

Supplementary Materials: A method for calculating the height of stable boundary layer based on wind profile lidar and turbulent fluxes *Remote Sensing* 2021, 13, 3596. <https://doi.org/10.3390/rs13183596>.

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S1. Introduction of UAT-2:

three wind components (u , v , w) in the orthonormal system $x - y - z$ can be obtained, and the velocity (U) measured by the ultrasonic anemometer is

$$U = \sqrt{u^2 + v^2 + w^2} \quad (1)$$

in addition, virtual potential temperature (T_v) can be approximated by

$$T_v = c^2/403 \quad (2)$$

Where c is the velocity of sound in air. All expressed in SI units.

S2. The pre-processing of turbulent fluctuations:

Turbulent fluctuations of some quantity φ are defined as deviations from the local average $\bar{\varphi}$, in which case the decomposition of φ can be written as

$$\varphi = \bar{\varphi} + \varphi' \quad (3)$$

where $\bar{\varphi}$ is an average over timescale L . The local averaging length L determines which scale of the motions are included in the computed flux. The numerical choice of L is discussed below.

Momentum fluxes are then computed by averaging the product of the perturbations over L :

$$\overline{w'\mathbf{V}'} = (\overline{w'u'^2} + \overline{w'v'^2})^{1/2} \quad (4)$$

where the momentum flux $\overline{w'\mathbf{V}'}$ is computed by first forming the time series $w'\mathbf{V}'$ where u' and v' are computed as deviations from averages over local averaging length L .

The total vertical flux of horizontal momentum measured near the surface can be called friction velocity (u_*^2):

$$u_*^2 = \overline{w'\mathbf{V}'} \quad (5)$$

Surface Reynolds stress (τ) can be defined:

$$\tau = \bar{\rho}u_*^2 \approx u_*^2$$

Buoyancy fluxes can be computed by the same local averaging length L by two parts:

$$F_{\theta s} = \overline{w'\theta'} \quad (6) \quad F_{qs} = \overline{w'q'} \quad (7)$$

where $F_{\theta s}$ and F_{qs} are the temperature flux and the humidity flux computed through the EC (eddy covariance) method.

The near-surface buoyancy flux could be determined as follows:

$$F_{bs} = \frac{g}{\theta} F_{\theta s} + 0.61gF_{qs} \quad (8)$$

For our study, the orders of magnitude of humidity flux (F_{qs}) is smaller than temperature flux ($F_{\theta s}$). Therefore, Eq. (7) could be rewritten as heat fluxes (F_{θ}):

$$F_{\theta} = \frac{g}{\theta} F_{\theta s} \quad (9)$$

To calculate the fluxes at the earth's surface: the fluxes of momentum $u_*^2 = \tau_* = \tau \big|_{z=0}$ and $F_* = F_{\theta} \big|_{z=0}$ through the mean wind speed $U_1 = U \big|_{z=Z_1}$ and potential temperature $\theta_1 = \theta \big|_{z=Z_1}$ at a given level, Z_1 , above surface. The friction velocity

$$u_* = \left[\overline{w'u'^2} + \overline{w'v'^2} \right]^{1/4} \quad (10)$$

All the turbulent fluxes were based on eddy covariance.

S3. The computation of the dimensionless stability parameter:

$$\mu_0 = u_*/fL_{MO} \quad (11)$$

for details see Clarke and Hess (1973).

S4. Z test:

Based the tests concerning the sample correlation coefficients on the statistics (see Freund (1962)):

$$Z = 0.5(n-3)^{\frac{1}{2}} \ln[(1+r)/(1-r)] \quad (12)$$

where n is the number of samples and r is the correlation coefficient. Z values can be looked upon as a value assumed by a random variable having approximately the standard normal distribution (based on the statistical table: when Z value exceeds $Z(\alpha = 0.025) = 1.960$, for 95% confidence interval, there is a significant correlation between two variables).

Figures S1-S5:

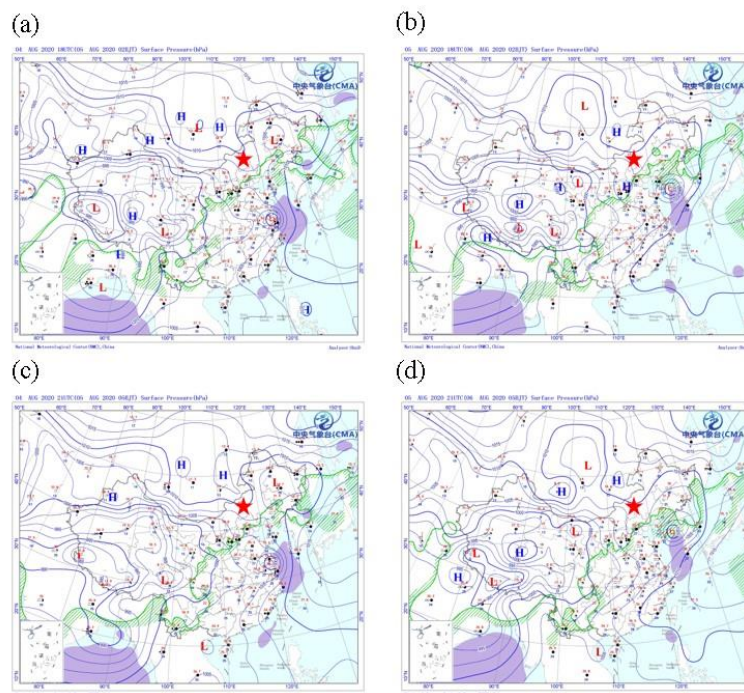


Figure S1. Synoptic weather maps during the observation experiment were provided by Central Weather Observatory of China at sea level at (a, b) 02:00 BT, (c, d) 05:00 BT, 5-6 August 2020. The high-pressure systems were clearly seen from the sea level pressure field (The red star represents the experiment site). Blue lines represent the sea level pressure.

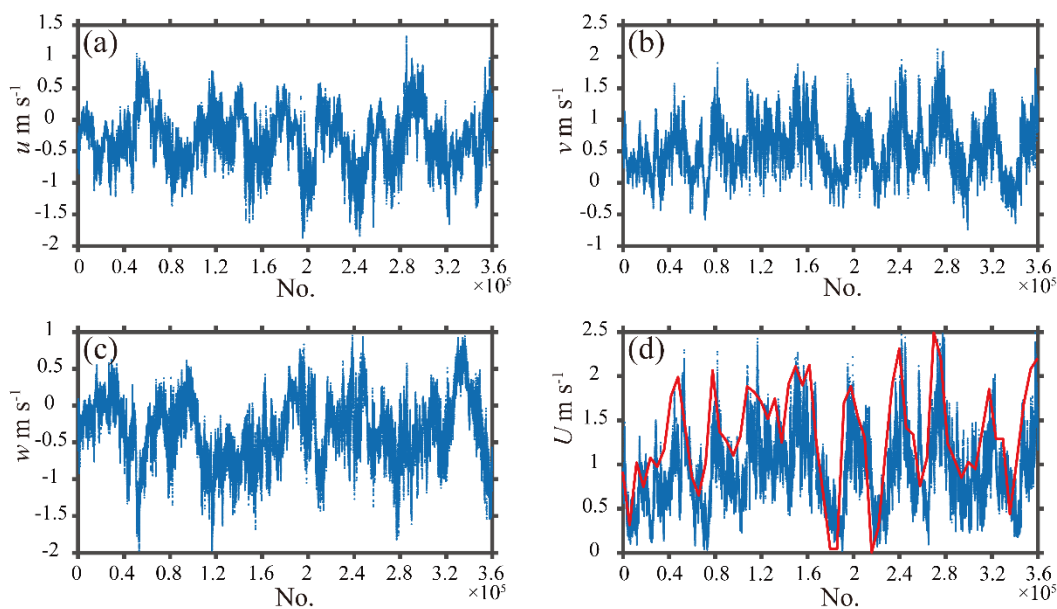


Figure S2. Three wind components (a) u , (b) v , (c) w and (d) the velocity U calculated by UAT-2 raw data after quality control with the method of $5\sigma^2$. This shown data was measured from 09:00:00 to 10:00:00 in 5 August 2020 (one hour) with the time resolution of 100 Hz frequency. The red solid line (d) was measured by auto weather station (AWS).

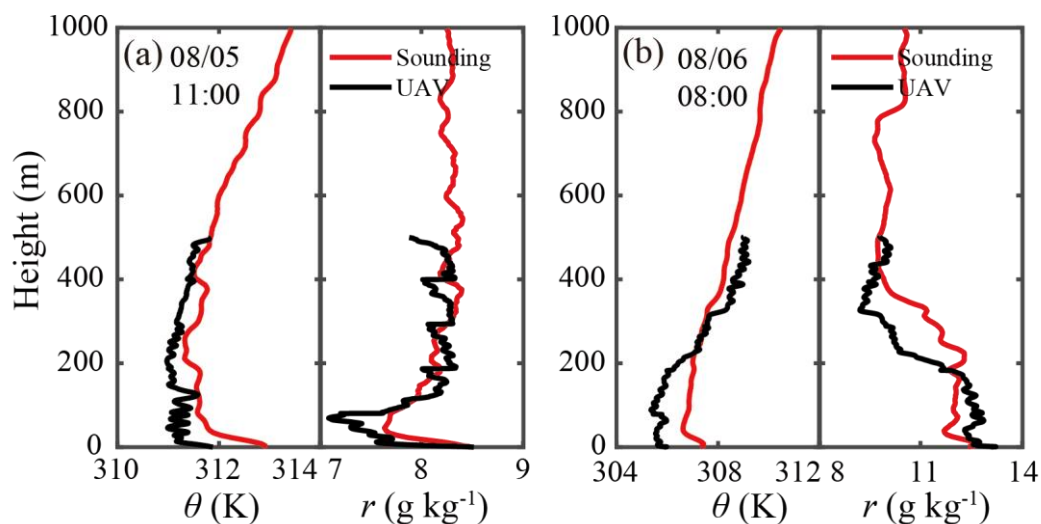


Figure S3. A comparison of vertical profile of potential temperature (θ) and water vapor mixing ratio (r) between Site A and Site B at (a) 10:00 LST on August 5, and (b) 08:00 LST on August 6.

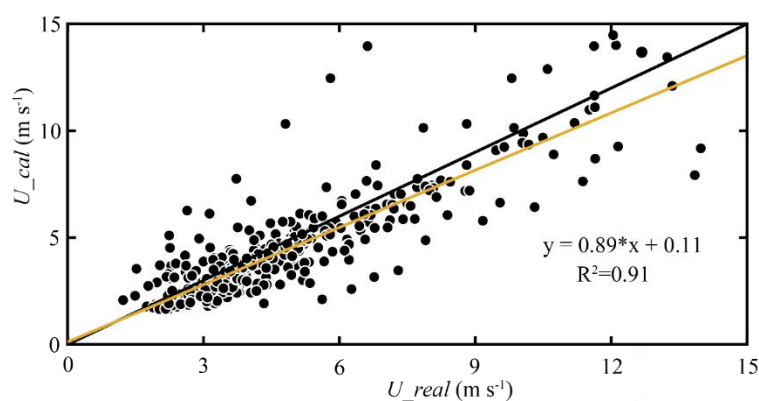


Figure S4. Scatter diagram showing the values of wind profile U_{real} measured by FC-II and calculated U_{cal} . Correlation coefficient was $R^2 = 0.91$. The yellow line represented first-order linear fitting.

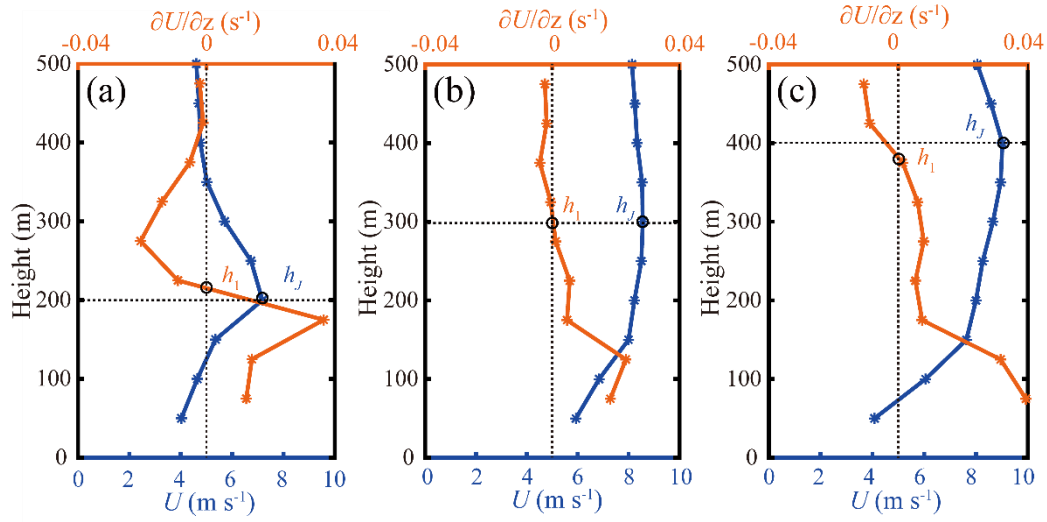


Figure S5. Samples profiles shown by blue profiles and orange profiles of U and $\partial U/\partial z$ obtained at (a) 20:50 LST 4 August 2020; (b) 19:40 LST 6 August 2020; 19:20 LST 6 August 2020. Black symbols indicate the BL height diagnostics: h_j ---- the height of wind profile maximum that indicated as horizontal dotted lines; h_1 ---- the height of zero crossing in the shear profile that indicated as perpendicular dotted lines.

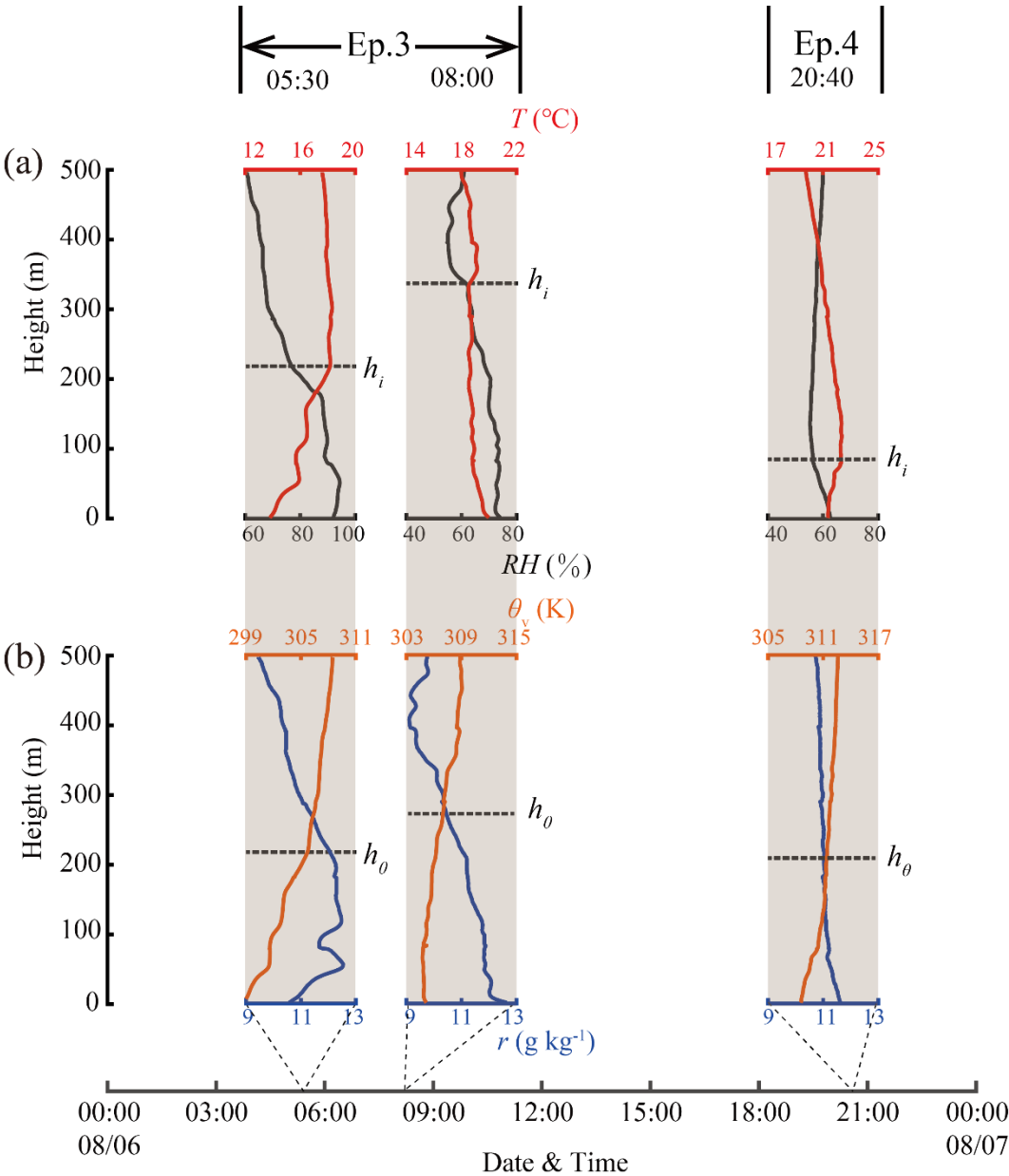


Figure S6. Same as Fig. 10, but for three times (05:30; 08:00; 20:40) were chosen that were marked in brown as background.

Table S1-S3:

Table S1. (a) Operating parameters of FC-II

Operating parameters	Values
Laser wavelength	1550 nm
Vertical range	30-1500 m
Range resolution	50 m
Detection period	3 s
Accuracy of wind speed	0.40 m s ⁻¹

Accuracy of wind direction	3°
Environmental temperature range	-40 °C-50 °C
Measuring blind aera	30 m
Wind range	0°-359°, 1 m s ⁻¹ -40 m s ⁻¹

(b) Operating parameters of UAT-2

Operating parameters	Values
Sample frequency	100 Hz
Wind range	0°-359°, 0 m s ⁻¹ -40 m s ⁻¹
Accuracy of wind speed	0.01 m s ⁻¹
Accuracy of wind direction	0.1°
Environmental temperature range	-30 °C-60 °C
Accuracy of wind direction	3°
Power supply mode	DC: 9 V-18 V

Table S2. Correlation coefficients between calculated observed rates of SBL and determined by the prognostic rate equations. Total number of data used for this calculation is $n=36$. The corresponding Z values are in parentheses

	$r[\Delta h, \lambda(h)]$	$r\left[\Delta h, \frac{\partial \lambda_1}{\partial t}\right]$	$r\left[\Delta h, \frac{\partial \lambda_2}{\partial t}\right]$	$r\left[\Delta h, \frac{\partial \lambda_3}{\partial t}\right]$
Correlation coefficients	0.02	0.49	0.59	0.64
calculated using Eq. (15)	(0.11)	(3.03)	(2.27)	(2.88)

Table S3. Classification of h by using dimensionless stability parameter μ_0

$\infty > \mu_0 > 78$	extremely stable	$78 > \mu_0 > 45$	very stable
$45 > \mu_0 > 27$	Moderately stable	$27 > \mu_0 > 5$	Slightly stable

1. Clarke, R. H.; Hess, G. D. Appropriate Scaling for Velocity and Temperature in Planetary Boundary Layer. *Journal of the Atmospheric Sciences*, **1973**, 30, 1346-1353.
2. Freund, J. E. Mathematical Statistics, Prentice-Hall, **1962**; pp. 308-312.