

Supplementary file

Table S1. Simulation assumptions for evaluating pitch estimation

| Nominal $\gamma_x / {}^\circ$ | Time interval | Date and Location | Initial yaw / {}^\circ | Real γ_x Min~Max (Mean) / {}^\circ |
|-------------------------------|-------------------|---------------------------|------------------------|--|
| 0 | 18:12:00-18:53:00 | 1/6/2020 20°N 120°E | 203.87 | 0~4.87(2.29) |
| 15 | | | 218.87 | 14.03~17.43 (15.35) |
| 30 | | | 233.87 | 28.65~32.14 (30.24) |
| 45 | | | 248.87 | 43.52~47.02 (45.20) |
| 60 | | | 263.87 | 58.44~61.95 (60.18) |
| 75 | | | 278.87 | 73.39~76.90 (75.16) |
| 90 | | | 293.87 | 88.15~90.00 (89.11) |

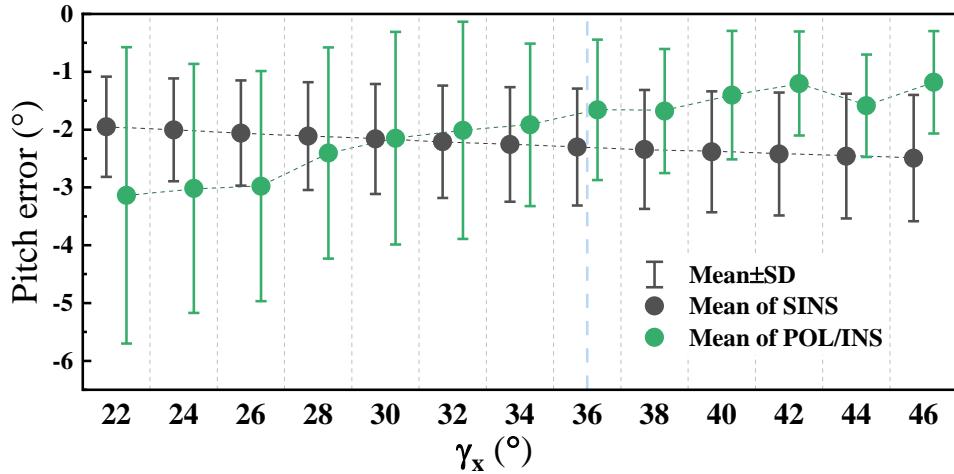


Figure S1. The pitch estimation error of INS/POL for different γ_x in the range from 22° to 46°

46°

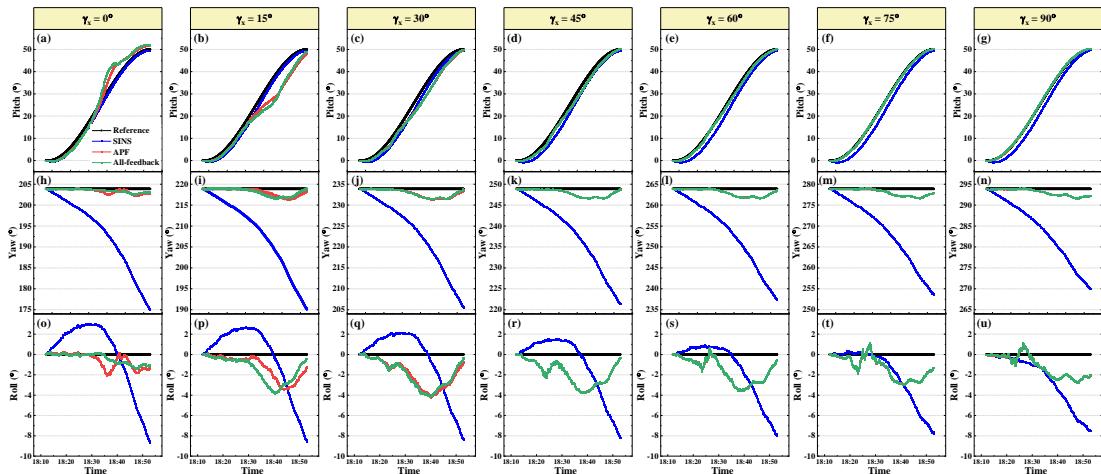


Figure S2. 3D attitude angles estimation curves for different γ_x .

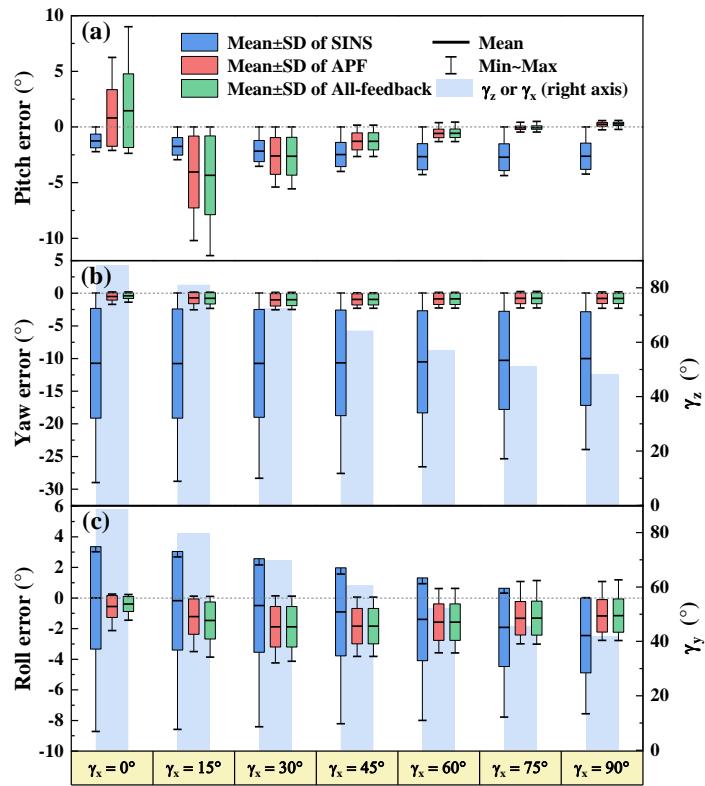


Figure S3. 3D attitude errors for different γ_x .

Table S2. Simulation assumptions for evaluating roll estimation

| Nominal γ_y /° | Time interval | Date and Location | Initial yaw /° | Real γ_y Min~Max (Mean) /° |
|-----------------------|-------------------|-------------------|----------------|-----------------------------------|
| 0 | | | 293.87 | 0~4.87(2.29) |
| 15 | | | 278.87 | 13.88~17.21 (15.06) |
| 30 | | 1/6/2020 | 263.87 | 28.47~31.94 (29.95) |
| 45 | 18:12:00~18:53:00 | 20°N | 248.87 | 43.33~46.82 (44.91) |
| 60 | | 120°E | 233.87 | 58.25~61.75 (59.88) |
| 75 | | | 218.87 | 73.19~76.70 (74.87) |
| 90 | | | 203.87 | 88.15~90.00 (89.11) |

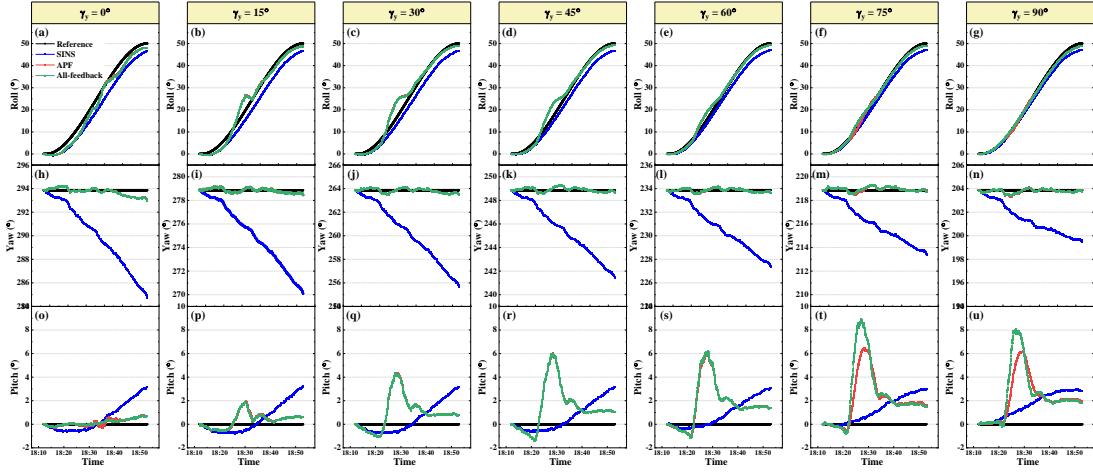


Figure S4. 3D attitude angle estimation curves for different γ_y .

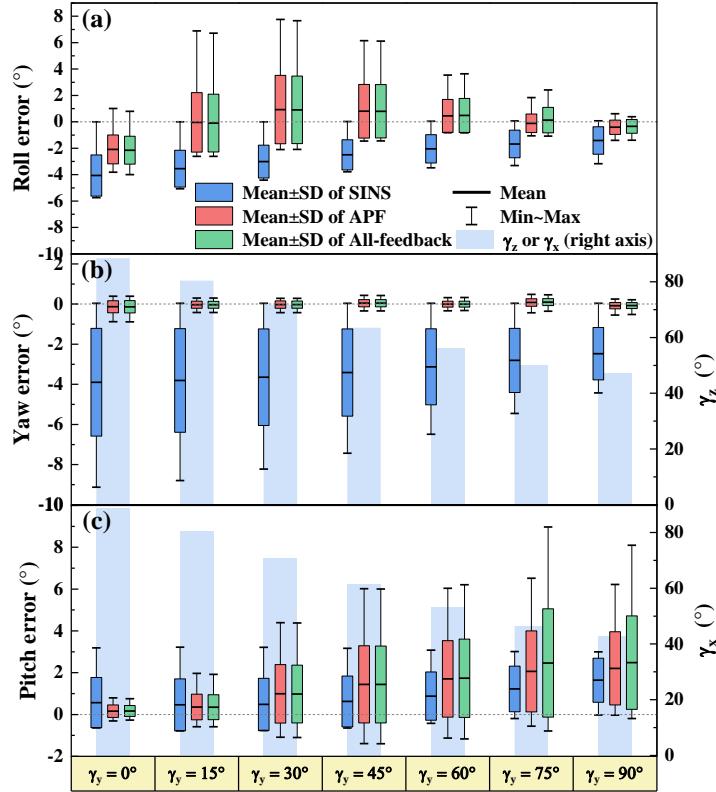


Figure S5. 3D attitude errors for different γ_y .

State-transition matrix Φ :

$$\Phi = \begin{bmatrix} \Phi_N & \Phi_S \\ \mathbf{0}_{6 \times 9} & \mathbf{0}_{6 \times 6} \end{bmatrix}_{15 \times 15}$$

The nonzero elements in Φ_N are expressed as follows:

$$\begin{aligned}
\boldsymbol{\Phi}_N(1,2) &= \omega_{ie} \sin L + \frac{v_E}{R_N + h} \tan L, \quad \boldsymbol{\Phi}_N(1,3) = -\left(\omega_{ie} \cos L + \frac{v_E}{R_N + h}\right), \quad \boldsymbol{\Phi}_N(1,5) = -\frac{1}{R_M + h}, \\
\boldsymbol{\Phi}_N(1,9) &= \frac{v_N}{(R_M + h)^2}, \quad \boldsymbol{\Phi}_N(2,1) = -(\omega_{ie} \sin L + \frac{v_E}{R_N + h} \tan L), \quad \boldsymbol{\Phi}_N(2,3) = -\frac{v_N}{R_M + h}, \\
\boldsymbol{\Phi}_N(2,4) &= \frac{1}{R_N + h}, \quad \boldsymbol{\Phi}_N(2,7) = -\omega_{ie} \sin L, \quad \boldsymbol{\Phi}_N(2,9) = -\frac{v_E}{(R_N + h)^2}, \quad \boldsymbol{\Phi}_N(3,1) = \omega_{ie} \cos L + \frac{v_E}{R_N + h}, \\
\boldsymbol{\Phi}_N(3,2) &= \frac{v_N}{R_M + h}, \quad \boldsymbol{\Phi}_N(3,4) = \frac{1}{R_N + h} \tan L, \quad \boldsymbol{\Phi}_N(3,7) = \omega_{ie} \cos L + \frac{v_E}{R_N + h} \sec^2 L, \quad \boldsymbol{\Phi}_N(4,2) = -f_U, \\
\boldsymbol{\Phi}_N(3,9) &= -\frac{v_E}{(R_N + h)^2} \tan L, \quad \boldsymbol{\Phi}_N(4,3) = f_N, \quad \boldsymbol{\Phi}_N(4,4) = \frac{v_N}{R_M + h} \tan L - \frac{v_U}{R_M + h}, \\
\boldsymbol{\Phi}_N(4,5) &= 2\omega_{ie} \sin L + \frac{v_E}{R_N + h} \tan L, \quad \boldsymbol{\Phi}_N(4,6) = -2\omega_{ie} \cos L - \frac{v_E}{R_N + h}, \\
\boldsymbol{\Phi}_N(4,7) &= 2\omega_{ie} v_N \cos L + \frac{v_E v_N}{R_N + h} \sec^2 L + 2\omega_{ie} v_U \sin L, \quad \boldsymbol{\Phi}_N(4,9) = \frac{v_E v_U - v_E v_N \tan L}{(R_N + h)^2}, \quad \boldsymbol{\Phi}_N(5,1) = f_U, \\
\boldsymbol{\Phi}_N(5,3) &= -f_E, \quad \boldsymbol{\Phi}_N(5,5) = -\frac{v_U}{R_M + h}, \quad \boldsymbol{\Phi}_N(5,4) = -(2\omega_{ie} \sin L + \frac{v_E}{R_N + h} \tan L), \quad \boldsymbol{\Phi}_N(5,6) = -\frac{v_N}{R_M + h}, \\
\boldsymbol{\Phi}_N(5,7) &= -(2\omega_{ie} v_E \cos L + \frac{v_E^2}{R_N + h} \sec^2 L) v_E, \quad \boldsymbol{\Phi}_N(5,9) = \frac{v_N v_U + v_E^2 \tan L}{(R_N + h)^2}, \quad \boldsymbol{\Phi}_N(6,1) = -f_N, \\
\boldsymbol{\Phi}_N(6,2) &= -f_E, \quad \boldsymbol{\Phi}_N(6,4) = 2(\omega_{ie} \cos L + \frac{v_E}{R_N + h}), \quad \boldsymbol{\Phi}_N(6,5) = \frac{2v_N}{R_M + h}, \quad \boldsymbol{\Phi}_N(6,7) = -2\omega_{ie} v_E \sin L, \\
\boldsymbol{\Phi}_N(6,9) &= -\frac{v_E^2 + v_N^2}{(R_N + h)^2}, \quad \boldsymbol{\Phi}_N(7,5) = \frac{1}{R_M + h}, \quad \boldsymbol{\Phi}_N(7,9) = -\frac{V_N}{(R_M + h)^2}, \quad \boldsymbol{\Phi}_N(8,4) = \frac{\sec L}{R_N + h}, \\
\boldsymbol{\Phi}_N(8,7) &= \frac{v_E}{R_N + h} \sec L \tan L, \quad \boldsymbol{\Phi}_N(8,9) = -\frac{v_E}{(R_N + h)^2} \sec L, \quad \boldsymbol{\Phi}_N(9,6) = 1
\end{aligned}$$

where, R_M is the radius of curvature of meridian, R_N is the radius of curvature in prime vertical, ω_{ie} is the earth's rotation angular velocity, L is the local longitude, h is the height of vehicle, v_E, v_N, v_U are 3D velocities and f_E, f_N, f_U are the outputs of tri-axis accelerometer.

$\boldsymbol{\Phi}_s$ is expressed as

$$\boldsymbol{\Phi}_s = \begin{bmatrix} C_b^n & 0_{3 \times 3} \\ 0_{3 \times 3} & C_b^n \\ 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}_{9 \times 6}$$