

Supplementary Information

Mechanical Performance and Dye Adsorption of Lignin/Poly(ethylene glycol) Diglycidyl Ether/Sorbitol Polyglycidyl Ether Hydrogels

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1. Synthesis mechanism of LP, LSnPn, and LS.
2. Enlarged FTIR spectra of hydrogels.
3. Gel fraction and swelling ratio of the hydrogels.
4. Rheological behavior of the hydrogels.
5. Adsorption isotherm analysis of MB.
6. Geometry-based analysis of mass-transfer limitation.

1. Synthesis mechanism of LP, LSnP_n, and LS.

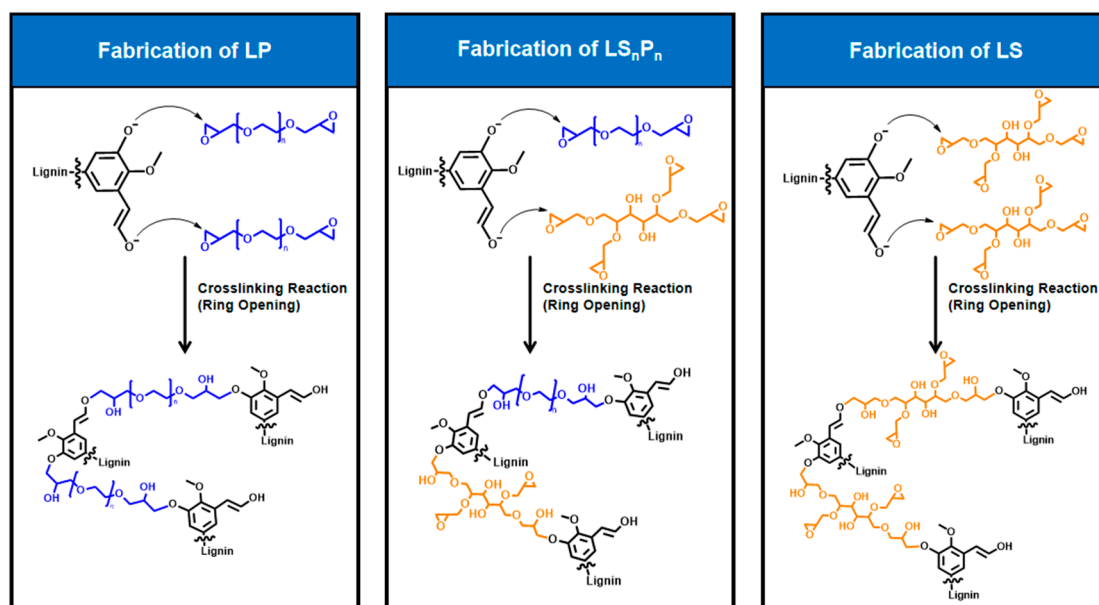


Figure S1. Schematic illustration of the synthesis mechanism of LP, LSnP_n, and LS.

2. Enlarged FTIR spectra of hydrogels.

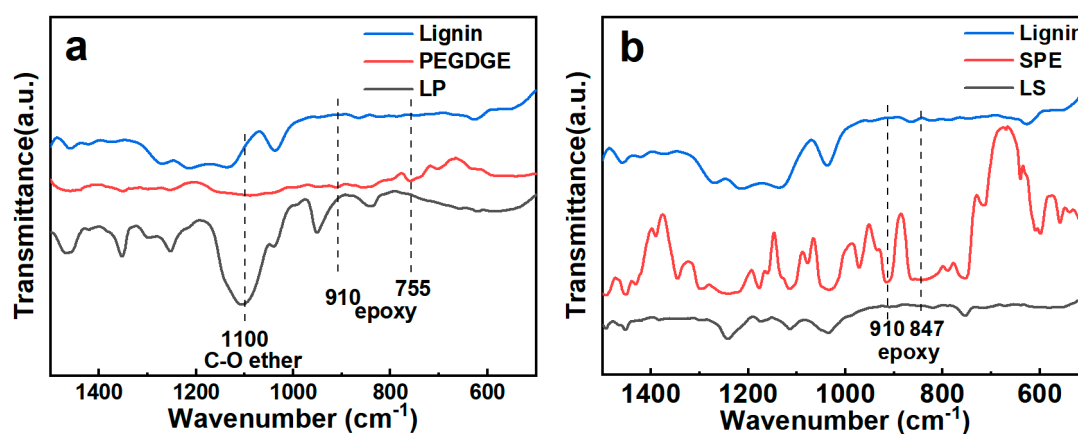


Figure S2. Enlarged FTIR spectra of (a) lignin, PEGDGE, and LP, and (b) lignin, SPE, and LS.

3. Gel fraction and swelling ratio of the hydrogels

Table S1. Gel fraction and swelling ratio of hydrogels.

Sample	Gel fraction (%) ¹	Swelling ratio (%)
LP	54.90 ± 0.02	429.17 ± 0.11
LS ₁ P ₉	62.59 ± 0.04	305.40 ± 0.09
LS ₂ P ₈	65.92 ± 0.03	181.87 ± 0.11
LS ₃ P ₇	67.03 ± 0.01	152.19 ± 0.06
LS ₄ P ₆	70.69 ± 0.03	114.25 ± 0.05
LS ₅ P ₅	75.16 ± 0.01	95.59 ± 0.03
LS ₆ P ₄	72.21 ± 0.01	71.20 ± 0.01
LS ₇ P ₃	69.76 ± 0.02	66.49 ± 0.03
LS ₈ P ₂	67.50 ± 0.01	38.45 ± 0.01
LS ₉ P ₁	63.03 ± 0.01	22.81 ± 0.01
LS	61.71 ± 0.01	22.40 ± 0.02

¹Gel fraction (GF) was determined by Soxhlet extraction in water at 110 °C for 4 h and calculated as $GF = w_d / w_0 \times 100\%$, where w_0 and w_d are the dry masses of the samples before and after extraction.

4. Rheological behavior of the hydrogels

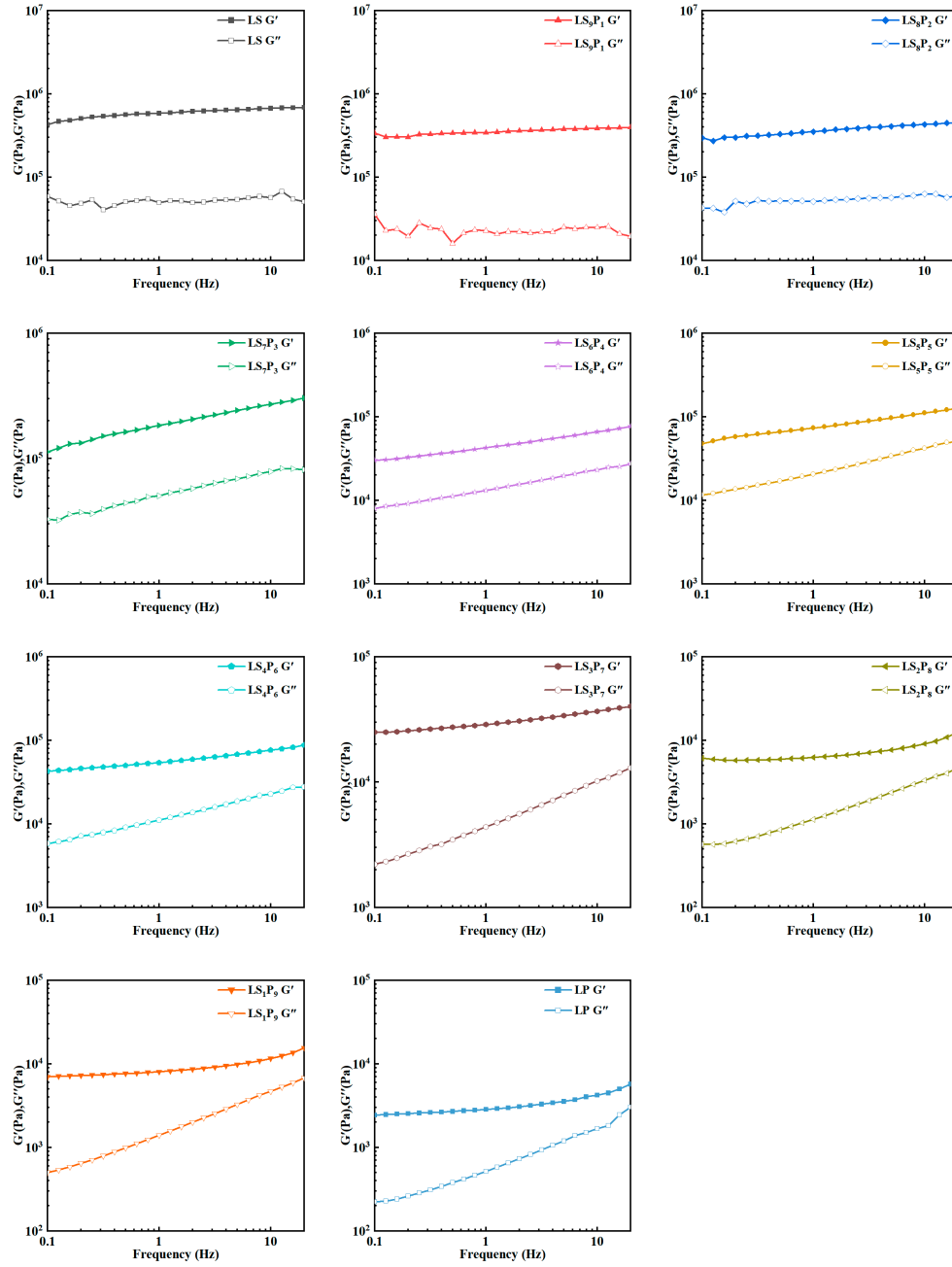


Figure S3. Rheological properties of the hydrogels.

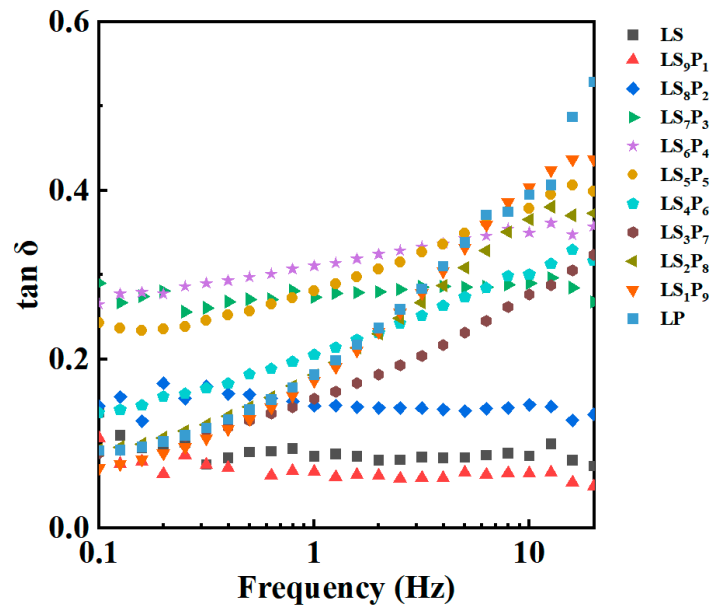


Figure S4. Frequency dependence of $\tan \delta$ (G''/G') for hydrogels.

5. Adsorption isotherm analysis of MB

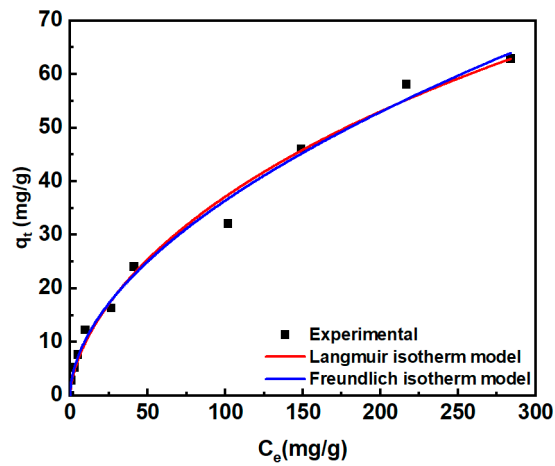


Figure S5. Langmuir and Freundlich isotherm models of MB adsorption isotherm.

Table S2. Characteristic parameters of the adsorption isotherm.

Langmuir isotherm	R^2	0.985
	k_L ($L \text{ mg}^{-1}$)	0.006
	$q_m(\text{cal})$ (mg g^{-1})	265.88
Freundlich isotherm	R^2	0.991
	k_F (mg g^{-1})	3.018
	n	1.849

6. Geometry-based analysis of mass-transfer limitation

1) Effective external surface area

(a) intact hydrogel: cylinder (radius: 1.25 cm, thickness: 1.5 cm)

$$V_{\text{bulk}} = \pi r^2 h = \pi \times 1.25^2 \times 1.5 = 7.36 \text{ cm}^3$$

$$S_{\text{bulk}} = 2\pi rh + 2\pi r^2 = 2\pi \times 1.25 \times 1.5 + 2\pi \times 1.25^2 = 21.59 \text{ cm}^2$$

$$\left(\frac{S}{V}\right)_{\text{bulk}} = 2.93 \text{ cm}^{-1}$$

(b) cut hydrogel: around cuboid shape (length: 0.5 cm, width: 0.4 cm, thickness: 0.15 cm)

$$V_{\text{small}} = lwt = 0.5 \times 0.4 \times 0.15 = 0.03 \text{ cm}^3$$

$$S_{\text{small}} = 2(lw + lt + wt) = 2(0.5 \times 0.4 + 0.5 \times 0.15 + 0.4 \times 0.15) = 0.67 \text{ cm}^2$$

$$\left(\frac{S}{V}\right)_{\text{small}} = 22.33 \text{ cm}^{-1}$$

$$\frac{\left(\frac{S}{V}\right)_{\text{small}}}{\left(\frac{S}{V}\right)_{\text{bulk}}} \approx 7.6$$

The cut hydrogel has an $\sim 7.6\times$ higher geometric surface-area-to-volume ratio than the intact hydrogel. This increases the effective external contact area and generally reduces external mass-transfer resistance under the same agitation conditions.

2) Diffusion length (thickness/size effect) and diffusion time scaling

For diffusion in a hydrogel, the characteristic diffusion time scales approximately as:

$$t_d \propto \frac{L^2}{D_{\text{eff}}}$$

where L is a characteristic diffusion length and D_{eff} is the effective diffusivity of methylene blue (MB) inside the hydrogel network. For the same material, D_{eff} is comparable, so geometry mainly affects t_d through L^2 .

(a) Cut hydrogel

The diffusion path is typically controlled by the half-thickness:

$$L_{\text{small}} = \frac{t}{2} = \frac{1.5 \text{ mm}}{2} = 0.75 \text{ mm}$$

(b) Intact hydrogel half-thickness:

$$L_{\text{bulk},z} = \frac{h}{2} = \frac{1.5\text{cm}}{2} = 0.75\text{cm} = 7.5\text{mm}$$

Radial length:

$$L_{\text{bulk},r} = r = 1.25\text{cm} = 12.5\text{mm}$$

The slower direction tends to dominate uptake, so both estimates are provided.

(c) Predicted diffusion time ratio (intact vs cut hydrogel)

If axial diffusion controls:

$$\frac{t_{\text{bulk}}}{t_{\text{small}}} \approx \left(\frac{L_{\text{bulk},z}}{L_{\text{small}}} \right)^2 = \left(\frac{7.5}{0.75} \right)^2 = 100$$

If radial diffusion controls:

$$\frac{t_{\text{bulk}}}{t_{\text{small}}} \approx \left(\frac{L_{\text{bulk},r}}{L_{\text{small}}} \right)^2 = \left(\frac{12.5}{0.75} \right)^2 \approx 278$$

So, the intact hydrogel needs 100-278 times longer diffusion time compared to the cut hydrogel.