

Viscoelastic response of elastohydrodynamically lubricated compliant contacts below glass-transition temperature

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A-Background

EHL was a subject of intense research for decades when the hard and soft modes of EHL were distinguished as Piezoviscous-elastic (P-E) [5] and Piezoviscous-rigid (P-R), and Isoviscous-elastic (I-E) [32] and Isoviscous rigid (I-R), respectively. For the definition of individual EHL modes, Johnson [21], Hamrock and Dowson [19], Esfahanian and Hamrock [22], Hooke et al. [18,33] and others [29–30] derived hydrodynamic maps of EHL modes, which put in the contrast the operating conditions relative to viscosity g_v and elasticity g_E parameters, see Eq. (SA1–SA2), for different contact geometries—line, circular ($k = 1$) and elliptical ($k < 1$ or $k > 1$) contacts. Both parameters were supplemented by the Archard's parameter of the system g_4 , see Eq. (SA3), which additionally defines the transition region (TR) between P-E and I-E modes to the interval $g_4 \in \langle 0.1, 2.2 \rangle$.

$$g_v = \frac{\bar{G} \bar{W}^3}{\bar{U}^2} \quad (\text{SA1})$$

$$g_E = \frac{\bar{W}^{8/3}}{\bar{U}^2} \quad (\text{SA2})$$

$$g_4 = g_v g_E^{-1.5} \quad (\text{SA3})$$

where $\bar{U} = \frac{U \eta_0}{E_R R'}$; $\bar{W} = \frac{W}{E_R R'^2}$, $\bar{G} = E_R \alpha$. All symbols are stated in [19,21] and in the Nomenclature of this paper.

Briefly, the classical EHL theory [3–8] is characterized by three crucial mechanisms influencing the formation of the central (h_c) and minimum (h_m) film thickness in the contact. The first two mechanisms cover the pressure–viscosity effect [34–35] and the hydrodynamic effect of the fluid according to the Reynolds equation [13,36] and thus represent the rheological response of the lubricant to the operating conditions. These mechanisms usually have a positive impact on the increase in fluid-film thickness in the EHL regime.

The third mechanism is represented by the constitutive material response of the solids, which commonly results in the elastic deformation of the rubbing surfaces after the non-conformal contact is loaded [37–38]. This is typical for the P-E mode of EHL where the elastic deformation of high elastic modulus (E) solids, such as metals, ceramics or glass, leads to high contact pressures (p) up to several gigapascals ($p \approx 3\text{--}4$ GPa) [39–40]. Hence, the high contact pressure has a significant effect on the pressure–viscosity relationship, and thus on the increase in film thickness in dependence on the used lubricant characterized by the pressure–viscosity coefficient α [40–41]. The rigid modes of EHL are beyond the focus of this paper due to the low relevance of the third mechanism.

Contrary to the P-E mode, the I-E mode of EHL is distinguished by low E of contact solids, such as elastomers and polymers, and the contact is rather compliant [32,42]. Moreover, the low contact pressure (usually $p < 1$ GPa) is insufficient to change the dynamic viscosity (η) in this mode. Moreover, the low stiffness of compliant solids, attributed to E , contributes to the significant increase in width of contact strip for the line contacts [43] and the contact radius (R_c) for circular [20,27,44] / elliptical [18,28] contacts when the contact is loaded.

In practical applications, such as transmissions and mechanisms, the contact of the meshing gears made of different materials is frequently provided, where E of the steel pinion is significantly higher than for the wheel gear, usually made from the engineering [11] and high-performance [12] polymers. Thus, the contact pair represents the rigid (hard) and compliant (soft) solids. With respect to this, the operation of the contact in the TR region [22,30] between the P-E and I-E modes of the EHL regime is expected; however, only limited research in this field of EHL has been performed so far. Moreover, the validity of the prediction models [5,18–20] of the film thickness corresponding to the P-E and I-E modes of EHL for the TR region has not been sufficiently validated yet.

Moreover, the viscoelastic response of a compliant material to the formation of film thickness [45–46] has been often neglected. The prediction formulae for dimensionless H_c as well as H_m film thickness in circular contact related to the P-E and I-E modes for H&D model [5,19] are described by Eqs. (SA4) and (SA5) and Eqs. (SA6) and (SA7), respectively. Alternatively, these prediction formulae could be expressed using the g_v (Eq. (SA1)) and g_E (Eq. (SA2)) parameters.

P-E mode:

$$H_c = \frac{h_c}{R'} = 1.9 \bar{U}^{0.67} \bar{W}^{-0.067} \bar{G}^{0.53} = 2.55 g_v^{0.53} g_E^{0.13} \left(\frac{\bar{U}}{\bar{W}} \right)^2 \quad (\text{SA4})$$

$$H_m = \frac{h_m}{R'} = 1.8 \bar{U}^{0.68} \bar{W}^{-0.073} \bar{G}^{0.49} = 1.69 g_v^{0.49} g_E^{0.17} \left(\frac{\bar{U}}{\bar{W}} \right)^2 \quad (\text{SA5})$$

I-E mode:

$$H_c = \frac{h_c}{R'} = 3.3 \bar{U}^{0.64} \bar{W}^{-0.22} = 5.08 g_E^{0.67} \left(\frac{\bar{U}}{\bar{W}} \right)^2 \quad (\text{SA6})$$

$$H_m = \frac{h_m}{R'} = 2.8 \bar{U}^{0.65} \bar{W}^{-0.21} = 3.28 g_E^{0.67} \left(\frac{\bar{U}}{\bar{W}} \right)^2 \quad (\text{SA7})$$

A phenomenon of viscoelastic behavior is achieved after the material exhibits a non-linear (non-Hookean) response in time in relation to the given constant stress or strain corresponding to the stress–strain dependence [47–48]. The viscoelastic response of the material is frequently numerically simulated by standard linear solid models (SLS) such as Maxwell's, Zener's and Kelvin–Voight's models, where dampers and springs in serial or parallel arrangement invoke a time-varying response [9,49]. Thus, the original elastic modulus is substituted by the relaxation or complex modulus with regard to the temperature or time (or frequency) domain, respectively. The complex modulus E^* includes two main components: the elastic part, the storage modulus (E') as well as the viscous part, the loss modulus (E''), see Eq. (SA8).

$$E^* = E' + i E'' \quad (\text{SA8})$$

$$\tan \delta = \frac{\sin \delta}{\cos \delta} = \frac{E''}{E'} \quad (\text{SA9})$$

The ratio between them represents the damping / loss factor $\tan \delta$, see Eq. (SA9), corresponding to the stored energy in the material and the loss energy converted (energy

dissipated due to internal friction of molecules in the polymer structure) to the heat after the polymer is tempered from glassy state ($T \ll T_g$) and viscoelastic state ($T \leq T_g$) to the rubbery plateau where $T > T_g$. The individual states define a macromolecular arrangement, and the reciprocal motion of molecules of the polymer chains (bonds) relative to the internal polymer structure and operating conditions [50]. According to the internal structure, the amorphous (PMMA, PC), semi-crystalline (PA66, POM, PEEK) and crystalline polymers demonstrate a significantly different behavior [51–52] in individual states with the rising temperature where the amorphous polymers prove a substantial temperature reliance rather than semi-crystalline and crystalline ones.

Furthermore, the constitutive temperature dependent behavior of polymers is characterized by the relaxation transitions labeled α , β , γ and δ in a decreasing order to the temperature where the main-chain and individual side-chains of polymer internal structure are rearranged, and the material demonstrates a different viscoelastic response [25]. The primary α -relaxation relates to the glass-transition temperature T_g of polymer in the viscoelastic state when the motion of molecules of the main-chain (chain backbone) is enhanced [17,53]. However, β , γ and δ —secondary relaxations—take place in the interval below T_g in the glassy state and they especially result from the side-chains molecular motions of the polymer [54] as well as heteroatom motions in the main-chain [55]. Simultaneously, the mobility of the main-chain (chain backbone) molecules decreases relative to the α -relaxation state but does not fully vanish [56]. The internal energy for initialization of the primary or secondary relaxation transitions of molecules corresponds to the viscoelastic activation energy (E_A) according to the inverse alphabetical order, i.e., $E_{A, \beta} < E_{A, \alpha}$ [57].

In practice, the E' and E'' moduli are often experimentally determined by the creep and relaxation tests using the nanoindentation [9,58] or by a dynamic mechanical analysis (DMA) [31,59] where the δ represents the phase angle between stress and strain corresponding to the interval of $\delta \in (0, \pi/2)$. With increasing value of $\tan \delta(f, T)$, the polymer exhibits more compliance (inverted parameter to the stiffness) in the mechanical cyclic loading, and thus, the material capability to damp the impacts and vibrations is maximal [60].

The influence of the time (or frequency f) of loading and temperature T can be combined to obtain the complex material response of the polymers by the time-temperature superposition principle (TTSP) [61,62]. With respect to this, the Arrhenius (Eq. (SA10)) and the William–Landel–Ferry (WLF) (Eq. (SA11)) equations are predominantly used in the temperature interval $T < T_g$ and $T \geq T_g$, respectively, to determine the time-temperature transformation by horizontal (a_T) and vertical (b_T) shift factors [54,57]. This is associated to the selected reference temperature T_R . However, the b_T is often omitted in TTSP.

$$\ln a_T = \frac{E_A}{R} \left(\frac{1}{T} - \frac{1}{T_R} \right) \quad (\text{SA10})$$

$$\log a_T = - \frac{C_1 (T - T_R)}{C_2 + (T - T_R)} \quad (\text{SA11})$$

where C_1 , C_2 are constants, for WLF, appropriate to the individual polymers, and R (8.164 J mol⁻¹ K⁻¹) is a universal gas constant.

When the influence of time (or frequency) and temperature on the E^* is reciprocally equivalent, the polymer is considered as thermorheologically simple covering a single relaxation time spectra $H(\tau)$ [61] where τ is the relaxation time. Hence, the viscoelastic deformation is governed only by a single molecular process. Consequently, the sole master-curve (MC) of the E^* or $\tan \delta$ covering the wide temperature and time (or frequency) interval can be derived for the individual thermorheologically simple polymers by a_T . This provides necessary input data to include the viscoelastic response of the material into the prediction models of fluid-film thickness, especially in the I-E mode of EHL. Otherwise, the polymer is thermorheologically complex [63], and the TTSP is not applicable [56].

B—Generalized Maxwell model (Prony series)Equilibrium relaxation modulus— E_∞ **Table SB1.** Parameters of the generalized Maxwell model – nDMA ($E_\infty = 3.43$ GPa)

Modes $j=1\dots n$	Linear spring modulus, E_j (MPa)	Relaxation time, τ_j (s)
1	8.27E+02	9.10E-07
2	3.80E+02	2.58E-05
3	1.57E+02	2.14E-04
4	7.10E+02	7.35E-04
5	3.37E+02	1.03E-02
6	5.24E+02	5.53E-02
7	4.81E+02	5.71E-01

Table SB2. Parameters of the generalized Maxwell model –DMA ($E_\infty = 0.81$ GPa)

Modes $j=1\dots n$	Linear spring modulus, E_j (MPa)	Relaxation time, τ_j (s)
1	4.98E+01	1.84E-01
2	1.35E+02	7.70E-03
3	1.90E+01	3.09E+00
4	2.97E+01	3.10E+00
5	2.90E+01	1.44E+01
6	1.28E-05	9.45E+02
7	1.23E+02	1.00E+03

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