S1 The proofs of Lemmas, Theorems and Propositions etc.

Proof of Lemma 1. (1) When $D \geqslant S$, $\pi(Q_i)$ can be expressed as $\pi(Q_i) = \max_{p_i \geq 0}\{p_i S - \delta(a_i - bp_i - S)\}$. The manufacturer's profit increases with p_i, and we have that $\tilde{p}_i = (a_i - S)/b$. (2) When $D < S$, $\pi(Q_i)$ can be expressed as $\pi(Q_i) = \max_{p_i \geq 0}\{p_i(a_i - bp_i)\}$, $\partial^2\pi/\partial p_i^2 < 0$. By the first-order condition $\tilde{p}_i = a_i/2b, D < S$ is equivalent to $S > a_i/2$.

Proof of Lemma 2. (1) When $Q_{2i} > A_i$, the manufacturer's problem can be formulated as $\max_{Q_{1i},Q_{2i}\geqslant 0}\left\{\theta\left(\frac{a_i^2}{4b} - w_{1i}Q_{1i} - w_{2i}Q_{2i}\right) + (1-\theta)\left(\frac{a_i^2}{4b} - w_{2i}Q_{2i}\right)\right\}$. The manufacturer's profit decreases with Q_{2i}; therefore, $\tilde{Q}_{2i} \leq A_i$.

(2) When $Q_{2i} \leq A_i, Q_{1i} + Q_{2i} > A_i$, the manufacturer's problem can be formulated as $\max_{Q_{1i},Q_{2i}\geqslant 0}\left\{\theta\left(\frac{a_i^2}{4b} - w_{1i}Q_{1i} - w_{2i}Q_{2i}\right) + (1-\theta)\left(\frac{a_i - Q_{2i}}{b}Q_{2i} - w_{2i}Q_{2i}\right)\right\}$. The manufacturer's profit decreases with Q_{1i}; therefore, $\tilde{Q}_{1i} + \tilde{Q}_{2i} \leq A_i$.

(3) When $Q_{2i} \leq A_i, Q_{1i} + Q_{2i} \leq A_i$, the manufacturer's problem can be formulated as $\max_{Q_{1i},Q_{2i}\geqslant 0}\left\{\theta\left[\frac{a_i - Q_{1i} - Q_{2i}}{b}(Q_{1i} + Q_{2i}) - w_{1i}Q_{1i} - w_{2i}Q_{2i}\right] + (1-\theta)\left(\frac{a_i - Q_{2i}}{b}Q_{2i} - w_{2i}Q_{2i}\right)\right\}$. The Hessian matrix $H = \begin{bmatrix} -\frac{2\theta}{b} & -\frac{2\theta}{b} \\ -\frac{2\theta}{b} & -\frac{2}{b} \end{bmatrix}$ is negative definite, and there are extreme values. From the first-order condition, $\tilde{Q}_{1i} = \frac{a_i - bw_1 - 2\tilde{Q}_{2i}}{2}$ and $\tilde{Q}_{2i} = \frac{a_i - bw_2 - 2\tilde{Q}_{1i}\theta}{2}$, and solving the quadratic equation by association yields $\tilde{Q}_{1i} = \frac{bw_2 - bw_1}{2(1-\theta)}$ and $\tilde{Q}_{2i} = \frac{a_i - \theta a_i + b\theta w_1 - bw_2}{2(1-\theta)}$.

Proof of Theorem 1. We prove Theorem 1 by drawing on the proof procedure by Shan et al. [19]. According to Lemma 2, the decision plane is divided into three regions (see Figure S1): $w_{1i} < w_{2i} < (a_i - \theta a_i + b\theta w_{1i})/b$, $w_{2i} \leq w_{1i}$, and $w_{2i} \geq (a_i - \theta a_i + b\theta w_{1i})/b$. As shown in Figure S1, Region D is the interval for dual-sourcing, and S(1) and S(2) are the intervals for single-sourcing. First, we solve the problem of Supplier 1.

(1) When $w_{1i} < w_{2i} < (a_i - \theta a_i + b\theta w_{1i})/b$, Supplier 1's problem can be formulated as follows: $\max_{w_{1i}}\left\{w_{1i}\theta\frac{bw_{2i} - bw_{1i}}{2(1-\theta)}\right\}$. By the first-order condition, $w_{1i} = w_{2i}/2$;

(2) When $w_{2i} \leq w_{1i}, \tilde{Q}_{1i} = 0$, Supplier 1 sets wholesale price: $w_{1i} = (a_i + bc_r)/2b$;

(3) When $w_{2i} \geq (a_i - \theta a_i + b\theta w_{1i})/b$, $\max_{w_{1i}}\left\{w_{1i}\theta\frac{a_i - bw_{1i}}{2}\right\}$. By the first-order condition, $w_{1i} = a_i/2b$.

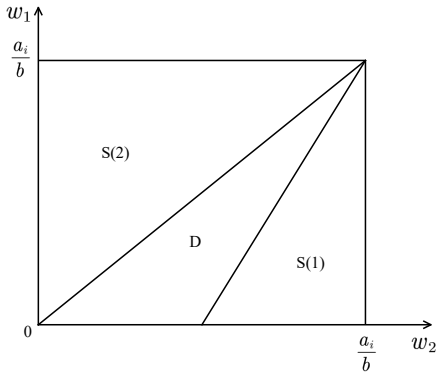


Figure S1. The decision plane.

We denote the intersection of $w_{1i} = w_{2i}/2$ and $w_{2i} = (a_i - \theta a_i + b\theta w_{1i})/b$ as (w_{1i}^a, w_{2i}^a),$w_{1i}^a = \frac{a_i - \theta a_i}{b(2-\theta)}$, and $w_{2i}^a = \frac{2(a_i - \theta a_i)}{b(2-\theta)}$, so $w_{1i}^a < \frac{a_i}{2b}$. The reaction curve of w_{1i} about w_{2i} is given in the following:

(1) When $c_r \le \frac{2a_i(1-\theta)}{b(2-\theta)}$,

$$w_{1i} = \begin{cases} \dfrac{w_{2i}}{2}, & c_r \le w_{2i} < \dfrac{2(a_i - \theta a_i)}{b(2-\theta)}, \\ \dfrac{bw_{2i} - a_i + \theta a_i}{b\theta}, & \dfrac{2(a_i - \theta a_i)}{b(2-\theta)} \le w_{2i} < \dfrac{(2-\theta)a_i}{2b}, \\ \dfrac{a_i}{2b}, & w_{2i} \ge \dfrac{(2-\theta)a_i}{2b}. \end{cases}$$

(2) When $\frac{2a_i(1-\theta)}{b(2-\theta)} < c_r \le \frac{(2-\theta)a_i}{2b}$,

$$w_{1i} = \begin{cases} \dfrac{bw_{2i} - a_i + \theta a_i}{b\theta}, & c_r \le w_{2i} < \dfrac{(2-\theta)a_i}{2b}, \\ \dfrac{a_i}{2b}, & w_{2i} \ge \dfrac{(2-\theta)a_i}{2b}. \end{cases}$$

(3) When $c_r > \frac{(2-\theta)a_i}{2b}$,$w_{1i} = \frac{a_i}{2b}$.

The reaction curve of w_{1i} with respect to w_{2i} when $c_r \le \frac{2a_i(1-\theta)}{b(2-\theta)}$ is shown in the following.

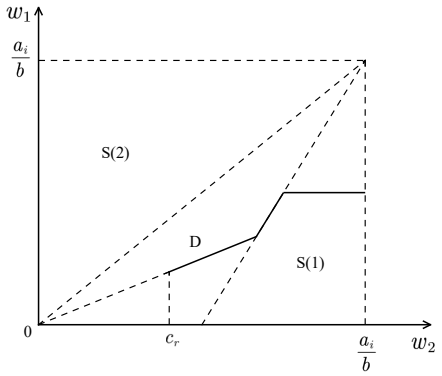


Figure S2. The reaction curve of w_{1i} with respect to w_{2i} when $c_r \le \frac{2a_i(1-\theta)}{b(2-\theta)}$.

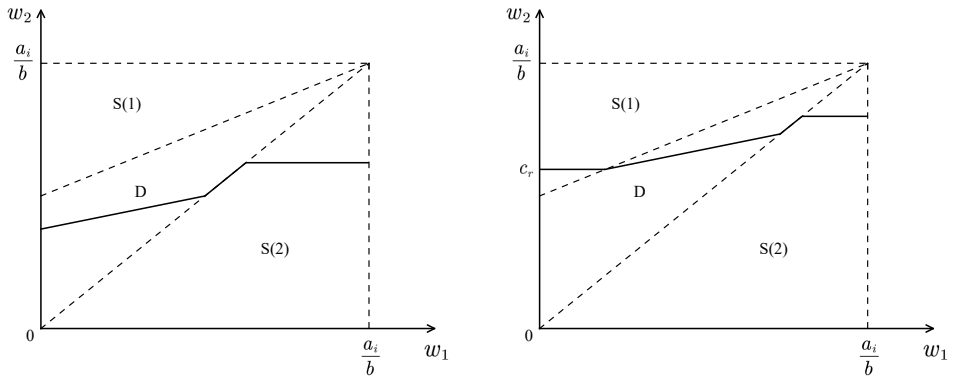
Next, we solve the problem of Supplier 2.

(1) When $w_{1i} < w_{2i} < (a_i - \theta a_i + b\theta w_{1i})/b$, Supplier 2's problem can be formulated as follows: $\max_{w_{2i}} \left\{(w_{2i} - c_r)\frac{a_i - \theta a_i + b\theta w_{1i} - bw_{2i}}{2(1-\theta)}\right\}$. By the first-order condition, $w_{2i} = \frac{a_i - \theta a_i + bc_r + b\theta w_{1i}}{2b}$;

(2) When $w_{2i} \le w_{1i}$, Supplier 2's problem can be formulated as follows: $\max_{w_{2i}} \left\{(w_{2i} - c_r)\frac{a_i - bw_{2i}}{2}\right\}$, $w_{2i} = \frac{a_i + bc_r}{2b}$;

(3) When $w_{2i} \ge (a_i - \theta a_i + b\theta w_{1i})/b$, Supplier 2 sets the wholesale price: $w_{2i} = c_r$.

We denote the intersection of $w_{2i} = \frac{a_i - \theta a_i + bc_r + b\theta w_{1i}}{2b}$ and $w_{2i} = \frac{a_i - \theta a_i + b\theta w_{1i}}{b}$ as $\left(w_{1i}^b, w_{2i}^b\right)$, $w_{1i}^b = \frac{\theta a_i + bc_r - a_i}{b\theta}$, and $w_{2i}^b = c_r$; we denote the intersection of $w_{2i} = \frac{a_i - \theta a_i + bc_r + b\theta w_{1i}}{2b}$ and $w_{2i} = w_{1i}$ as (w_{1i}^c, w_{2i}^c), $w_{1i}^c = \frac{a_i - \theta a_i + bc_r}{b(2-\theta)}$, and $w_{2i}^c = \frac{a_i - \theta a_i + bc_r}{b(2-\theta)}$, and we have $c_r < w_{2i}^c < \frac{a_i + bc_r}{2b}$. The reaction curve of w_{2i} with respect to w_{1i} is given in the following (see Figure S3).



(a) $0 < c_r \le \frac{a_i - \theta a_i}{b}$ **(b)** $c_r > \frac{a_i - \theta a_i}{b}$

Figure S3. The reaction curve of w_{2i} with respect to w_{1i} for different values of c_r.

(1) When $0 < c_r \le \frac{a_i - \theta a_i}{b}$,

$$w_{2i} = \begin{cases} \frac{a_i - \theta a_i + bc_r + b\theta w_{1i}}{2b}, & 0 \le w_{1i} < \frac{a_i - \theta a_i + bc_r}{b(2-\theta)}, \\ w_{1i}, & \frac{a_i - \theta a_i + bc_r}{b(2-\theta)} \le w_{1i} < \frac{a_i + bc_r}{2b}, \\ \frac{a_i + bc_r}{2b}, & w_{1i} \ge \frac{a_i + bc_r}{2b}. \end{cases}$$

(2) When $c_r > \frac{a_i - \theta a_i}{b}$,

$$w_{2i} = \begin{cases} c_r, & 0 \le w_{1i} < \frac{\theta a_i + bc_r - a_i}{b\theta}, \\ \frac{a_i - \theta a_i + bc_r + b\theta w_{1i}}{2b}, & \frac{\theta a_i + bc_r - a_i}{b\theta} \le w_{1i} < \frac{a_i - \theta a_i + bc_r}{b(2-\theta)}, \\ w_{1i}, & \frac{a_i - \theta a_i + bc_r}{b(2-\theta)} \le w_{1i} < \frac{a_i + bc_r}{2b}, \\ \frac{a_i + bc_r}{2b}, & w_{1i} \ge \frac{a_i + bc_r}{2b}. \end{cases}$$

In summary, based on the response curves of w_{1i} and w_{2i}, by intersecting the response curves, we obtain the unique Nash equilibrium of the wholesale price with responsive pricing when the manufacturer shares information, i.e., $(\widetilde{w}_{1i}^{RS}, \widetilde{w}_{2i}^{RS})$. In addition, when $a_l < a_h/a_l < 2/(2-\theta)$, this satisfies $(2-\theta)a_h/2b < a_l/b$.

Proof of Theorem 2. When the manufacturer does not share information, the two suppliers only have the prior distribution of the demand potential, i.e., $a_i = (1/2)a_h + (1/2)a_l$, and they obtain a unique Nash equilibrium based on this prior distribution. The proof procedure is similar to that for Theorem 1.

Proof of Corollary 2. This can be proved by comparing the relationship between the magnitudes of $\widetilde{w}_{2i}^{RN}$ and $(a_i - \theta a_i + b\theta\widetilde{w}_{1i}^{RN})/b$ when $a_i = a_h$ and $a_i = a_l$.

Proof of Lemma 3. (1) When $Q_{2i} > D$, the manufacturer's problem can be formulated as $\max_{Q_{1i},Q_{2i},p_i\geqslant 0}\{\theta[p_i(a_i - bp_i) - w_{1i}Q_{1i} - w_{2i}Q_{2i}] + (1-\theta)[p_i(a_i - bp_i) - w_{2i}Q_{2i}]\}$. The manufacturer's profit decreases with Q_{2i}, $\tilde{Q}_{2i} \leq D$.

(2) When $Q_{2i} \leq D$, $Q_{1i} + Q_{2i} > D$, the manufacturer's problem can be formulated as $\max_{Q_{1i},Q_{2i},p_i\geqslant 0}\{\theta[p_i(a_i - bp_i) - w_{1i}Q_{1i} - w_{2i}Q_{2i}] + (1-\theta)(p_iQ_{2i} - w_{2i}Q_{2i})\}$. The manufacturer's profit decreases with Q_{1i}, $\tilde{Q}_{1i} + \tilde{Q}_{2i} \leq D$.

(3) When $Q_{2i} \leq D$, $Q_{1i} + Q_{2i} \leq D$, the manufacturer's problem can be formulated as $\max_{Q_{1i},Q_{2i},p_i\geqslant 0}\{\theta[p_i(Q_{1i} + Q_{2i}) - w_{1i}Q_{1i} - w_{2i}Q_{2i}] + (1-\theta)(p_iQ_{2i} - w_{2i}Q_{2i})\}$. The manufacturer's profit decreases with p_i, so $\tilde{p}_i = \left(a_i - \tilde{Q}_{1i} - \tilde{Q}_{2i}\right)/b$.

Substituting $\tilde{p}_i$ into Equation (7) yields

$$\max_{Q_{1i},Q_{2i}\geqslant 0}\left\{\theta\left[\frac{a_i - Q_{1i} - Q_{2i}}{b}(Q_{1i} + Q_{2i}) - w_{1i}Q_{1i} - w_{2i}Q_{2i}\right] + (1-\theta)\left[\frac{a_i - Q_{1i} - Q_{2i}}{b}Q_{2i} - w_{2i}Q_{2i}\right]\right\}$$

The Hessian matrix $H = \begin{bmatrix} -\frac{2\theta}{b} & -\frac{1+\theta}{b} \\ -\frac{1+\theta}{b} & -\frac{2}{b} \end{bmatrix}$ is not negative definite and has no extreme values. The first-order condition on Q_{2i} yields $\tilde{Q}_{2i} = \begin{cases} 0, & Q_{1i} > \frac{a_i - bw_{2i}}{1+\theta}, \\ \frac{a_i - Q_{1i} - \theta Q_{1i} - bw_{2i}}{2}, & Q_{1i} \leq \frac{a_i - bw_{2i}}{1+\theta}. \end{cases}$

(1) When $Q_{1i} > \frac{a_i - bw_{2i}}{1+\theta}$, the manufacturer's problem is $\max_{Q_{1i}\geqslant 0}\left\{\theta\left(\frac{a_i - Q_{1i}}{b}Q_{1i} - w_{1i}Q_{1i}\right)\right\}$; therefore, $\tilde{Q}_{1i} = \frac{a_i - bw_{1i}}{2}$. $Q_{1i} > \frac{a_i - bw_{2i}}{1+\theta}$ is equivalent to $w_{1i} < \frac{\theta a_i - a_i + 2bw_{2i}}{b+b\theta}$.

(2) When $Q_{1i} \leq \frac{a_i - bw_{2i}}{1+\theta}$, the manufacturer's problem is $\max_{Q_{1i}\geqslant 0}\left\{\frac{(a_i - Q_{1i})^2 + (a_i - Q_{1i} - 2bw_{1i})2\theta Q_{1i} - 2bw_{2i}(a_i - Q_{1i}) + (bw_{2i} + \theta Q_{1i})^2}{4b}\right\}$. It is known that the profit function is a convex function with respect to Q_{1i}. The optimal profit is obtained only at the two endpoints of the interval, i.e., $Q_{1i} = 0$ or $Q_{1i} = \frac{a_i - bw_{2i}}{1+\theta}$. Since $\frac{\mathrm{d}^2\pi_M(Q_1)}{\mathrm{d}Q_{1i}^2} = \frac{(1-\theta)^2}{2b} > 0$, $\frac{\mathrm{d}\pi_M(Q_1)}{\mathrm{d}Q_{1i}}$ is monotonically increasing. When $w_{1i} < \frac{\theta a_i - a_i + bw_{2i} + b\theta w_{2i}}{2b\theta}$, $\frac{\mathrm{d}\pi_M(Q_{1i})}{\mathrm{d}Q_{1i}}$ is greater than 0 at $Q_{1i} = 0$, and $\pi_M(Q_{1i})$ is monotonically increasing. The profit function takes its maximum value at $Q_{1i} = \frac{a_i - bw_{2i}}{1+\theta}$. When $w_{1i} > \frac{\theta a_i - a_i + 2bw_{2i}}{b+b\theta}$, $\frac{\mathrm{d}\pi_M(Q_{1i})}{\mathrm{d}Q_{1i}}$ is less than 0 at $Q_{1i} =$

$\frac{a_i-bw_{2i}}{1+\theta}$, and $\pi_M(Q_{1i})$ is monotonically decreasing. The profit function takes its maximum value at $Q_1=0$. Thus, there exists a $\bar{w}_{1i}\in\left[\frac{\theta a_i-a_i+bw_{2i}+b\theta w_{2i}}{2b\theta},\frac{\theta a_i-a_i+2bw_{2i}}{b+b\theta}\right]$ such that $\pi_M(0)=\pi_M\left(\frac{a_i-bw_{2i}}{1+\theta}\right)$, and we have $\tilde{Q}_{1i}=\begin{cases}\frac{a_i-bw_{2i}}{1+\theta}, & w_{1i}<\bar{w}_{1i},\\ 0, & w_{1i}\geq\bar{w}_{1i}.\end{cases}$

Combining (1) and (2) shows that $\tilde{Q}_{1i}=0$ when $w_{1i}\geq\frac{\theta a_i-a_i+2bw_{2i}}{b+b\theta}$ and $\tilde{Q}_{1i}=\frac{a_i-bw_{1i}}{2}$ when $w_{1i}\leq\bar{w}_{1i}$. When $\bar{w}_{1i}<w_{1i}\leq\frac{\theta a_i-a_i+2bw_{2i}}{b+b\theta}$, we need to compare the sizes of $\pi_M(0)$ and $\pi_M\left(\frac{a_i-bw_{1i}}{2}\right)$ to determine the equilibrium decision for that interval.

Let $f(w_{1i})=\pi_M(0)-\pi_M\left(\frac{a_i-bw_{1i}}{2}\right)$. $\pi_M\left(\frac{a_i-bw_{1i}}{2}\right)>\pi_M\left(\frac{a_i-bw_{2i}}{1+\theta}\right)$ when $\bar{w}_{1i}<w_{1i}<\frac{\theta a_i-a_i+2bw_{2i}}{b+b\theta}$, so $f(\bar{w}_{1i})<0.\frac{\mathrm{d}f(w_{1i})}{\mathrm{d}w_{1i}}=\frac{a_i-bw_{1i}}{2}\theta>0$, and $f(w_{1i})$ increases with w_{1i}. This is because when $w_{1i}=\frac{\theta a_i-a_i+2bw_{2i}}{b+b\theta}$, $\pi_M\left(\frac{a_i-bw_{1i}}{2}\right)=\pi_M\left(\frac{a_i-bw_{2i}}{1+\theta}\right)<\pi(0)$, so $f\left(\frac{\theta a_i-a_i+2bw_{2i}}{b+b\theta}\right)>0$. Hence, there exists $\hat{w}_{1i}\in\left(\bar{w}_{1i},\frac{\theta a_i-a_i+2bw_{2i}}{b+b\theta}\right)$ such that $f(\hat{w}_{1i})=0$. When $w_{1i}>\hat{w}_{1i}$, $\tilde{Q}_{1i}=0$; when $w_{1i}\leq\hat{w}_{1i}$, $\tilde{Q}_{1i}=\frac{a_i-bw_{1i}}{2}$.

In summary, the manufacturer's equilibrium purchasing decision from Supplier 1 is $\tilde{Q}_{1i}=\begin{cases}\frac{a_i-bw_{1i}}{2}, w_{1i}\leq\hat{w}_{1i},\\ 0, \quad w_{1i}>\hat{w}_{1i}.\end{cases}$

Due to $\hat{w}_{1i}=\frac{\sqrt{\theta}a_i-a_i+bw_{2i}}{b\sqrt{\theta}}$, when $0<w_{2i}\leq\frac{a_i}{b}\left(1-\sqrt{\theta}\right)$, we have $\hat{w}_{1i}\leq 0$, $\tilde{Q}_{1i}=0$; when $\frac{a_i}{b}\left(1-\sqrt{\theta}\right)<w_{2i}<\frac{a_i}{b}$, we have $\tilde{Q}_{1i}=\begin{cases}\frac{a_i-bw_1}{2}, w_{1i}\leq\hat{w}_{1i},\\ 0, \quad w_{1i}>\hat{w}_{1i}.\end{cases}$

Proof of Theorem 3. The proof of Theorem 3 proceeds similarly to that of Theorem 1.

Proof of Theorem 4. The proof of Theorem 4 proceeds similarly to that of Theorem 2.

Proof of Corollary 3. Similarly to the proof process for Corollary 2, the proof can be obtained by comparing the magnitude relationships between $\widetilde{w}_2^{CN}$ and $a_i\left(1-\sqrt{\theta}\right)/b$, $\widetilde{w}_1^{CN}$, and $\hat{w}_{1i}$ when $a_i=a_h$ and $a_i=a_l$ are present.

Proof of Proposition 1. When $1<\frac{a_h}{a_l}<\min\left\{\frac{4-4\theta+\theta^2}{4-4\theta},\frac{2}{2-\theta}\right\}$, $\bar{C}_{1h}^{RS}<\hat{C}_{1l}^{RS}$. Thus, the manufacturer's expected profit with the RS strategy for different values of c_r is as follows. For $0<c_r\leq\bar{C}_{1l}^{RS}$, $\widetilde{\Pi}_M^{RS}=\frac{(4+5\theta)(a_h^2+a_l^2)-2b(4+\theta)(a_h+a_l)c_r}{8b(4-\theta)^2}+\frac{bc_r^2(4-3\theta)}{4(4-\theta)^2(1-\theta)}$. For $\bar{C}_{1l}^{RS}<c_r\leq\bar{C}_{1h}^{RS}$, $\widetilde{\Pi}_M^{RS}=\frac{(4+5\theta)a_h^2-2b(4+\theta)a_hc_r}{8b(4-\theta)^2}+\frac{bc_r^2(16+6\theta^2-20\theta-\theta^3)}{8\theta(4-\theta)^2(1-\theta)}+\frac{a_l^2-2ba_lc_r}{8b\theta}$. For $\bar{C}_{1h}^{RS}<c_r\leq\hat{C}_{1l}^{RS}$, $\widetilde{\Pi}_M^{RS}=\frac{(a_h-bc_r)^2+(a_l-bc_r)^2}{8b\theta}$. For $\hat{C}_{1l}^{RS}<c_r\leq\hat{C}_{1h}^{RS}$, $\widetilde{\Pi}_M^{RS}=\frac{(a_h-bc_r)^2}{8b\theta}+\frac{\theta a_l^2}{32b}$. For $c_r>\hat{C}_{1h}^{RS}$, $\widetilde{\Pi}_M^{RS}=\frac{\theta(a_h^2+a_l^2)}{32b}$.

Similarly, the expected profit with the RN strategy is as follows. For $0<c_r\leq\frac{(1-\theta)[(6-\theta)a_l-(2-\theta)a_h]}{2b(2-\theta)}$, $\widetilde{\Pi}_M^{RN}=\frac{bc_r^2(4-3\theta)-(1-\theta)(4+\theta)(a_h+a_l)c_r}{4(4-\theta)^2(1-\theta)}+\frac{(20-3\theta+\theta^2)(a_h^2+a_l^2)-2(12-\theta)(1-\theta)a_ha_l}{16b(4-\theta)^2}$. For $\frac{(1-\theta)[(6-\theta)a_l-(2-\theta)a_h]}{2b(2-\theta)}<c_r\leq\bar{C}_1^{RN}$, $\widetilde{\Pi}_M^{RN}=\frac{bc_r^2(4-2\theta-\theta^2)}{8(4-\theta)^2(1-\theta)}-\frac{(12-2\theta+\theta^2)a_hc_r-(4-10\theta+\theta^2)a_lc_r}{8(4-\theta)^2}+\frac{(36+2\theta-3\theta^2+\theta^3)a_h^2-2(1-\theta)(12+6\theta-\theta^2)a_ha_l+(4+42\theta-11\theta^2+\theta^3)a_l^2}{32b(4-\theta)^2}$. For $\bar{C}_1^{RN}<c_r\leq\hat{C}_1^{RN}$; $\widetilde{\Pi}_M^{RN}=$

$\frac{(2+\theta+\theta^2)(a_h^2+a_l^2)+2a_ha_l(2-\theta-\theta^2)}{32b\theta}+\frac{bc_r^2-c_r(a_h+a_l)}{4\theta}$. For $\hat{C}_1^{RN}<c_r\le\frac{4a_h-3\theta a_h+\theta a_l}{4b}$, $\tilde{\Pi}_M^{RN}=\frac{(16-14\theta-\theta^2)a_h^2+(10\theta-9\theta^2)a_l^2}{128b(1-\theta)}+\frac{16b^2c_r^2-8b\theta a_lc_r-(4-6\theta)\theta a_ha_l-8b(4-3\theta)a_hc_r}{128b(1-\theta)}$. For $c_r>\frac{4a_h-3\theta a_h+\theta a_l}{4b}$, $\tilde{\Pi}_M^{RN}=\frac{\theta(5a_h^2-6a_ha_l+5a_l^2)}{64b}$.

Proposition 1 is then obtained by comparing the sizes of $\tilde{\Pi}_M^{RS}$ and $\tilde{\Pi}_M^{RN}$ in each interval.

Proof of Proposition 2. The manufacturer's expected profit with the CS strategy is as follows. When $0<c_r\le\frac{a_l}{b}(1-\sqrt{\theta})$, $\tilde{\Pi}_M^{CS}=\frac{(a_h-bc_r)^2+(a_l-bc_r)^2}{32b}$. When $\frac{a_l}{b}(1-\sqrt{\theta})<c_r\le\frac{a_h}{b}(1-\sqrt{\theta})$, $\tilde{\Pi}_M^{CS}=\frac{(a_h-bc_r)^2+\theta a_l^2}{32b}$. When $\frac{a_h}{b}(1-\sqrt{\theta})<c_r<\frac{a_l}{b}$, $\tilde{\Pi}_M^{CS}=\frac{\theta(a_h^2+a_l^2)}{32b}$.

The manufacturer's expected profit with the CN strategy is as follows. When $0<c_r\le\frac{(a_h+a_l)}{2b}(1-\sqrt{\theta})$, $\tilde{\Pi}_M^{CN}=\frac{(a_l-3a_h+2bc_r)^2+8\sqrt{\theta}a_l(a_h-2\sqrt{\theta}a_h+a_l-2bc_r)}{128b}$. When $\frac{(a_h+a_l)}{2b}(1-\sqrt{\theta})<c_r\le\frac{(2-\sqrt{\theta})(a_h+a_l)}{4b}$, $\tilde{\Pi}_M^{CN}=\frac{\theta(a_h-3a_l)^2+\left[(2+\sqrt{\theta})a_h-(2-\sqrt{\theta})a_l\right]^2}{128b}$. When $c_r>\frac{(2-\sqrt{\theta})(a_h+a_l)}{4b}$, $\tilde{\Pi}_M^{CN}=\frac{\theta(a_h-3a_l)^2+16(a_h-bc_r)^2}{128b}$.

Proposition 2 is then obtained by comparing the sizes of $\tilde{\Pi}_M^{CS}$ and $\tilde{\Pi}_M^{CN}$ for each interval.