Electronic Supporting Material

Mitigating the Scintillation Effect on GPS Signals Using MP and ROTI

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This document has two parts

1. The Kalman Filter method, which is used in the main part of this paper, is summarized. For further details the interested reader should consult the references provided.
2. Detailed results for the observations-removal strategy presented in the main body of our paper are provided. Whereas the results are summarized in the main body of the manuscript, detailed results for each scintillation day and each station are presented here.

S1. KF and Extended KF(EKF) formulae

The state is predicted using the dynamic model and state at the previous epoch [1]:

$$x_{k/k-1} = \Phi x_{k-1/k-1} \tag{S1}$$

where x is the state; $k/k-1$ represents the prediction at the epoch k from the epoch $k-1$; and $k-1/k-1$ represents the update at epoch $k-1$; Φ is the dynamic model. The corresponding covariance matrix is also predicted as:

$$P_{k/k-1} = \Phi P_{k-1/k-1}\Phi^T + Q_{k-1} \tag{S2}$$

where P is the covariance matrix; ϕ^T is the transpose of the dynamic model; Q is the process noise covariance. The innovation vector is then formed as:

$$y_k = z_k - H_k x_{k/k-1} \tag{S3}$$

where y is the innovation vector; z is the measurement; H is the observation model. Next, the Kalman gain is calculated as follow:

$$K_k = P_{k/k-1}H_k^T(H_k P_{k/k-1}H_k^T + R_k)^{-1} \tag{S4}$$

where K is the Kalman gain; H^T is the transpose of the observation model; R is the measurement error. Next, updating is applied to the state and corresponding covariance matrix in equations (S5) and (S6) respectively:

$$x_{k/k} = x_{k/k-1} - K_k y_k \tag{S5}$$

$$P_{k/k} = (I - K_k H_k)P_{k/k-1} \tag{S6}$$

where I is the identity matrix. Next, the iteration is implemented for all the observations. However, KF can only be applied for linear estimation, which is not suitable for GNSS applications. Thus, EKF can be applied instead where the dynamic and observation models are replaced with nonlinear ones [1]. Therefore, formulae (S1) and (S3) are replaced with formulae (S7) and (S8):

$$x_{k/k-1} = \phi(x_{k-1/k-1}) \tag{S7}$$

$$y_k = z_k - h(x_{k/k-1}) \tag{S8}$$

where ϕ and h are the nonlinear dynamic and observation models respectively. Moreover, Φ and H_k for the computation of the covariance matrix are defined with Jacobians instead:

$$\Phi = \left.\frac{\partial \phi}{\partial x}\right|_{x_{k-1/k-1}} \tag{S9}$$

$$H_k = \left.\frac{\partial h}{\partial x}\right|_{x_{k/k-1}} \tag{S10}$$

S2. Adaptive robust KF (ARKF)

Positioning can be performed by applying EKF and using the a priori information of the process and measurement noise. However, it may not be possible to acquire the a priori information [2], as was the case in our study. Thus, we used ARKF to overcome this problem in PPPH [3]. The only difference between ARKF and EKF is that an adaptive factor (also referred to as the inflation factor) is applied to the Kalman gain and measurement noise covariance [2]. Thus, the equation (S4) was replaced with equations (S11)-(S14):

$$K_k{}' = \frac{1}{\alpha_k} P_{k/k-1}H_k^T(\frac{1}{\alpha_k} H_k P_{k/k-1}H_k^T + R_k{}')^{-1} \tag{S11}$$

$$\alpha_k = \begin{cases} 1, & |\bar{Y}_k| \le c_0 \\ \frac{c_0}{|\bar{Y}_k|}(\frac{c_1 - |\bar{Y}_k|}{c_1 - c_0})^2, & c_0 < |\bar{Y}_k| \le c_1 \\ 0, & |\bar{Y}_k| > c_1 \end{cases} \tag{S12}$$

$$\overline{Y}_k = \frac{\sum_{i=1}^{n_k} {y_k}^2}{\sum_{i=1}^{n_k} \sigma_{y_k}^2}$$

$$R_k{}' = R_k/\gamma_k \tag{S13}$$

$$\gamma_k = \begin{cases} 1, & |v_k| \le k_0 \\ \frac{k_0}{|v_k|}\left(\frac{k_1 - |v_k|}{k_1 - k_0}\right)^2, & c_0 < |v_k| \le k_1 \\ 0, & |v_k| > k_1 \end{cases} \tag{S14}$$

where α_k and γ_k are inflation factors; c_0, c_1, k_0 and k_1 are constants with empirical values: 1.5<c_0<3.0, 3.0<c_1<8.0, 1.5<k_0<3.0 and 3.0<k_1<8.0; σ_{y_k} is the standard deviation of y_k; n_k is the innovation numbers; v_k is the standardized residual. With the inflation factors varying from 0 to 1, the influence of outliers in measurements was compensated.

Table S1. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SAO0P on 2017-09-07. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SAO0P	20170907	0.0144	0.0180	σ_ϕ	0.0145	**0.0175**
				S4	0.0170	0.0203
				σ_ϕ, S4	0.0214	0.0234
				ROTI	**0.0115**	0.0192
				MP2	**0.0044**	**0.0057**
				MP1	0.0270	0.0307
				MP2, ROTI	**0.0031**	**0.0050**
				MP1, ROTI	**0.0073**	**0.0078**
				MP1, MP2	**0.0015**	**0.0036**
				MP1, MP2, ROTI	**0.0014**	**0.0042**

Table S2. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SAO0P on 2017-09-08. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SAO0P	20170908	0.6245	0.6422	σ_ϕ	**0.0981**	**0.1255**
				S4	**0.3661**	**0.4013**
				σ_ϕ, S4	**0.1046**	**0.1261**
				ROTI	**0.1828**	**0.2087**
				MP2	0.6771	0.6954
				MP1	**0.4566**	**0.4961**
				MP2, ROTI	**0.0718**	**0.0780**
				MP1, ROTI	**0.1756**	**0.1979**
				MP1, MP2	0.7069	0.7238
				MP1, MP2, ROTI	**0.0885**	**0.0946**

Table S3. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SAO0P on 2017-09-08. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SAO0P	20170913	0.1395	0.1581	σ_ϕ	**0.0345**	**0.0518**
				S4	**0.0283**	**0.0321**
				σ_ϕ, S4	**0.0248**	**0.0259**
				ROTI	**0.0172**	**0.0308**
				MP2	**0.1012**	**0.1573**
				MP1	**0.0559**	**0.0635**
				MP2, ROTI	**0.0182**	**0.0210**
				MP1, ROTI	**0.0345**	**0.0360**
				MP1, MP2	**0.0880**	**0.1472**
				MP1, MP2, ROTI	**0.0270**	**0.0289**

Table S4. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SJCU on 2017-09-04. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SJCU	20170904	0.0260	0.0345	σ_ϕ	0.0435	0.0452
				S4	**0.0205**	**0.0279**
				σ_ϕ, S4	0.0429	0.0460
				ROTI	**0.0248**	**0.0330**
				MP2	0.0340	0.0465
				MP1	**0.0254**	**0.0276**
				MP2, ROTI	0.0342	0.0474
				MP1, ROTI	0.0262	**0.0284**
				MP1, MP2	0.0418	0.0490
				MP1, MP2, ROTI	0.0417	0.0492

Table S5. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SJCU on 2017-09-07. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SJCU	20170907	0.0247	0.0311	σ_ϕ	0.0285	**0.0308**
				S4	0.0313	0.0321
				σ_ϕ, S4	0.0388	0.0395
				ROTI	0.0315	0.0320
				MP2	0.0329	0.0399
				MP1	0.0298	**0.0301**
				MP2, ROTI	0.0339	0.0401
				MP1, ROTI	0.0300	**0.0303**
				MP1, MP2	0.0331	0.0373
				MP1, MP2, ROTI	0.0343	0.0373

Table S6. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SJCU on 2017-09-08. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SJCU	20170908	0.1355	0.3454	σ_ϕ	0.2160	**0.2731**
				S4	1.5847	1.6025
				σ_ϕ, S4	0.2205	**0.2808**
				ROTI	0.2461	**0.3328**
				MP2	0.3595	0.3801
				MP1	0.2776	**0.3248**
				MP2, ROTI	**0.1201**	**0.1418**
				MP1, ROTI	0.2142	**0.3107**
				MP1, MP2	0.1731	**0.2521**
				MP1, MP2, ROTI	**0.1129**	**0.1249**

Table S7. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SJCU on 2017-09-13. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SJCU	20170913	0.0219	0.0329	σ_ϕ	**0.0188**	**0.0282**
				S4	**0.0157**	**0.0269**
				σ_ϕ, S4	**0.0193**	**0.0298**
				ROTI	0.0646	0.0850
				MP2	0.0302	0.0564
				MP1	0.0258	0.0370
				MP2, ROTI	0.0795	0.1280
				MP1, ROTI	0.0402	0.0551
				MP1, MP2	0.0261	0.0502
				MP1, MP2, ROTI	0.0787	0.1242

Table S8. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SNA0P on 2016-02-18. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SNA0P	20160218	0.1699	0.2148	σ_ϕ	**0.1671**	**0.2125**
				S4	0.1877	0.2263
				σ_ϕ, S4	**0.1583**	**0.2064**
				ROTI	0.2319	0.2828
				MP2	0.2433	0.2835
				MP1	0.2174	0.2651
				MP2, ROTI	0.3193	0.3426
				MP1, ROTI	0.2889	0.3164
				MP1, MP2	0.2036	0.2424
				MP1, MP2, ROTI	0.3450	0.3676

Table S9. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SNA0P on 2016-04-02. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SNA0P	20160402	0.0087	0.0329	σ_ϕ	**0.0068**	**0.0280**
				S4	**0.0059**	**0.0282**
				σ_ϕ, S4	**0.0066**	**0.0283**
				ROTI	**0.0072**	**0.0324**
				MP2	**0.0078**	**0.0295**
				MP1	**0.0064**	**0.0285**
				MP2, ROTI	**0.0070**	**0.0290**
				MP1, ROTI	**0.0054**	**0.0283**
				MP1, MP2	**0.0073**	**0.0291**
				MP1, MP2, ROTI	**0.0073**	**0.0289**

Table S10. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SNA0P on 2016-04-13. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SNA0P	20160413	0.8233	0.8521	σ_ϕ	**0.1799**	**0.2260**
				S4	**0.4178**	**0.6136**
				σ_ϕ, S4	**0.1834**	**0.2262**
				ROTI	**0.1427**	**0.2025**
				MP2	**0.5525**	**0.5626**
				MP1	0.8517	0.9361
				MP2, ROTI	**0.1161**	**0.1800**
				MP1, ROTI	**0.0687**	**0.1724**
				MP1, MP2	**0.1997**	**0.2997**
				MP1, MP2, ROTI	**0.0801**	**0.1760**

Table S11. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SNA0P on 2016-05-09. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SNA0P	20160509	0.4466	0.5796	σ_ϕ	**0.3020**	**0.3191**
				S4	**0.4084**	**0.5487**
				σ_ϕ, S4	**0.3018**	**0.3191**
				ROTI	**0.2238**	**0.2431**
				MP2	**0.4407**	**0.5573**
				MP1	**0.4017**	**0.4720**
				MP2, ROTI	**0.2139**	**0.2496**
				MP1, ROTI	**0.2758**	**0.2930**
				MP1, MP2	**0.2794**	**0.3374**
				MP1, MP2, ROTI	**0.2229**	**0.2591**

Table S12. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SNA0P on 2016-06-06. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SNA0P	20160606	0.5636	1.1350	σ_ϕ	**0.5182**	**0.5730**
				S4	1.6784	2.0880
				σ_ϕ, S4	0.6953	**0.7354**
				ROTI	**0.1968**	**0.2298**
				MP2	2.0577	2.3840
				MP1	1.0227	1.4342
				MP2, ROTI	**0.3771**	**0.3993**
				MP1, ROTI	**0.1517**	**0.1971**
				MP1, MP2	1.7578	2.1342
				MP1, MP2, ROTI	**0.3778**	**0.4000**

Table S13. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SNA0P on 2016-07-28. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SNA0P	20160728	0.0099	0.0207	σ_ϕ	0.0105	0.0309
				S4	**0.0091**	**0.0204**
				σ_ϕ, S4	0.0122	0.0274
				ROTI	0.0105	0.0213
				MP2	0.0151	0.0352
				MP1	0.0136	0.0301
				MP2, ROTI	0.0168	0.0356
				MP1, ROTI	0.0131	0.0300
				MP1, MP2	0.0147	0.0356
				MP1, MP2, ROTI	0.0145	0.0354

Table S14. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SNA0P on 2016-10-13. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SNA0P	20161013	0.0010	0.0228	σ_ϕ	0.0247	0.0507
				S4	0.0023	0.0344
				σ_ϕ, S4	0.0256	0.0518
				ROTI	0.0197	0.0624
				MP2	0.0246	0.0625
				MP1	0.0039	0.0326
				MP2, ROTI	0.0121	0.0568
				MP1, ROTI	0.0257	0.0669
				MP1, MP2	0.0307	0.0694
				MP1, MP2, ROTI	0.0171	0.0564

Table S15. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SAO0P on 2017-09-07. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SAO0P	20170907	0.0144	0.0180	σ_ϕ	0.0165	0.0204
				S4	**0.0121**	**0.0167**
				σ_ϕ, S4	0.0160	0.0203
				ROTI	**0.0124**	0.0192
				MP2	0.0148	**0.0178**
				MP1	0.0175	0.0213
				MP2, ROTI	**0.0130**	0.0196
				MP1, ROTI	0.0179	0.0224
				MP1, MP2	0.0179	0.0215
				MP1, MP2, ROTI	0.0183	0.0226

Table S16. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SAO0P on 2017-09-08. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SAO0P	20170908	0.6245	0.6422	σ_ϕ	**0.4126**	**0.4540**
				S4	**0.1664**	**0.2517**
				σ_ϕ, S4	**0.4672**	**0.5482**
				ROTI	**0.1202**	**0.1464**
				MP2	**0.5154**	**0.5376**
				MP1	**0.5769**	**0.5887**
				MP2, ROTI	**0.1174**	**0.1420**
				MP1, ROTI	**0.1101**	**0.1337**
				MP1, MP2	**0.6117**	**0.6208**
				MP1, MP2, ROTI	**0.1069**	**0.1310**

Table S17. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SAO0P on 2017-09-08. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SAO0P	20170913	0.1395	0.1581	σ_ϕ	**0.0608**	**0.0617**
				S4	**0.0556**	**0.0657**
				σ_ϕ, S4	**0.0387**	**0.0416**
				ROTI	**0.0165**	**0.0349**
				MP2	0.1536	0.1683
				MP1	**0.1018**	**0.1202**
				MP2, ROTI	**0.0223**	**0.0359**
				MP1, ROTI	**0.0239**	**0.0382**
				MP1, MP2	**0.1270**	0.1773
				MP1, MP2, ROTI	**0.0269**	**0.0387**

Table S18. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SJCU on 2017-09-04. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SJCU	20170904	0.0260	0.0345	σ_ϕ	0.0265	**0.0282**
				S4	0.0268	0.0353
				σ_ϕ, S4	**0.0256**	**0.0272**
				ROTI	**0.0255**	**0.0339**
				MP2	0.0272	**0.0289**
				MP1	**0.0245**	**0.0306**
				MP2, ROTI	0.0269	**0.0286**
				MP1, ROTI	**0.0243**	**0.0302**
				MP1, MP2	0.0270	**0.0282**
				MP1, MP2, ROTI	0.0268	**0.0279**

Table S19. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SJCU on 2017-09-07. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SJCU	20170907	0.0247	0.0311	σ_ϕ	0.0282	0.0298
				S4	0.0268	0.0331
				σ_ϕ, S4	0.0294	**0.0310**
				ROTI	0.0304	0.0311
				MP2	0.0302	**0.0306**
				MP1	0.0292	**0.0300**
				MP2, ROTI	0.0308	**0.0310**
				MP1, ROTI	0.0290	**0.0295**
				MP1, MP2	0.0287	**0.0291**
				MP1, MP2, ROTI	0.0292	**0.0295**

Table S20. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SJCU on 2017-09-08. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SJCU	20170908	0.1355	0.3454	σ_ϕ	0.1638	0.3521
				S4	0.4893	0.5559
				σ_ϕ, S4	0.1638	0.3521
				ROTI	0.1986	**0.2357**
				MP2	0.3026	0.4002
				MP1	**0.0632**	**0.0839**
				MP2, ROTI	**0.0386**	**0.0473**
				MP1, ROTI	0.1979	**0.2342**
				MP1, MP2	**0.0779**	**0.2221**
				MP1, MP2, ROTI	**0.0511**	**0.0572**

Table S21. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SJCU on 2017-09-13. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SJCU	20170913	0.0219	0.0329	σ_ϕ	0.0240	**0.0328**
				S4	**0.0153**	**0.0275**
				σ_ϕ, S4	**0.0193**	**0.0292**
				ROTI	0.0332	0.0440
				MP2	0.0264	0.0402
				MP1	0.0226	0.0339
				MP2, ROTI	**0.0210**	0.0383
				MP1, ROTI	0.0331	0.0440
				MP1, MP2	0.0264	0.0402
				MP1, MP2, ROTI	**0.0210**	0.0383

Table S22. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SNA0P on 2016-02-18. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SNA0P	20160218	0.1699	0.2148	σ_ϕ	**0.1645**	0.2167
				S4	**0.1689**	**0.2118**
				σ_ϕ, S4	**0.1658**	0.2182
				ROTI	0.2287	0.2658
				MP2	0.1992	0.2315
				MP1	0.1703	**0.2113**
				MP2, ROTI	0.1983	0.2389
				MP1, ROTI	0.1909	0.2350
				MP1, MP2	0.2092	0.2577
				MP1, MP2, ROTI	0.1874	0.2322

Table S23. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SNA0P on 2016-04-02. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SNA0P	20160402	0.0087	0.0329	σ_ϕ	**0.0059**	**0.0281**
				S4	**0.0062**	**0.0287**
				σ_ϕ, S4	**0.0059**	**0.0281**
				ROTI	**0.0077**	**0.0326**
				MP2	**0.0066**	**0.0319**
				MP1	**0.0064**	**0.0298**
				MP2, ROTI	**0.0061**	**0.0315**
				MP1, ROTI	**0.0060**	**0.0296**
				MP1, MP2	**0.0069**	0.0331
				MP1, MP2, ROTI	**0.0064**	**0.0328**

Table S24. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SNA0P on 2016-04-13. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SNA0P	20160413	0.8233	0.8521	σ_ϕ	**0.0831**	**0.1741**
				S4	**0.5556**	**0.5855**
				σ_ϕ, S4	**0.0788**	**0.1580**
				ROTI	**0.2972**	**0.3955**
				MP2	0.8869	0.9404
				MP1	**0.7885**	0.8586
				MP2, ROTI	**0.3655**	**0.4579**
				MP1, ROTI	**0.3101**	**0.3843**
				MP1, MP2	0.8351	0.8954
				MP1, MP2, ROTI	**0.3358**	**0.4303**

Table S25. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SNA0P on 2016-05-09. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SNA0P	20160509	0.4466	0.5796	σ_ϕ	**0.2487**	**0.2673**
				S4	**0.3828**	**0.5038**
				σ_ϕ, S4	**0.2486**	**0.2672**
				ROTI	**0.2345**	**0.2629**
				MP2	**0.4373**	0.6193
				MP1	**0.3512**	**0.4326**
				MP2, ROTI	**0.2269**	**0.2532**
				MP1, ROTI	**0.3006**	**0.3249**
				MP1, MP2	**0.3344**	**0.4559**
				MP1, MP2, ROTI	**0.2963**	**0.3186**

Table S26. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SNA0P on 2016-06-06. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SNA0P	20160606	0.5636	1.1350	σ_ϕ	0.5722	**0.7549**
				S4	0.5700	1.2371
				σ_ϕ, S4	0.7753	**1.0596**
				ROTI	**0.5522**	**0.6354**
				MP2	0.6488	**1.1009**
				MP1	1.1436	1.6746
				MP2, ROTI	1.3408	1.4053
				MP1, ROTI	**0.1624**	**0.3385**
				MP1, MP2	0.8409	1.3097
				MP1, MP2, ROTI	1.5458	1.5911

Table S27. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SNA0P on 2016-07-28. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SNA0P	20160728	0.0099	0.0207	σ_ϕ	0.0111	0.0217
				S4	**0.0092**	**0.0204**
				σ_ϕ, S4	0.0106	0.0215
				ROTI	0.0109	0.0214
				MP2	0.0114	0.0218
				MP1	0.0128	0.0252
				MP2, ROTI	0.0117	0.0220
				MP1, ROTI	0.0157	0.0255
				MP1, MP2	0.0126	0.0275
				MP1, MP2, ROTI	0.0126	0.0256

Table S28. Change of the height and 3D positioning errors through the observation removal strategy based on MT at SNA0P on 2016-10-13. Improved RMSEs (m) are highlighted in bold.

Station	Date	Original RMSE		Permutations and combinations	Observation-removed RMSE	
		height	3D		height	3D
SNA0P	20161013	0.0010	0.0228	σ_ϕ	0.0039	0.0238
				S4	**0.0006**	0.0230
				σ_ϕ, S4	0.0042	0.0241
				ROTI	0.0036	**0.0198**
				MP2	0.0097	0.0289
				MP1	**0.0008**	0.0335
				MP2, ROTI	0.0054	0.0242
				MP1, ROTI	0.0033	0.0298
				MP1, MP2	0.0043	0.0378
				MP1, MP2, ROTI	0.0038	0.0300

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