Supplementary Material: A Simple Model of Air Pollution, Compensation, and Within-Firm Inequality

S1 Environment and Timing

A risk-neutral firm hires one executive and n rank-and-file workers at nominal wage w. Capital K is predetermined.

Local air pollution P is exogenous to the firm. It affects both the level and the risk of output within the period.

Timing: P realizes → the firm offers an executive contract (c, a) (cash c, incentive intensity a) and chooses (w, n) → production and payoffs realize.

S2 Technology and Risk

Output is

$$Y = \theta(P)F(K,n) + \varepsilon\,, \qquad E[\varepsilon] = 0\,, \qquad Var(\varepsilon) = \sigma^2(P)\,,$$

With $\theta'(P) < 0$ (level effect) and $\sigma^{2\prime}(P) > 0$ (risk effect).

Profits (before executive compensation costs) are $Y - wn$.

S3 Contracts, Preferences, and Constraints

The executive has CARA utility over (stochastic) pay: $U = -exp[-r(c + a\pi)], r > 0$. The performance signal π is a linear function of profits (e.g.,$\pi = \alpha(Y - wn)$) and is normally distributed.

The cost of incentive intensity is $\phi(a)$ with $\phi'(a) > 0, \phi''(a) \geq 0$ (e.g.,dilution, agency or implementation costs).

Rank-and-file wages exhibit downward nominal rigidity: $w \geq \overline{w}$ (minimum wage).

A tighter cash position under higher P can be captured by a shadow cost of cash $\mu(P)$ with $\mu'(P) > 0$.

S4 Firm's Problem and First-Order Conditions

The firm maximizes the certainty equivalent of shareholders' payoff. Under the linear-normal-CARA structure (and absorbing the participation constraint into a constant), the objective is

$$max_{c,a,w,n}\ \underbrace{E[Y - wn] - c - \phi(a)}_{\text{expected profit}} - \underbrace{\tfrac{1}{2}ra^2Var(\pi)}_{risk\ premium} - \underbrace{\mu(P)c}_{\text{cash shadow cost}} \qquad \text{s.t. } w \geq \overline{w}$$

Because $\mathrm{Var}(\pi) = \alpha^2\sigma^2(P)$, the FOC for a yields

$$\phi'(a^*) + ra^*\alpha^2\sigma^2(P) = \text{benefit of incentives} \Rightarrow \frac{\partial a^*}{\partial P} < 0$$

i.e., higher pollution (greater earnings risk) lowers optimal incentive intensity.

The FOC for c implies $1 + \mu(P)$ is the effective marginal cost of cash; with $\mu'(P) > 0$,

$\frac{\partial c^*}{\partial P} < 0$ (cash conservation under pollution shocks).

With $w \geq \overline{w}$ typically binding, adjustment occurs on the employment margin:

$\frac{\partial n^*}{\partial P} < 0$, $\frac{\partial w^*}{\partial P} = 0$ (at the floor).

S5 Implications for Labor Share and Pay Dispersion

Define the labor income share as $LS = \frac{wn}{Y}$ (or an equivalent compensation-to-output measure).

Since $\theta(P) \downarrow$ and $n^*(P) \downarrow$, $\frac{\partial LS}{\partial P} < 0$. Executive total pay is $\mathrm{ExecPay} = c^* + a^*E[\pi]$, so with $a^* \downarrow$

and $c^* \downarrow$while w is rigid, $\frac{\partial}{\partial \mathrm{P}}\left(\frac{\mathrm{ExecPay}}{\mathrm{w}}\right) < 0$, i.e., within-firm pay dispersion narrows because the top falls more while the bottom is constrained—what we term spurious equality: a reduction in dispersion arising from top-end compression, not from welfare gains for lower-paid workers, and typically accompanied by headcount cuts and weaker innovation.

S6 Testable Predictions (Mapping to the Empirics)

The model implies that higher pollution P lowers the firm's labor income share LS by depressing expected productivity and shifting adjustment to employment rather than wages; the effect should be stronger where employment is the primary margin—namely in labor-intensive firms and firms with weaker unions. Increased risk under higher P compresses risk-sensitive executive incentives and cash pay, while rank-and-file wages remain downward-rigid, so within-firm pay gaps narrow via "larger cuts at the top." Consistent with health and mobility responses, higher P should also induce outflows of high-skill labor (reducing the share of highly educated employees), contributing further to the decline in LS. Under tighter risk and cash constraints, firms are expected to defer long-horizon projects, yielding fewer patents—especially invention-quality output. Finally, in more capital-intensive firms, part of the shock can be absorbed through utilization and scheduling, so the negative impact of P on LS is attenuated.

This appendix provides the formal backbone for the paper's mechanism: pollution increases uncertainty and lowers expected productivity, leading to executive incentive and cash pay compression, wage-floor-induced adjustment on employment, a lower labor share, and narrower within-firm pay gaps driven by the top rather than improvements at the bottom.