frac{\partial p_i^{**}}{\partial \varepsilon} > 0$, which implies that p_i^{**} increases with ε.

Next, we proceed to examine the effect of parameters λ and ε on t_i^{**}. The first-order derivative of t_i^{**} in Equation (62) with respect to λ is given by:

$$\begin{aligned}\frac{\partial t_i^{**}}{\partial \lambda} &= \frac{-2\lambda\alpha B_0\cdot[k-(1-\mu)c]\cdot[2s_1\varepsilon+s_2-\varepsilon(2-B+\mu)](2-B+\mu)}{[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]^2} \\ &= t_i^{**}\cdot\frac{-2\lambda(2-B+\mu)}{F}>0\end{aligned} \tag{S92}$$

This reveal that t_i^{**} increase with λ.

The first-order derivative of t_i^{**} in Equation (62) with respect to ε is expressed as:

$$\frac{\partial t_i^{**}}{\partial \varepsilon} = \alpha B_0[k-(1-\mu)c]\cdot\frac{H\cdot F-[H\varepsilon+s_2]\cdot[2\alpha s_1(2-B-\mu)]}{[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]} \tag{S93}$$

where $H=2s_1-(2-B+\mu)$.

We can prove that $s_1<(2-B+\mu)$. In fact, from Assumption 2, $2-B-\mu>0$. Therefore, it can be seen from Formula (54) that $s_1=(2-B+\mu)[(1-\mu)^2+\mu B]=(2-B+\mu)[1-2\mu+\mu^2+\mu B]=(2-B+\mu)[1-\mu(2-B-\mu)]<(2-B+\mu)$. Thus, $-s_2\cdot s_1<-(2-B+\mu)s_2$. Adding $2s_1s_2$ to both sides of the above inequality, we have $s_2\cdot s_1<[2s_1-(2-B+\mu)]s_2$, that is, $s_2\cdot s_1<Hs_2$. Adding $Hs_1\varepsilon$ to both sides of the above inequality, we get $(H\varepsilon+s_2)\cdot s_1<H(s_1\varepsilon+s_2)=H(M+N)$, and hence,

$$(H\varepsilon+s_2)\cdot 2\alpha s_1(2-B-\mu)<H2\alpha(M+N)(2-B-\mu) \tag{S94}$$

Clearly, $H\cdot\lambda^2(2-B+\mu)+H2\alpha(M+N)(2-B-\mu)>(H\varepsilon+s_2)\cdot 2\alpha s_1(2-B-\mu)$. This is, $H\cdot F-[H\varepsilon+s_2]\cdot[2\alpha s_1(2-B-\mu)]>0$. Thus, $\frac{\partial t_i^{**}}{\partial \varepsilon}>0$, which implies that t_i^{**} increase with ε.

Lastly, we analyze the effects of the parameters λ and ε on Q_1^{**} and Q_2^{**}. Differentiating Q_1^{**} and Q_2^{**} in Equation (63) with respect to λ yields

$$\frac{\partial Q_1^{**}}{\partial \lambda}=\frac{\partial Q_2^{**}}{\partial \lambda}=\frac{-\alpha(M+N)[k-c(1-\mu)]\cdot 2\lambda(2-B+\mu)}{[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]^2}>0 \tag{S95}$$

Thus, Q_1^{**} and Q_2^{**} increase with λ.

Differentiating Q_1^{**} and Q_2^{**} in Equation (63) with respect to ε yields

$$\frac{\partial Q_1^{**}}{\partial \varepsilon}=\frac{\partial Q_2^{**}}{\partial \varepsilon}=\frac{\alpha\cdot s_1[k-c(1-\mu)]\cdot\lambda^2(2-B+\mu)}{[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]^2} \tag{S96}$$

It follows from Assumption 3 that $\frac{\partial Q_1^{**}}{\partial \varepsilon}=\frac{\partial Q_2^{**}}{\partial \varepsilon}>0$, which implies that Q_1^{**} and Q_2^{**} increase with ε.

By replacing ε with 0, w_i^{**} becomes w_i^{*}. Because w_i^{**} increase with ε, we have $w_i^{**} > w_i^{*}$. Similarly, it can be seen that $e^{**} > e^{*}$, $p_i^{**} > p_i^{*}$, $t_i^{**} > t_i^{*}$, $Q_1^{**} > Q_1^{*}$, and $Q_2^{**} > Q_2^{*}$. □

S2.11. Proof of Proposition 4

Proof. First, consider the effect of λ on π_s^{**}. Differentiating π_s^{**} with respect to λ yields

$$\frac{\partial \pi_s^{**}}{\partial \lambda} = \frac{2\alpha d[k-c(1-\mu)](2-B+\mu)\lambda}{[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]^2} \tag{S97}$$

Because of Lemma 2, Inequality (4), and Assumption 3, we have $\frac{\partial \pi_s^{**}}{\partial \lambda} > 0$.

Next, consider the effect of ε on π_s^{**}. Differentiating π_s^{**} with respect to ε results in

$$\frac{\partial \pi_s^{**}}{\partial \varepsilon} = \frac{2\alpha^2 d[k-c(1-\mu)](2-B-\mu)s_1}{[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]^2} \tag{S98}$$

Based on Assumptions 2 and 3 and Inequalities (25) and (54), we have $\frac{\partial \pi_s^{**}}{\partial \varepsilon} > 0$. By setting ε as 0, π_s^{**} becomes π_s^{*}. Because π_s^{**} increase with ε, we have $\pi_s^{**} > \pi_s^{*}$. □

S2.12. Proof of Proposition 5

Differentiating $\pi_{f_1}^{**}(=\pi_{f_2}^{**})$ with respect to λ gives

$$\begin{aligned} \frac{\partial \pi_{f_1}^{**}}{\partial \lambda} = \frac{\partial \pi_{f_2}^{**}}{\partial \lambda} &= \frac{4\alpha^2 \cdot [k-(1-\mu)c]^2 \cdot L \cdot \lambda(2-B+\mu)}{[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]^3} \\ &= \frac{4\lambda(2-B+\mu)\cdot \pi_{f_1}^{**}}{\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)} > 0 \end{aligned} \tag{S99}$$

From Equation (S99), we can conclude that $\pi_{f_1}^{**}$ and $\pi_{f_2}^{**}$ increase with λ.

Next, consider the effects of ε on $\pi_{f_1}^{**}$ and $\pi_{f_2}^{**}$. For this, differentiating L with respect to ε and taking into account that $M+N=s_1\varepsilon+s_2$ from Lemma 5, we obtain:

$$\begin{aligned} &\frac{\partial L}{\partial \varepsilon} = (2-B)(M+N)s_1 + E(M+N) + \varepsilon E s_1 - \varepsilon B[s_1-(2-B+\mu)]^2 \\ &= \{(2-B){s_1}^2 + 2Es_1 - B[s_1-(2-B+\mu)]^2\}\varepsilon + (3-2B)s_1s_2 - s_2s_2 \\ &= [(4-4B){s_1}^2 - 2s_1s_2 + 2s_1B(2-B+\mu) - B(2-B+\mu)^2]\varepsilon + (3-2B)s_1s_2 - {s_2}^2 \end{aligned} \tag{S100}$$

Thus, it is easy to see that $\frac{\partial L}{\partial \varepsilon} = 2l_1\varepsilon + l_2 = L_d$. Then, we can find that

$$\frac{\partial\pi_{f_1}^{**}}{\partial\varepsilon}=\frac{\partial\pi_{f_2}^{**}}{\partial\varepsilon}=\alpha^2\cdot[k-(1-\mu)c]^2\cdot\frac{L_d\cdot F^2-L\cdot 2\cdot F\cdot[2\alpha s_1(2-B-\mu)]}{[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]^4}$$
$$=\alpha^2\cdot[k-(1-\mu)c]^2\cdot\frac{F[L_d\cdot F-4L\cdot\alpha s_1(2-B-\mu)]}{[\lambda^2(2-B+\mu)+2\alpha(M+N)(2-B-\mu)]^4} \tag{S101}$$

By noting that $F<0$ from Formulas (S62) and (68), and $L>0$ from Formulas (66) and (S78), it is concluded that $\pi_{f_1}^{**}$ and $\pi_{f_2}^{**}$ increase with ε if either the conditions (a) or (b) hold. In this case, it can be seen that $\pi_{f_1}^{**}>\pi_{f_1}^{*}$ and $\pi_{f_2}^{**}>\pi_{f_2}^{*}$ by setting $\varepsilon=0$ in $\pi_{f_1}^{**}$ and $\pi_{f_2}^{**}$.

Equation (S101) also implies that $\pi_{f_1}^{**}$ and $\pi_{f_2}^{**}$ decrease with ε, given that the conditions (c) and (d) are satisfied. In this scenario, it is apparent that $\pi_{f_1}^{**}<\pi_{f_1}^{*}$ and $\pi_{f_2}^{**}<\pi_{f_2}^{*}$ by setting $\varepsilon=0$ in $\pi_{f_1}^{**}$ and $\pi_{f_2}^{**}$.

Therefore, we can conclude that the outcomes described in (2) and (3) are valid. □

S3. Mathematical Derivation in Remark 1

$M+N>0$ and $M-N>0$ will lead to a contradiction:

In fact, if $M+N>0$ and $M-N>0$, then ε needs to satisfy these two conditions, i.e., $\varepsilon>\frac{2-B-\mu^2+\mu(B-1)}{2-B+\mu(B-1)(3-B)+\mu^3}$ and $\varepsilon>\frac{(2-B)-\mu^2-\mu(B-1)}{(2-B)-\mu(B-1)(3-B)-\mu^3}$. In order to ensure that the first leading principal minor is positive, we have proven that $\varepsilon<\frac{2-B-\mu^2}{2-B}$. Thus, the following two conditions must hold simultaneously: (1) $\frac{2-B-\mu^2+\mu(B-1)}{2-B+\mu(B-1)(3-B)+\mu^3}<\frac{2-B-\mu^2}{2-B}$ and (2) $\frac{(2-B)-\mu^2-\mu(B-1)}{(2-B)-\mu(B-1)(3-B)-\mu^3}<\frac{2-B-\mu^2}{2-B}$. Based on condition (1), it follows that $(2-B)[2-B-\mu^2+\mu(B-1)]<(2-B-\mu^2)[2-B+\mu(B-1)(3-B)+\mu^3]$.

Consequently, we have

$$\mu(2-B)(B-1)<(2-B)[\mu(B-1)(3-B)+\mu^3]-\mu^2[\mu(B-1)(3-B)+\mu^3] \tag{S102}$$

On the other hand, built on condition (2), it follows that $(2-B)[(2-B)-\mu^2-\mu(B-1)]<[2-B-\mu^2][(2-B)-\mu(B-1)(3-B)-\mu^3]$. Therefore, we obtain

$$(2-B)[-\mu(B-1)]<[2-B][-\mu(B-1)(3-B)-\mu^3]-\mu^2[-\mu(B-1)(3-B)-\mu^3] \tag{S103}$$

However, this contradicts Equation (S102). □

S4. Range of the Altruistic Preference Degree

The proof that the condition $\varepsilon<\varepsilon_1$ in Corollary 1 is tighter than $\varepsilon<\varepsilon_0$ in Proposition 2:

Indeed, it is clear that $E+\sqrt{E^2+(2-B)B[s_1-(2-B+\mu)]^2}>0$. Thus, according to Equation (66), we have $\varepsilon_2<\frac{-s_2(2-B)}{(2-B)s_1}=\frac{-s_2}{s_1}$. In addition, we can prove that

$$\varepsilon_2<\frac{-s_2}{s_5}=\frac{-s_2}{2s_1-(2-B+\mu)+(2-B+\mu)(2-B-\mu)} \tag{S104}$$

In fact, because $0<B<2$ from Assumption 2, we have $(1-B)^2=1-2B+B^2=1-B(2-B)<1$. Thus, $(1-B)^2(2-B+\mu)^2(2-B-\mu)^2<(2-B+\mu)^2(2-B-\mu)^2$. Hence,

$$[s_1-\delta_1+(1-B)\delta_1\delta_2]^2<[s_1-\delta_1]^2+2(1-B)[s_1-\delta_1]\delta_1\delta_2+\delta_1^2\delta_2^2 \tag{S105}$$

where $\delta_1=2-B+\mu$ and $\delta_2=2-B-\mu$.

Considering that

$$\begin{aligned}&E^2+(2-B)B[s_1-(2-B+\mu)]^2\\&=\{(1-B)[s_1-\delta_1]+\delta_1\delta_2\}^2+(2-B)B[s_1-\delta_1]^2\\&=[s_1-\delta_1]^2+2(1-B)[s_1-\delta_1]\delta_1\delta_2+(\delta_1\delta_2)^2\end{aligned} \tag{S106}$$

According to Equation (66), it follows from Equations (S105) and (S106) that

$$s_1-\delta_1+(1-B)\delta_1\delta_2\le|s_1-\delta_1+(1-B)\delta_1\delta_2|<\sqrt{E^2+(2-B)B[s_1-(2-B+\mu)]^2} \tag{S107}$$

In view of $s_1-\delta_1+(1-B)\delta_1\delta_2=(2-B)[s_1-\delta_1+\delta_1\delta_2]-[(1-B)(s_1-\delta_1)+\delta_1\delta_2]$, and $(1-B)(s_1-\delta_1)+\delta_1\delta_2=E$ by using Equation (66), it follows from Equation (S107) that

$$(2-B)[s_1-\delta_1+\delta_1\delta_2]<E+\sqrt{E^2+(2-B)B[s_1-(2-B+\mu)]^2} \tag{S108}$$

Adding $(2-B)s_1$ to both sides of Equation (S108), we have

$$(2-B)[2s_1-\delta_1+\delta_1\delta_2]<(2-B)s_1+E+\sqrt{E^2+(2-B)B[s_1-(2-B+\mu)]^2} \tag{S109}$$

It follows from Equation (S109) that

$$\frac{(2-B)}{(2-B)s_1+E+\sqrt{E^2+(2-B)B[s_1-(2-B+\mu)]^2}}<\frac{1}{2s_1-\delta_1+\delta_1\delta_2} \tag{S110}$$

Multiplying $-s_2$ on both sides of Equation (S110), Equation (S104) follows. □

S5. Data Collection and Calculation Basis (Including Key Parts in English)

S5.1. Data Collection and Parameter Estimation

S5.1.1. Container Shipping Quotation

The container shipping quotations come from the following two companies in Tianjin, China.
Name of Freight Forwarder 1: Tianjin Yuzhou International Freight Forwarding Co., Ltd, Tianjin, China.
Web Site: https://inter.chinawutong.com/fcl/204399.html (accessed on 12 July 2023)
Webpage Snapshot: see next page (Figure S2).

Name of Freight Forwarder 2: Tianjin McKinley International Freight Forwarding Co., Ltd, Tianjin, China.
Web Site: http://www.tjxg.cn/html/sea_detail_5068.html (accessed on 12 July 2023)
Webpage Snapshot: see next page (Figure S3).
Based on the above websites, we can obtain the following quotation table.

Table S1. Container shipping quotation from Tianjin Port in China to Haiphong in Vietnam.

	20 GP	40 GP	40 HQ/ 40 HC
Freight Forwarder 1	USD 250	USD 350	USD 350
Freight Forwarder 2	USD 250	USD 400	USD 400

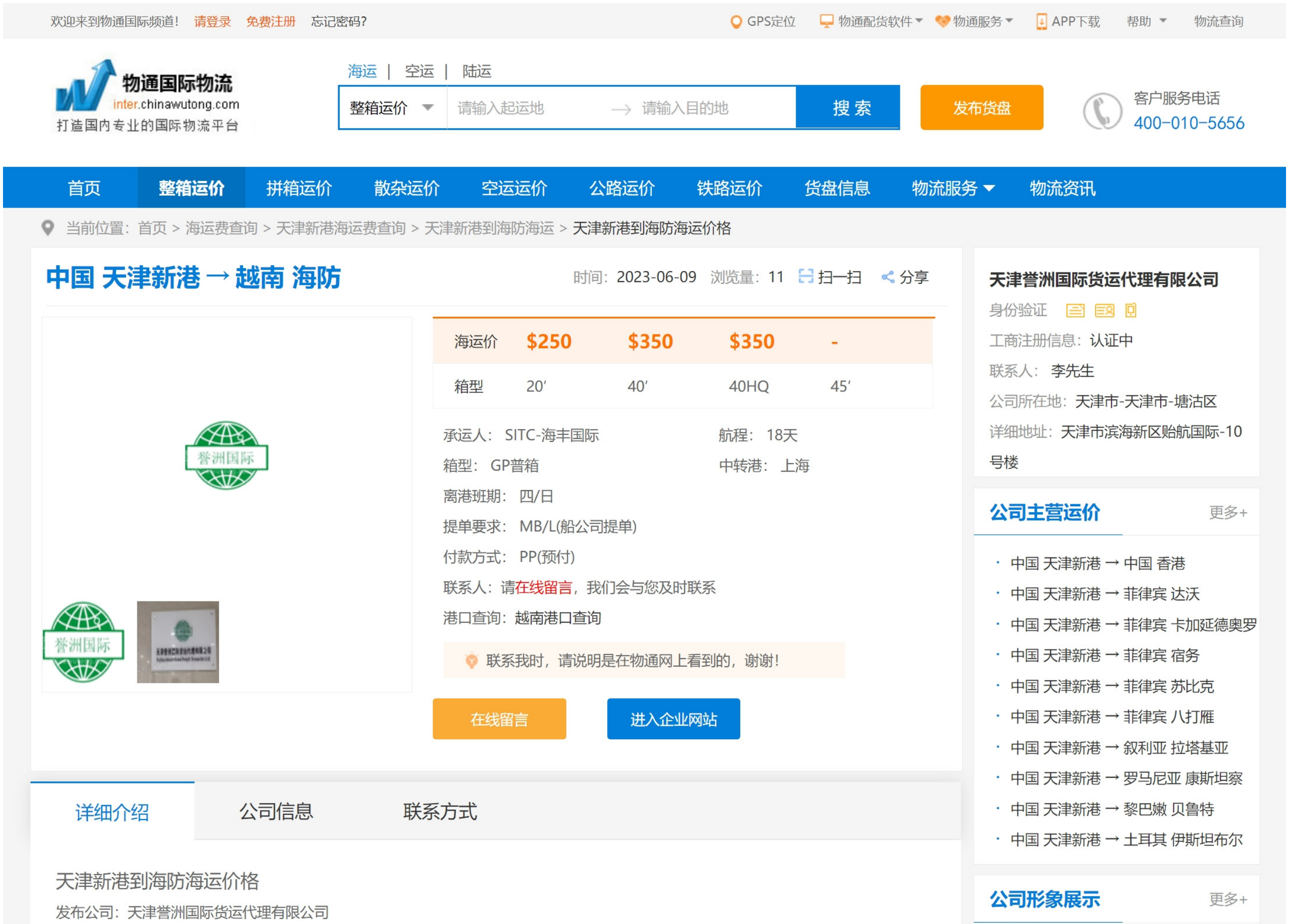

Figure S2. Webpage Snapshot of Freight Forwarder 1. Key Parts in English for Figure S2: Figure S2 is Vietnam sea freight container quotation and shipping schedule of Freight Forwarder 1 from Tianjin Port to Haiphong, Vietnam. Full container load sea freight quotation (USD) are 250 per 20 GP, 350 per 40 GP and 350 per 40 HQ, respectively. Quotation date: 6 June 2023. The length of the voyage is about 18 days.

Figure S3. Webpage Snapshot of Freight Forwarder 2. Key Parts in English for Figure S3: Figure S3 is Vietnam sea freight container quotation and shipping schedule of Freight Forwarder 2 from Tianjin Port to Haiphong, Vietnam. Full container load sea freight quotation (USD) are 250 per 20 GP, 400 per 40 GP and 400 per 40 HC, respectively. Quotation date: 12 July 2023; Expiration date: 27 July 2023. The length of the voyage is about 15 days.

S5.1.2. Estimation of the Shipping Company's Marginal Cost

Because the cost often belongs to the trade secrets of an enterprise, we have to estimate it.

From the water transport logistics website, the profit margin of freight forwarders is about 20 to 30 percent (http://www.shuishangwuliu.com/jiabanjixie/148461.html, accessed on 12 July 2023), and the profit margin in the shipping industry is around 15 to 25 percent (http://www.shuishangwuliu.com/chanpinfenlei/29056.html, accessed on 12 July 2023).

Thus, by using Table A1, based on the container shipping quotation (USD 250 per standard container (20 GP)) from one port of departure to one port of destination, we estimate that the wholesale price is about USD 150 per standard container according to the profit rate of freight forwarders, and then estimate that the cost of the shipping company is about USD 150 per standard container. That, $c \approx$ USD150 per standard container.

S5.1.3. Estimation of the Basic Market Demand

According to the official website of Tianjin Port Group, in the first half of 2023, the cargo throughput of Tianjin Port Group was 241 million tons; the container throughput was 11.353 million standard containers (https://www.ptacn.com/contents/17/1715.html, accessed on 12 June 2023).

Tianjin Port has 140 container routes (https://www.ptacn.com/channels/12.html, accessed on 12 June 2023).

According to the website of fobshanghai, Tianjin has 1200 sea freight forwarders, including branches established by companies from other regions (Accessed from https://link.fobshanghai.com/info/tianjinhaiyunhuodai.html, accessed on 12 June 2023).

Therefore, through a simple calculation, a freight forwarding company faces an annual basic demand of about 135 standard containers.

Specifically, $k \approx [(1135.3 \div 0.12) \div 140] \times 2 \approx 135$.

S5.2. Determinations of $\varepsilon \in (0, 0.328)$ *and* $\lambda \in J = (0, 1.61)$

In Supplementary Material Section 4, we have proved that the condition $\varepsilon < \varepsilon_1$ in Corollary 1 is tighter than $\varepsilon < \varepsilon_0$ in Proposition 2. Thus, the range of ε is determined by ε_1 in Remark 2.

By using Formula (28), we have $\lambda \in (0, 2.3664)$ by direct calculation, and by using Formula (58), we have $\lambda \in J = (0, 1.61)$ by calculation. Thus, $\lambda \in J = (0, 1.61)$. In fact, from Formula (58), we have

$$\lambda^2 < \frac{-2\alpha(M+N)(2-B-\mu)}{2-B+\mu}$$

Denote $y = \dfrac{-2\alpha(M+N)(2-B-\mu)}{2-B+\mu}$. Note that $M+N = s_1\varepsilon + s_2$ by Lemma 5. Therefore, $\dfrac{\partial y}{\partial \varepsilon} = \dfrac{-2\alpha s_1(2-B-\mu)}{2-B+\mu}$. Because $s_1 > 0$ by Lemma 5, we have $\dfrac{\partial y}{\partial \varepsilon} < 0$. Thus, $\lambda_{\min} = \sqrt{\dfrac{-2\alpha(M+N)(2-B-\mu)}{2-B+\mu}}\Bigg|_{\varepsilon=0.328} = 1.61$.