Bio-electrochemical system depollution capabilities and monitoring applications: Models, applicability, advanced bio-based concept for predicting pollutant degradation and microbial growth kinetics via gene regulation modelling

Table S1: Nomenclature and model parameter values of Tsipa et al. model

Symbols	Description	Estimated value	Units
α_{BenB}	BenB degradation and dilution due to cellular volume increase	33.352	h^{-1}
α_{BenRi}	BenRi degradation and dilution due to cellular volume increase	60	h^{-1}
α_{Pm}	mRNA degradation rate of *Pm*	1.613	h^{-1}
α_{PbenA}	mRNA degradation rate of *PbenA*	5.583	h^{-1}
α_{PbenR}	mRNA degradation rate of *PbenR*	5.865	h^{-1}
α_{Pr}	mRNA degradation rate of *Pr*	2.483	h^{-1}
α_{Ps}	mRNA degradation rate of *Ps*	2.107	h^{-1}
α_{Pu}	mRNA degradation rate of *Pu*	2.528	h^{-1}
α_{XylM}	XylM degradation and dilution due to cellular volume increase	60	h^{-1}
α_{XylRi}	XylRi degradation and dilution due to cellular volume increase	60	h^{-1}
α_{XylSi}	XylSi degradation and dilution due to cellular volume increase	6	h^{-1}
α_{XylU}	XylU degradation and dilution due to cellular volume increase	0.056	h^{-1}
β_b	maximum specific growth rate of biomass based on XylM	0.087	h^{-1}
β_{BenB}	translation rate based on *PbenA* mRNA	$9.11x10^2$	mM h^{-1}
β_{BenRi}	maximal translation rate of BenRi	$24.58x10^2$	mM h^{-1}
β_{PbenA}	maximal mRNA expression of *PbenA*	52.887	h^{-1}
β_{PbenR}	maximal mRNA expression of *PbenR*	4.212	h^{-1}
β_{Pm}	maximal expression level of *Pm*	0.927	h^{-1}
β_{Pr}	maximal expression level of *Pr*	0.343	h^{-1}
β_{Ps}	maximal expression level of *Ps*	28.47	h^{-1}
β_{Pu}	maximal expression level of *Pu*	44.812	h^{-1}
β_{XylM}	translation rate based on *Pm* mRNA	0.002	mM h^{-1}
β_{XylSi}	translation rate based on *Ps* mRNA	$10.69x10^2$	mM h^{-1}
β_{XylRi}	maximal translation rate of *Pr* mRNA	$5.7\ x10^2$	mM h^{-1}
β_{XylU}	translation rate based on *Pu* mRNA	$16.98\ x10^2$	mM h^{-1}
$\beta_{XylU,m\text{-}xyl}$	maximum *m*-xylene metabolic quotient based on XylU	0.018	$g_{tol}\ g_{biomass}\ h^{-1}$
$\beta_{XylU,tol}$	maximum toluene metabolic quotient based on XylU	0.018	$g_{tol}\ g_{biomass}\ h^{-1}$
$K_{BenB,b}$	saturation constant of BenB	19.543	mM
$K_{BenRa,PbenA}$	activation coefficient of *PbenA* by BenRa	2.02	mM
$K_{BenRa,PbenR}$	activation coefficient of *PbenR* by BenRa	1.645	mM
$K_{BenRa,Pm}$	activation coefficient of *Pm* by BenR	9.208	mM
$K_{XylM,b}$	saturation constant of XylM	25.103	mM

K_{XylSi}	activation coefficient of *Pm* by XylSi	17.984	mM
$K_{XylRa,Ps}$	activation coefficient of *Ps* by XylRa	5.44	mM
$K_{XylRa,Pu}$	activation coefficient of *Pu* by XylRa	3.626	mM
K_{XylRi}	repression coefficient of *Pr*	11.035	mM
$K_{XylU,m\text{-}xyl}$	saturation constant for XylU due to *m*-xylene	3.863	mM
$K_{XylU,tol}$	saturation constant for XylU due to toluene	3.863	mM
$MW_{m\text{-}xyl}$	molecular weight of *m*-xylene	106.2	g mol^{-1}
MW_{tol}	molecular weight of toluene	92.14	g mol^{-1}
$r_{R,BenR}$	dissociation constant of BenR	53.46	h^{-1}
$r_{R,XylS}$	dissociation constant of XylS	37.88	h^{-1}
$r_{R,XylR}$	dissociation constant of XylR	4.2 $x10^2$	h^{-1}

Table S2: Parameter values estimated for the double Monod, Mankad and Bungay, SKIP and the sum kinetics with competitive enzymatic interactions models using.

Symbols	**Description**	**Double Monod**	**Mankad and Bungay**	**SKIP**	**Competitive enzymatic interactions**	**Units**
$I_{m\text{-}xyl,tol}$	degree of inhibition of *m*-xylene to toluene	-	-	0.004	-	-
$I_{tol,m\text{-}xyl}$	degree of inhibition of toluene to *m*-xylene	-	-	0.0003	-	-
$K_{m\text{-}xyl}$	saturation constant of *m*-xylene	0.1	0.7	0.7	0.7	(mM)
K_{tol}	saturation constant of toluene	0.1	0.46	0.035	0.45	(mM)
μ_{max}	maximal specific growth rate	0.033	0.079	-	-	(h^{-1})
$\mu_{max,tol}$	maximal specific growth rate of toluene	-	-	0.018	0.097	(h^{-1})
$\mu_{max,m\text{-}xyl}$	maximal specific growth rate of *m*-xylene	-	-	0.093	0.042	(h^{-1})
$Y_{m\text{-}xyl}$	yield coefficient of *m*-xylene	0.99	0.99	0.99	0.99	(g/g)
Y_{tol}	yield coefficient of toluene	0.99	0.99	0.99	0.99	(g/g)