A Novel Design of a Hybrid Solar Double-Chimney Power Plant for Generating Electricity and Distilled Water
(Supplementry Materials)

Detailed mathematical model and model validation.

Nomenclature

$\overline{Nu}$	Nusselt number
$\overline{\mathrm{m}}$	Mass flow rate, (kg/sec)
A	Area: (m^2)
c_p	Specific heat capacity, (J/kg.K)
D	Diameter, (m)
Dh	Hydraulic diameter, (m)
d_h	Hydraulic diameter, (m)
F	Friction factor
F	Friction factor
G	Acceleration of gravity, (m/s^2)
H	Height, (m)
H	Heat transfer coefficient, (W/m^2. K)
hfg	Latent heat of water evaporation, (W/m^2. K)
I	Solar irradiation intensity, (W/m^2)
I	Enthalpy
K	Air thermal conductivity
Pelc	Electrical Out Power, (W)
Pr	Prandlt number
Q	Heat transfer rate, (W/m^2)
Q_{out}	The heat transfer between the chimney and the ambient, (W)

R	Radius, (m)
Re	Reynolds number
Rw	Water Pool Radius
Sc	Schmidt number
Sh_D	Sherwood number
T	Temperature, (K)
V	Wind velocity (m/s)
Z	Available wind data height
Zo	The roughness height of the glass
Z_R	The height at which the velocity needed to estimate

Greek Symbols

α	Absorptivity
Ε	Emissivity
η	efficiency, %
μ	Air dynamic viscosity
ρ	density, kg/m^3
σ	Stefan- Boltzmann constant, $W/m^2. K^4$
τ	Transmissivity
Ω	Humidity ratio

Subscripts

abs	Absorber plate
air	Airflow
c	Convective heat transfer
cd	Condensated water
ch	Chimney
col	Collector roof

e	Evaporation
ent	Entrance
gls	Glass cover or convective heat transfer
out	Outside
r	Radiative heat transfer
rad	Radiation heat transfer
sky	Sky
wtr	Water

The proposed design has the following dimensions:

Table S1. Dimensions of the HSDCPP model.

Parameter	Dimension (m)
Collector Diameter	250
Collector Entrance Height	6
Chimney Height	270
Chimney Diameter	20

Refer to Figure S1 below as a reference diagram for the equations.

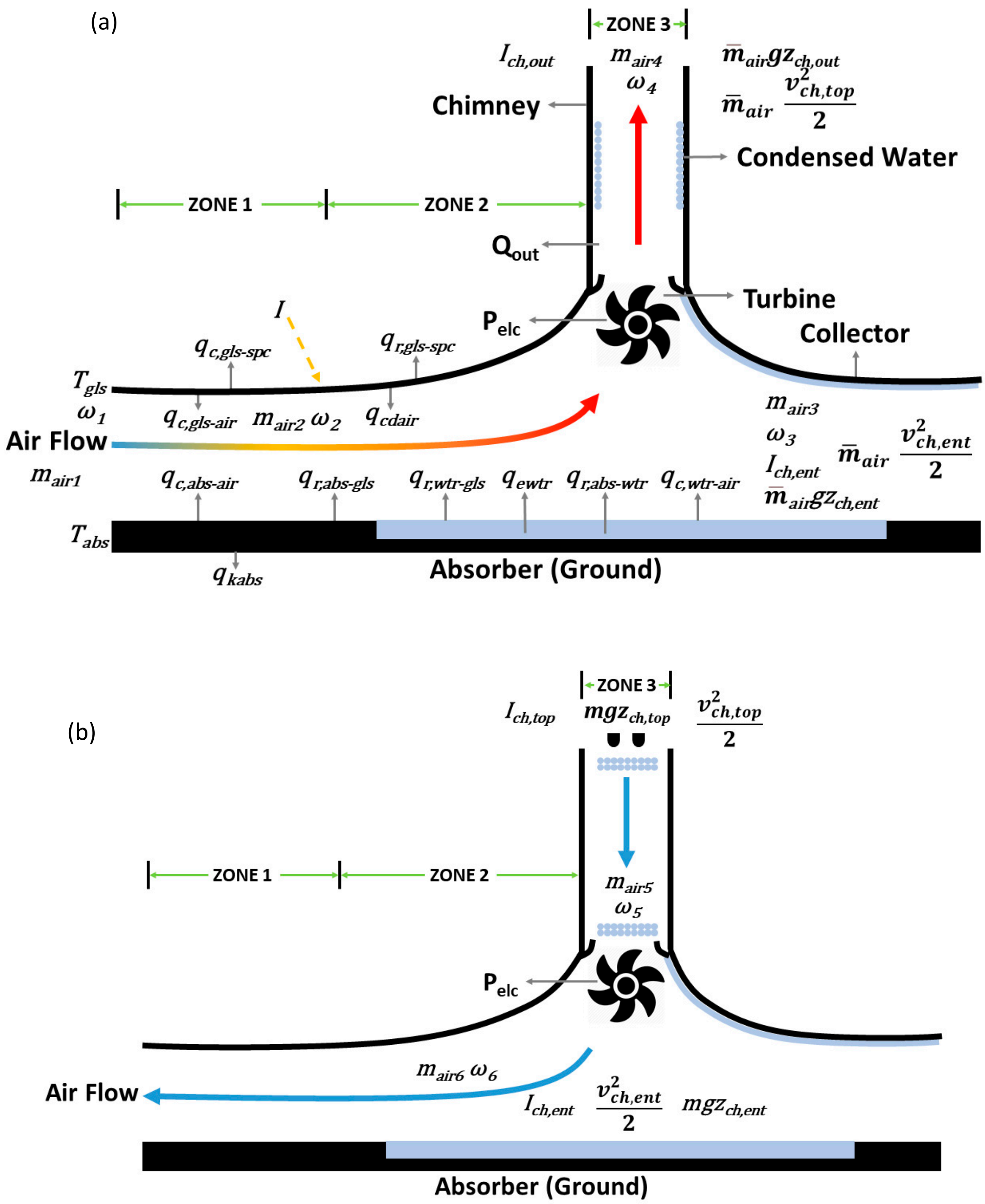

Figure S1. Energy balance environment. (**a**) is in the case of the SCPP mode, and (**b**) in the case of the CT mode.

Zone 1: Solar Air Heating

The energy balance equations for the air flowing in are as follows:

Airflow:

$$q_{c,gls-air} + q_{c,abs-air} = -\frac{c_{p,air}\overline{m}_{air}}{2\pi r}\frac{dT_{air}}{dr} \tag{S1}$$

where $\omega_1 = \omega_2$, from air mass balance equation

Absorber:

$$q_{r,abs-gls} + q_{c,abs-air} + q_{kabs} = \alpha_{abs}\tau_{gls}I \tag{S2}$$

Collector:

$$q_{c,gls-out} + q_{c,gls-air} + q_{r,gls-spc} = \alpha_{gls}I + q_{r,abs-gls} \tag{S3}$$

The convective heat transfer between the glass of the collector and the air flowing in the system is as follows:

$$q_{c,gls-air} = h_{c,gls-air}\,(T_{gls} - T_{air}) \tag{S4}$$

The convective heat transfer coefficient between the air under the collector and the glass is given according to the two following equations [1]:

$$h_{c,gls-air} = \frac{0.2106 + 0.0026\,V_{in}\left(\frac{T_m\rho_{air}}{\mu g(T_{gls} - T_{air})}\right)^{\frac{1}{3}}}{\left(\frac{\mu T_m}{(T_{gls} - T_{air})g\rho_{air}{}^2 C_P k^2}\right)^{\frac{1}{3}}} \tag{S5}$$

Where, T_m is the average temperature of T_{gls} and T_{air}, $\left[T_m = \left(\frac{T_{gls}+T_{air}}{2}\right)\right]$

$$h_{c,gls-air} = \frac{\left(\frac{f}{8}\right)(Re - 1000)Pr}{1 - 12.7\left(\frac{f}{8}\right)^{\frac{1}{2}}\left(Pr^{\frac{2}{3}} - 1\right)}\left(\frac{k}{d_h}\right) \tag{S6}$$

When $T_{air} > T_{gls}$, then the value of $h_{c,gls-air}$ is based on the higher value produced from Eqs. (S5) and Eqs. (S6). However, if $T_{gls} > T_{air}$, Eqs. (S6) is used.

The convictive heat transfer rate between the absorber and the air under the collector is as follows:

$$q_{c,abs-air} = h_{c,abs-air}\,(T_{abs} - T_{air}) \tag{S7}$$

The convective heat transfer coefficient between the air under the collector and the base is given according to the two following equations [1]:

$$h_{c,abs-air} = \frac{0.2106 + 0.0026\,V_{in}\left(\frac{T_m \rho_{air}}{\mu g (T_{abs} - T_{air})}\right)^{\frac{1}{3}}}{\left(\frac{\mu T_m}{(T_{abs} - T_{air}) g {\rho_{air}}^2 C_P k^2}\right)^{\frac{1}{3}}} \tag{S8}$$

Where T_m is the mean temperature of T_{abs} and T_{air}.

$$h_{c,abs-air} = \frac{\left(\frac{f}{8}\right)(Re - 1000)Pr}{1 - 12.7\left(\frac{f}{8}\right)^{\frac{1}{2}}\left(Pr^{\frac{2}{3}} - 1\right)}\left(\frac{k}{d_h}\right) \tag{S9}$$

When $T_{air} > T_{abs}$, then the value of $\mathrm{h_{c,abs-air}}$ is based on the higher value produced from Eqs. (S8) and Eqs. (S9). However, if $T_{abs} > T_{air}$, Eqs. (S6) is used.

The radiation heat transfer rate between the absorber and the solar collector is given as follows:

$$q_{r,abs-gls} = h_{r,abs-gls}\,(T_{abs} - T_{gls}) \tag{S10}$$

The radiative heat transfer coefficient is given follows [2]:

$$h_{r,abs-gls} = \frac{\sigma\left({T_{gls}}^2 + {T_{abs}}^2\right)\left(T_{gls} + T_{abs}\right)}{\frac{1}{\varepsilon_{gls}} + \frac{1}{\varepsilon_{abs}} - 1} \tag{S11}$$

Where $\sigma = 5.67 \times 10^{-8}\,\frac{w}{m^2 k}$.

The convective heat transfer rate between the collector and the outside environment (sky) is as follows:

$$q_{c,gls-sky} = h_{c,gls-sky} \left(T_{gls} - T_{sky}\right) \tag{S12}$$

The convective heat transfer coefficient ($h_{c,gls-sky}$) is given by [2]:

$$h_{c,gls-sky} = 2.8 + 3.0v_0 \tag{S13}$$

Where, v_0 is the wind speed above the horizontal glass of the collector.

The sky temperature is given by [3]:

$$T_{sky} = T_0 - 6 \tag{S14}$$

The radiation heat transfer rate between the collector and the sky is given as:

$$q_{r,gls-sky} = h_{r,gls-sky} \left(T_{gls} - T_{sky}\right) \tag{S15}$$

The heat transfer coefficient ($h_{r,gls-sky}$) is given as follows [2]:

$$h_{r,gls-sky} = \sigma\varepsilon_{gls} \left(\frac{{T_{gls}}^4 - {T_{sky}}^4}{T_{gls} - T_{sky}}\right) \tag{S16}$$

Zone 2: Water Evaporation

The energy balance equations for the seawter are as follows:

$$q_{c,abs-wtr} + \alpha_{wtr}\tau_{gls}I = q_{ewtr} + q_{r,wtr-air} + q_{c,wtr-air} + c_{p,wtr}\dot{m_{wtr}}\frac{dT_{wtr}}{dt} \tag{S17}$$

It was assumed that that there is no spatial change the water temperature for the above equation.

Energy balance equation for air flow:

$$q_{c,wtr-air} + q_{c,gls-air} = -\frac{c_{p,air}\dot{m_{air}}}{2\pi r}\frac{dT_{air}}{dr} \tag{S18}$$

Energy balance equation for the absorber:

$$\alpha_{gls}\tau_{wtr}\tau_{gls}I = q_{c,abs-wtr} + q_{kabs} \quad \text{(S19)}$$

Energy balance equation for collector roof:

$$q_{c,gls-air} + q_{r,wtr-gls} + \alpha_{gls}I = q_{r,gls-sky} + q_{c,gls-sky} \quad \text{(S20)}$$

The convictive heat transfer rate between the absorber and the seawater is as follows:

$$q_{c,abs-wtr} = h_{c,abs-wtr}\,(T_{abs} - T_{wtr}) \quad \text{(S21)}$$

Where, the heat transfer coefficient between the water and the base is given as:

$$h_{c,abs-wtr} = 135\frac{w}{m^2}\ [3] \quad \text{(S22)}$$

The evaporative heat transfer rate between the seawater and the air under the collector is as follows:

$$\dot{q}_{ew} = \dot{m}_{ev}\, h_{fg}\ [4] \quad \text{(S23)}$$

The heat transfer coefficient ($\dot{m}_{ev}$) is as follows [4]:

$$\dot{m}_{ev} = h_m A_{ch} \Delta\rho \quad \text{(S24)}$$

Where, $\Delta\rho$ can be found from:

$$\Delta\rho = \frac{\rho_{sat,Tairi} - \rho_{sat,Tairo}}{\ln\left(\frac{\rho_{sat,Twtr} - \rho_{sat,Tairo}}{\rho_{sat,Twtr} - \rho_{sat,Tairi}}\right)} \quad \text{(S25)}$$

$$\rho_{sat,Tairo} = \rho_{sat,Twtr} + (\rho_{sat,Tairi} - \rho_{sat,Twtr})e^{-\frac{h_m \rho_{air} A_c}{\dot{m}_{air}}} \quad \text{(S26)}$$

To calculate the humidity ratio at the entrance of the chimney, the following can be used:

$$w_3 = \frac{\rho_{sat,Tairo}}{\rho_{air}} \quad \text{(S27)}$$

h_m can be found using Sherwood number as follows:

$$Sh_D = \frac{h_m d_h}{D_{AB}} \tag{S28}$$

Where, $D_{AB} = 0.26 \times 10^{-4} \frac{m^2}{s}$.

$$Sh_D = \frac{\left(\frac{f}{8}\right)(Re_D - 1000)S_c}{1 + 12.7\left(\frac{f}{8}\right)^{\frac{1}{2}}\left(S_c^{\frac{2}{3}} - 1\right)} \tag{S29}$$

Given that $S_c = \frac{v}{D_{AB}}$.

$$Nu_D = \frac{\left(\frac{f}{8}\right)(Re_D - 1000)Pr}{1 + 12.7\left(\frac{f}{8}\right)^{\frac{1}{2}}\left(Pr^{\frac{2}{3}} - 1\right)} \tag{S30}$$

$$\overline{Nu_D} = Nu_D\left(\frac{C}{\frac{L}{d_h}}\right) \tag{S31}$$

$$f = (0.79\ lnRe - 1.64)^{-2} \tag{S32}$$

The radiation heat transfer rate between the water surface and the collector glass is as follows [3]:

$$q_{r,wtr-gls} = h_{r,wtr-gls}\,(T_{wtr} - T_{gls}) \tag{S33}$$

The heat transfer coefficient ($h_{r,wtr-gls}$) can be found from:

$$h_{r,wtr-gls} = \varepsilon_{eff}\sigma\left[{T_{wtr}}^2 + {T_{gls}}^2\right](T_{wtr} + T_{gls}) \tag{S34}$$

Where, ε_{eff} is as follows:

$$\varepsilon_{eff} = \left(\frac{1}{\varepsilon_{wtr}} + \frac{1}{\varepsilon_{gls}} - 1\right)^{-1} \tag{S35}$$

The convective heat transfer rate between the water and the air under the collector is as follows:

$$q_{c,wtr-air} = h_{c,wtr-air}\,(T_{wtr} - T_{air}) \quad \text{(S36)}$$

The heat transfer coefficient ($h_{c,wtr-air}$) is as follows:

$$h_{c,wtr-air} = \frac{0.2106 + 0.0026\,V_{in}\,\left(\frac{T_m \rho_{air}}{\mu g (T_{wtr} - T_{air})}\right)^{\frac{1}{3}}}{\left(\frac{\mu T_m}{(T_{wtr} - T_{air}) g \rho_{air}{}^2 C_P k^2}\right)^{\frac{1}{3}}} \quad \text{(S37)}$$

Where, $T_m = \left(\frac{T_{wtr}+T_{air}}{2}\right)$.

Humidification mass balance equation are as follows:

$$\frac{q_{ew}}{h_{fg}} = \dot{m}_v = \dot{m}_{air}(w_3 - w_2) \quad \text{(S38)}$$

The density of the air entering the chimney is given as:

$$\rho_{ent,ch} = \frac{\rho_{dair}(1 + w_3)}{(1 + 1.609\,w_3)} \quad \text{(S39)}$$

Zone 3: Solar Chimney

The energy balance equation at the entrance (bottom) and exit (top) of the chimney is as follows:

$$P_{elc} + Q_{out} = \overline{m}_{air}\left[\left(\frac{v_{ch,ent}^2}{2} + gz_{ch,ent} + i_{ch,ent}\right) - \left(\frac{v_{ch,out}^2}{2} + gz_{ch,out} + i_{ch,out}\right)\right] \quad \text{(S40)}$$

Where Q_{out} is the heat transfer between the chimney walls and the ambient outside the chimney.

The energy balance for the air inside the chimney is given by:

$$Q_{out} = \overline{m}_{air}\left[\left(i_{ch,ent} - i_{ch,out}\right) - (\omega_4 - \omega_3) i_{wtr}\right] \quad \text{(S41)}$$

The mass balance equation for the air in and out of the chimney is as follows:

$$\rho_{ch,ent} V_{ch} A_{in} = \rho_{out} V_{ch,out} A_{out} \tag{S42}$$

The enthalpy of the moist air entering the chimney is as follows:

$$i_{ch,ent} = T_a + w_3 \,(2501.3 + 1.86 T_a) \tag{S43}$$

Where T_a is the air temperature at the inlet (in C°).

To calculate the mass flow rate of the water condensation, the following equation is used:

$$\overline{m}_{wtr} = \overline{m}_{air}(\omega_4 - \omega_3) \tag{S44}$$

To accurately calculate the heat transfer rate between the chimney and ambient, the chimney was vertically divided into equal sections. This is to take into consideration the change in outside wind velocity along the height of the chimney. The following equations were applied to each section of the chimney[5]:

$$Q_{out} = hA\Delta T = hA\,(T_{air} - T_o) \tag{S45}$$

$$h = \overline{Nu_D} \frac{k}{D} \tag{S46}$$

$$\overline{Nu_D} = C Re_D^m \, Pr^n \left(\frac{Pr}{Pr}\right)^{\frac{1}{4}} \tag{S47}$$

Where m = 0.6 and n = 0.37.

$$Re_D = \frac{V(Z_R)D}{v} \tag{S48}$$

The wind velocity outside the chimney at different heights is estimated as follows [6]:

$$V(Z_R) = V(Z) \frac{\ln\left(\frac{Z_R}{Z_o}\right)}{\ln\left(\frac{Z_R}{Z_o}\right)} \tag{S49}$$

The mass flow rate of the water condensation, can be calculate as follows:

$$\overline{m}_{wtr} = \overline{m}_{air}(\omega_4 - \omega_3) \tag{S50}$$

To calculate the velocity of the air as it enters the IC, the following equation was used [7]:

$$V_{ch} = \sqrt{2gH_{ch}\frac{T_{ch,ent} - T_{out}}{T_{out}}} \tag{S51}$$

The output power produced by the turbine at the bottom of the IC was calculated as follows [7]:

$$P_{elc} = \frac{1}{2}\rho_{en,ch}C_f A_{ch} V_{ch}^3 \tag{S52}$$

where, C_f is the turbine efficiency, set at 0.42.

Cooling Tower

Water sprinklers installed at the top of each CT channel, spray a mist of water in the air. The mist of water is absorbed almost immediately by the hot air to form cool air (vapor).

To calculate the enthalpy of the vapor the following was used:

$$i_{vap} = i_{air} + \omega_{vap} i_{wtr} \tag{S53}$$

$$i_{air} = c_{p,air}\, T_{out} \tag{S54}$$

The water enthalpy can be calculated as follows:

$$i_{wtr} = c_{p,wtr}\, T_{wtr} \tag{S55}$$

The inlet enthalpy and exit enthalpy of the cooled air remains the same, because of this natural evaporation process. However, the temperature of the vapor decreases, due to the latent heat of vaporization. The change in the temperature can be calculated as follows:

$$c_{p,air}\, T_{out} + (\omega_{air} 2501.3 + T_{out} 1.86) = c_{p,air}\, T_{vap} + (\omega_{vap} 2501.3 + T_{vap} 1.86) \tag{S56}$$

To bring the water up to the sprinklers at the top of the cooling towers, water pumps are usually used to pump the water from a nearby reservoir. In the process of doing this, the water pumps consume some of the energy produced by the CT. The velocity of the air and power generated from the CT can be calculated from Equations (S51) and (S52) respectively.

Model Validation

Figure S1 depicts the thermal-radiative characteristics, based on [8] and [9] research work, for the glass, water, and base. The mathematical model for the solar chimney part was validated against the baseline results obtained by the well-known prototype in the literature, build and investigated by [10]. The validation results are shown in Figure S2 showing a 24-hour power production profile. As can be seen, the results from Haff and the results of the HSDCPP proposed design are almost identical. The model was validated based on the dimensions reported in Haff's work.

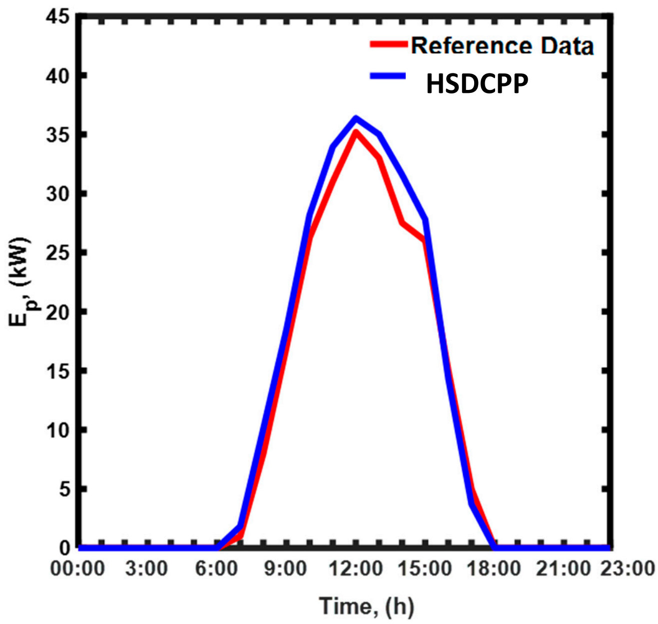

Figure S2. Model validation against the results obtained by [10]. Showing 24-hour electrical power production.

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