Supplementary Information

Dynamic Reconstruction of Vegetation Variables in Earth Observation: Beyond Gap-Filling Toward Constrained Inference

Supplementary Text S1. Synthetic FLEX–FLoX reconstruction experiment

Synthetic experiment design

A synthetic experiment was designed to illustrate how different reconstruction approaches can recover FLEX-like F760 dynamics from sparse satellite observations supported by high-frequency tower measurements. The experiment was intentionally simplified and is intended solely to illustrate the behaviour of representative reconstruction approaches rather than to provide a formal performance benchmark.

An hourly tower-scale FLoX F760 time series was generated for an 81-day period. The signal comprised a diurnal cycle, seasonal modulation, autocorrelated weather variability, abrupt cloudy or stress-like events, and measurement noise. A corresponding satellite-scale FLEX-like F760 reference was subsequently generated from the tower signal by introducing temporal smoothing, a short time lag, nonlinear scaling, slowly varying footprint or viewing-geometry effects, and independent pixel-scale perturbations. Consequently, the tower and satellite signals remained physically related while exhibiting realistic temporal and structural differences.

Sparse FLEX-like observations were simulated by sampling the satellite-scale reference every 27 days, consistent with the planned FLEX revisit cycle. All reconstruction methods estimated hourly satellite-scale F760 throughout the complete 81-day period.

Four representative reconstruction approaches were evaluated:

- Linear interpolation;
- Single-output Gaussian Process Regression (GPR);
- Multi-output Gaussian Process Regression (MOGPR);
- Ensemble Kalman Filter (EnKF).

Linear interpolation and GPR used only the sparse FLEX-like observations. MOGPR combined the sparse FLEX-like F760 observations with the continuous FLoX F760 measurements through a multi-output covariance model. Both GPR and MOGPR employed squared-exponential covariance functions with hyperparameters optimized by maximizing the marginal likelihood.

EnKF employed a simplified dynamic state-space model in which satellite-scale F760, tower F760, and a slowly varying scaling factor were updated sequentially using hourly FLoX observations together with sparse FLEX-like observations. The state evolution model assumed persistence with stochastic process noise, while hourly FLoX observations and sparse FLEX observations were assimilated using standard Kalman update equations.

Prediction uncertainties were obtained directly from the predictive variance provided by GPR and MOGPR, and from the ensemble spread in the EnKF. Linear interpolation was treated as a deterministic method and therefore did not provide predictive uncertainty estimates.

Performance assessment

Reconstruction performance was evaluated against the known satellite-scale reference using four complementary criteria:

- Root Mean Square Error (RMSE);
- Coefficient of determination (R^2);
- Empirical 95% prediction interval coverage;
- Qualitative computational cost.

The empirical 95% coverage was computed as the percentage of reconstructed time steps for which the true satellite-scale reference fell within the nominal prediction interval,

$$\hat{y}(t) \pm 1.96\,\sigma(t),$$

where $\hat{y}(t)$ denotes the reconstructed F760 estimate and $\sigma(t)$ its predictive standard deviation.

Coverage values close to 95% indicate well-calibrated predictive uncertainties. Values below 95% suggest overconfident uncertainty estimates, whereas values exceeding 95% indicate conservative uncertainty intervals.