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## Supplementary Materials

# Quantitative Evaluation of the Phase Function Effects on Light Scattering and Radiative Transfer in Dispersed Systems

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**Abstract:** The light scattering properties of particles play important roles in radiative transfer in many dispersed systems, such as turbid atmosphere, ocean water, nanofluids, composite coatings and so on. As one of the scattering property parameters, the scattering phase functions of particles are strongly dependent on the particle size, size distribution, morphology as well as the complex refractive indices of the particles and surrounding media. For the sake of simplicity, the empirical phase function models are widely used in many practical applications. In this work, we focus on the radiative transfer problem in dispersed systems composed of spherical particles and give quantitative analyses of the impact of scattering phase functions on the radiative transfer process. We fit the scattering phase functions of four different types of practical dispersed systems using four previously proposed empirical phase function models, including the Henyey–Greenstein (HG) model, Cornette Shanks (CS) model, Reynold and McCormick (RM) model, and two–term Reynolds–McCormick (TTRM) model. By comparing the radiative transfer characteristics (i.e. hemispherical reflectance, hemispherical transmittance and total absorptance) of dispersed layers calculated using the Monte Carlo method, the relative errors caused by using the empirical phase functions are systematically investigated. The results demonstrate that the HG, CS and RM models will cause obvious errors in the calculation of hemispherical reflectance in many cases. Meanwhile, the induced errors show no obvious regularity, but are related to the particle size and layer optical thickness. Due to the good fitting effect in both forward and back-ward directions, the TTRM model provides significantly higher performances to fit phase functions of all considered cases than the widely used single–term parametrizations. Moreover, for different particle sizes and layer optical thicknesses, the induced errors of TTRM model in radiative transfer characteristics are very small, especially for the case of polydisperse particles. Our results can be used to guide the design, analysis and optimization of dispersed systems in practical optics and photonics applications.

**Keywords:** scattering phase function; light scattering; radiative transfer; Mie scattering; Monte Carlo simulation

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**Table S1.** Values used as bound constraints in the fitting procedure for the RM and TTRM models.

| Model | Parameters | Lower Bound | Upper Bound |
|-------|------------|-------------|-------------|
| RM    | $\bar{g}$  | -1          | 1           |
|       | $\alpha$   | -5          | 5           |
|       | $\gamma$   | 0           | 1           |
| TTRM  | $g^1$      | 0           | 1           |
|       | $\alpha_1$ | -5          | 5           |
|       | $g^2$      | -1          | 0           |
|       | $\alpha_2$ | -5          | 5           |
|       |            |             |             |

**Table S2.** Radiative properties of monodisperse and polydisperse spherical particles for different particle size parameters.

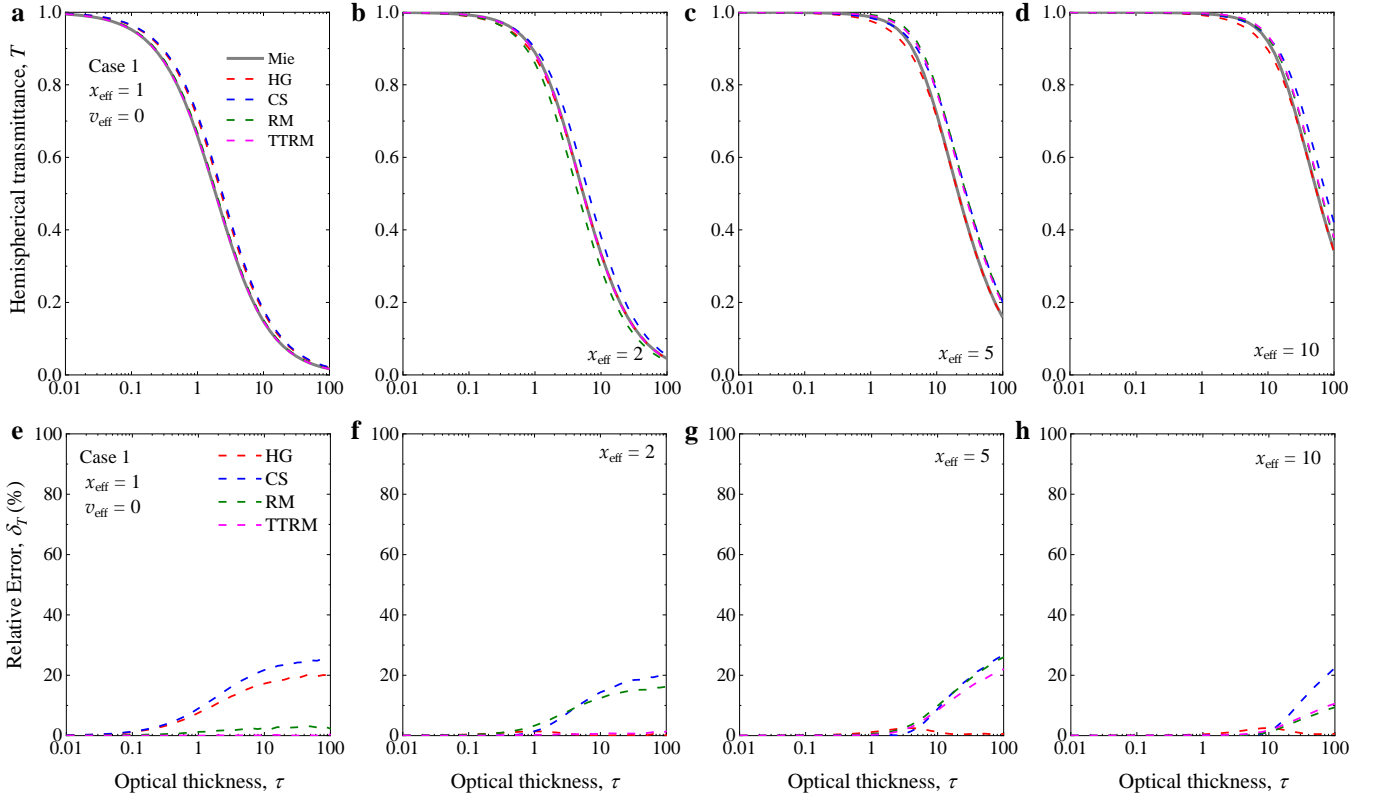
| Number | $\chi_{\text{eff}}$ | $\nu_{\text{eff}}$ | $\langle C_{\text{ext}} \rangle_r$ | $\langle C_{\text{sca}} \rangle_r$ | $\langle V \rangle_r$ | $\omega$  | $g$       |
|--------|---------------------|--------------------|------------------------------------|------------------------------------|-----------------------|-----------|-----------|
| Case 1 | 1.0                 | 0                  | 3.280E-04                          | 3.280E-04                          | 7.130E-03             | 1.000E+00 | 1.708E-01 |
|        | 2.0                 | 0                  | 9.044E-03                          | 9.044E-03                          | 5.704E-02             | 1.000E+00 | 6.358E-01 |
|        | 5.0                 | 0                  | 4.699E-01                          | 4.699E-01                          | 8.912E-01             | 1.000E+00 | 9.067E-01 |
|        | 10.0                | 0                  | 6.898E+00                          | 6.898E+00                          | 7.130E+00             | 1.000E+00 | 9.645E-01 |
|        | 1.0                 | 0.05               | 3.231E-04                          | 3.231E-04                          | 6.096E-03             | 1.000E+00 | 2.396E-01 |
|        | 2.0                 | 0.05               | 8.475E-03                          | 8.475E-03                          | 4.877E-02             | 1.000E+00 | 6.631E-01 |
|        | 5.0                 | 0.05               | 4.187E-01                          | 4.187E-01                          | 7.620E-01             | 1.000E+00 | 9.116E-01 |
|        | 10.0                | 0.05               | 5.905E+00                          | 5.905E+00                          | 6.096E+00             | 1.000E+00 | 9.648E-01 |
| Case 2 | 1.0                 | 0                  | 3.276E-02                          | 3.276E-02                          | 7.130E-03             | 1.000E+00 | 2.661E-01 |
|        | 2.0                 | 0                  | 7.652E-01                          | 7.652E-01                          | 5.704E-02             | 1.000E+00 | 5.068E-01 |
|        | 5.0                 | 0                  | 3.531E+00                          | 3.531E+00                          | 8.912E-01             | 1.000E+00 | 4.506E-01 |
|        | 10.0                | 0                  | 9.489E+00                          | 9.489E+00                          | 7.130E+00             | 1.000E+00 | 5.877E-01 |
|        | 1.0                 | 0.05               | 3.601E-02                          | 3.601E-02                          | 6.096E-03             | 1.000E+00 | 3.904E-01 |
|        | 2.0                 | 0.05               | 6.220E-01                          | 6.220E-01                          | 4.877E-02             | 1.000E+00 | 5.008E-01 |
|        | 5.0                 | 0.05               | 2.474E+00                          | 2.474E+00                          | 7.620E-01             | 1.000E+00 | 4.991E-01 |
|        | 10.0                | 0.05               | 9.242E+00                          | 9.242E+00                          | 6.096E+00             | 1.000E+00 | 6.000E-01 |
| Case 3 | 1.0                 | 0                  | 3.266E-01                          | 3.020E-01                          | 7.130E-03             | 9.245E-01 | 3.556E-01 |
|        | 2.0                 | 0                  | 1.602E-01                          | 1.065E-01                          | 5.704E-02             | 6.646E-01 | 1.335E-01 |
|        | 5.0                 | 0                  | 2.900E+00                          | 2.282E+00                          | 8.912E-01             | 7.869E-01 | 4.402E-01 |
|        | 10.0                | 0                  | 9.308E+00                          | 6.513E+00                          | 7.130E+00             | 6.998E-01 | 6.286E-01 |
|        | 1.0                 | 0.05               | 1.432E-01                          | 1.343E-01                          | 6.096E-03             | 9.378E-01 | 2.533E-01 |
|        | 2.0                 | 0.05               | 4.256E-01                          | 3.601E-01                          | 4.877E-02             | 8.460E-01 | 2.659E-01 |
|        | 5.0                 | 0.05               | 2.459E+00                          | 1.971E+00                          | 7.620E-01             | 8.015E-01 | 5.234E-01 |
|        | 10.0                | 0.05               | 9.073E+00                          | 6.678E+00                          | 6.096E+00             | 7.360E-01 | 6.468E-01 |
| Case 4 | 1.0                 | 0                  | 2.438E-01                          | 2.119E-01                          | 7.130E-03             | 8.692E-01 | 2.454E-02 |
|        | 2.0                 | 0                  | 7.482E-01                          | 6.668E-01                          | 5.704E-02             | 8.912E-01 | 3.950E-01 |
|        | 5.0                 | 0                  | 4.024E+00                          | 3.584E+00                          | 8.912E-01             | 8.907E-01 | 5.832E-01 |
|        | 10.0                | 0                  | 1.360E+01                          | 1.223E+01                          | 7.130E+00             | 8.993E-01 | 5.995E-01 |
|        | 1.0                 | 0.05               | 2.022E-01                          | 1.700E-01                          | 6.096E-03             | 8.409E-01 | 4.770E-02 |
|        | 2.0                 | 0.05               | 6.671E-01                          | 5.866E-01                          | 4.877E-02             | 8.794E-01 | 4.011E-01 |
|        | 5.0                 | 0.05               | 3.415E+00                          | 3.054E+00                          | 7.620E-01             | 8.941E-01 | 5.718E-01 |
|        | 10                  | 0.05               | 1.176E+01                          | 1.057E+01                          | 6.096E+00             | 8.990E-01 | 6.004E-01 |

**Table S3.** Parameters obtained from fits to the RM function

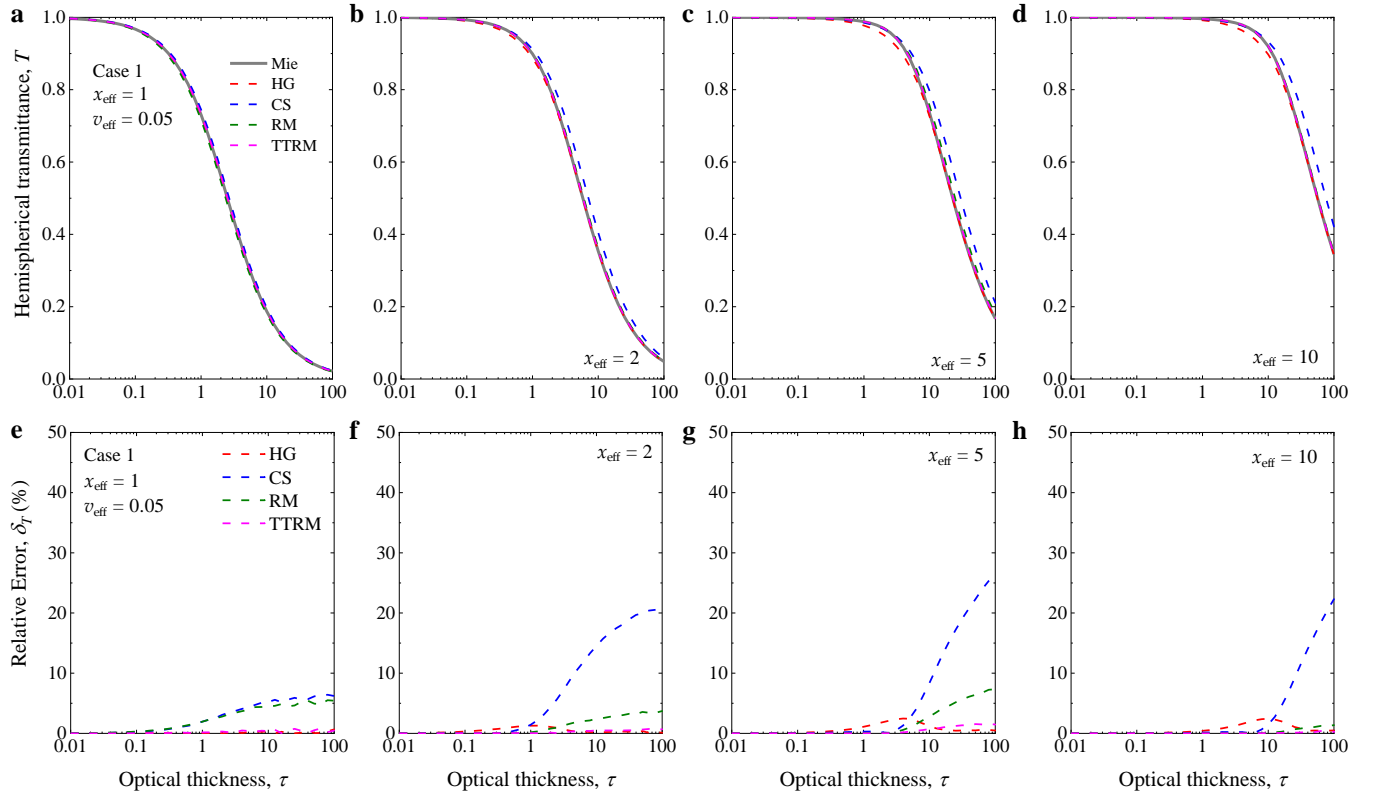
| Number | $\chi_{\text{eff}}$ | $\nu_{\text{eff}}$ | $\bar{g}$ | $\alpha$ | MSLE     |
|--------|---------------------|--------------------|-----------|----------|----------|
| Case 1 | 1.0                 | 0                  | 0.972     | −0.9382  | 1.05E−02 |
|        | 2.0                 | 0                  | −0.345    | −5       | 1.12E−02 |
|        | 5.0                 | 0                  | 0.6913    | 1.8142   | 1.63E−01 |
|        | 10.0                | 0                  | 0.8869    | 1.0419   | 9.77E−02 |
|        | 1.0                 | 0.05               | 0.6504    | −0.5085  | 3.24E−03 |
|        | 2.0                 | 0.05               | 0.3703    | 2.0705   | 5.19E−03 |
|        | 5.0                 | 0.05               | 0.7621    | 1.2186   | 1.97E−02 |
|        | 10.0                | 0.05               | 0.8718    | 1.0879   | 6.14E−03 |
| Case 2 | 1.0                 | 0                  | 0.5497    | −0.3614  | 3.12E−03 |
|        | 2.0                 | 0                  | 0.5101    | 0.4794   | 2.65E−03 |
|        | 5.0                 | 0                  | 0.949     | −0.3877  | 1.65E−01 |
|        | 10.0                | 0                  | 0.9639    | −0.2235  | 1.59E−01 |
|        | 1.0                 | 0.05               | 0.3304    | 0.6926   | 2.58E−03 |
|        | 2.0                 | 0.05               | 0.5863    | 0.1739   | 5.94E−03 |
|        | 5.0                 | 0.05               | 0.9293    | −0.3539  | 7.55E−02 |
|        | 10.0                | 0.05               | 0.9637    | −0.2484  | 1.02E−01 |
| Case 3 | 1.0                 | 0                  | 0.2833    | 0.7898   | 2.63E−03 |
|        | 2.0                 | 0                  | 0.2818    | −0.548   | 3.95E−02 |
|        | 5.0                 | 0                  | 0.9575    | −0.436   | 1.03E−01 |
|        | 10.0                | 0                  | 0.9599    | −0.1059  | 5.59E−02 |
|        | 1.0                 | 0.05               | 0.5373    | −0.3858  | 3.54E−03 |
|        | 2.0                 | 0.05               | 0.8413    | −0.5838  | 1.14E−02 |
|        | 5.0                 | 0.05               | 0.9285    | −0.3056  | 3.78E−02 |
|        | 10.0                | 0.05               | 0.9717    | −0.1958  | 4.48E−02 |
| Case 4 | 1.0                 | 0                  | 0.9474    | −0.9136  | 8.04E−03 |
|        | 2.0                 | 0                  | 0.7066    | −0.2852  | 2.44E−02 |
|        | 5.0                 | 0                  | 0.8877    | −0.0526  | 4.93E−02 |
|        | 10.0                | 0                  | 0.9747    | −0.2281  | 5.13E−02 |
|        | 1.0                 | 0.05               | 0.9231    | −0.886   | 6.11E−03 |
|        | 2.0                 | 0.05               | 0.6658    | −0.1754  | 4.40E−03 |
|        | 5.0                 | 0.05               | 0.9133    | −0.2028  | 1.52E−02 |
|        | 10                  | 0.05               | 0.9729    | −0.2281  | 2.72E−02 |

**Table S4.** Parameters obtained from fits to the TTRM function

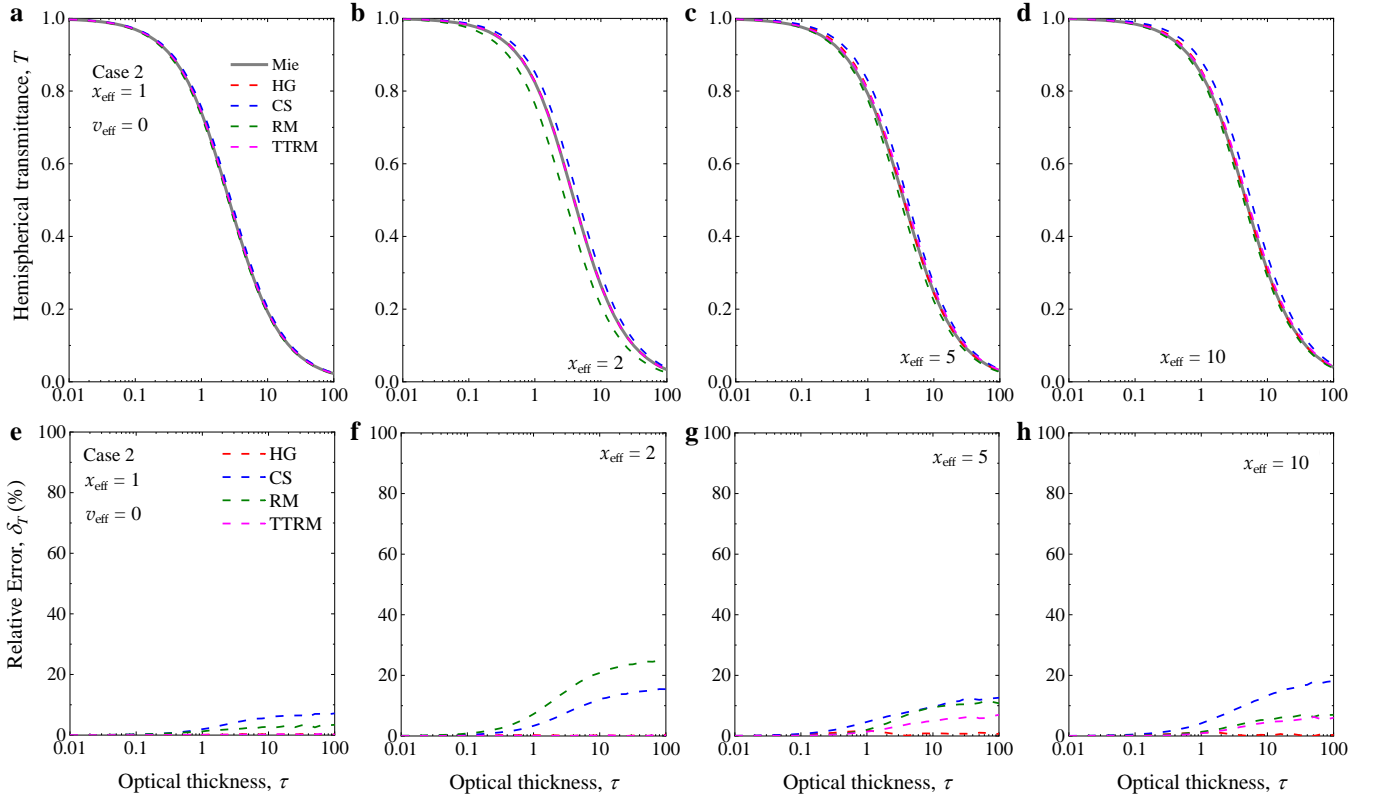
| Number | $x_{\text{eff}}$ | $v_{\text{eff}}$ | $g_1$  | $\alpha_1$ | $g_2$   | $\alpha_2$ | $\gamma$ | MSLE     |
|--------|------------------|------------------|--------|------------|---------|------------|----------|----------|
| Case 1 | 1.0              | 0                | 0.3001 | -3.7705    | -0.7618 | -3.3107    | 0.554    | 1.63E-07 |
|        | 2.0              | 0                | 0.2458 | 5          | -0.3433 | -5         | 0.422    | 6.96E-04 |
|        | 5.0              | 0                | 0.5202 | 5          | -0.7342 | -2.088     | 0.9693   | 1.07E-01 |
|        | 10.0             | 0                | 0.8626 | 1.2225     | -0.5429 | 5          | 0.9998   | 6.82E-02 |
|        | 1.0              | 0.05             | 0.3684 | -1.9561    | -0.5129 | -4.9581    | 0.4633   | 2.34E-06 |
|        | 2.0              | 0.05             | 0.2205 | 5          | -0.1436 | 4.9989     | 0.9859   | 3.99E-04 |
|        | 5.0              | 0.05             | 0.5634 | 3.3562     | -0.0561 | -5         | 0.98     | 2.00E-03 |
|        | 10.0             | 0.05             | 0.8249 | 1.5363     | -0.1098 | -5         | 0.9933   | 2.83E-03 |
| Case 2 | 1.0              | 0                | 0.3885 | -2.4812    | -0.5875 | -3.9237    | 0.3406   | 2.36E-06 |
|        | 2.0              | 0                | 0.2537 | 5          | -0.3736 | -1.6799    | 0.6003   | 1.76E-03 |
|        | 5.0              | 0                | 0.8195 | 0.3868     | -0.9275 | -0.3675    | 0.7516   | 5.14E-02 |
|        | 10.0             | 0                | 0.9414 | -0.061     | -0.7352 | 5          | 0.964    | 5.26E-02 |
|        | 1.0              | 0.05             | 0.3507 | -2.8393    | -0.4119 | -4.3149    | 0.1749   | 1.02E-05 |
|        | 2.0              | 0.05             | 0.3993 | 1.227      | -0.8373 | -0.6207    | 0.9184   | 6.45E-04 |
|        | 5.0              | 0.05             | 0.8306 | 0.0467     | -0.7966 | 0.2724     | 0.9039   | 2.43E-03 |
|        | 10.0             | 0.05             | 0.9334 | -0.0514    | -0.7927 | 1.5957     | 0.9573   | 4.64E-03 |
| Case 3 | 1.0              | 0                | 0.6573 | -2.99      | -0.6759 | -3.2291    | 0.1554   | 1.18E-06 |
|        | 2.0              | 0                | 0.0537 | 5          | -0.4674 | 5          | 0.9391   | 1.56E-02 |
|        | 5.0              | 0                | 0.8625 | 0.0731     | -0.8097 | -0.1295    | 0.7971   | 2.33E-02 |
|        | 10.0             | 0                | 0.8799 | 0.7397     | -0.6724 | -1.308     | 0.6659   | 4.02E-02 |
|        | 1.0              | 0.05             | 0.0944 | 4.8527     | -0.1624 | 4.9959     | 0.8934   | 1.82E-05 |
|        | 2.0              | 0.05             | 0.5923 | 0.0313     | -0.7805 | -0.5187    | 0.7695   | 4.81E-04 |
|        | 5.0              | 0.05             | 0.7001 | 1.2427     | -0.9421 | -0.8212    | 0.6329   | 1.54E-03 |
|        | 10.0             | 0.05             | 0.8882 | 0.6345     | -0.9906 | -0.8885    | 0.7106   | 4.32E-03 |
| Case 4 | 1.0              | 0                | 0.5245 | -2.8452    | -0.5333 | -3.6624    | 0.5276   | 3.44E-06 |
|        | 2.0              | 0                | 0.4002 | 0.7822     | -0.3784 | 5          | 0.9541   | 2.59E-03 |
|        | 5.0              | 0                | 0.605  | 2.9202     | -0.9687 | -1.2453    | 0.6019   | 2.63E-02 |
|        | 10.0             | 0                | 0.8812 | 0.8486     | -0.0107 | -5         | 0.6108   | 2.17E-02 |
|        | 1.0              | 0.05             | 0.9987 | -2.4576    | -0.2785 | -4.9332    | 0.4974   | 2.89E-05 |
|        | 2.0              | 0.05             | 0.5687 | 0.0779     | -0.449  | 5          | 0.9896   | 6.20E-04 |
|        | 5.0              | 0.05             | 0.7524 | 0.831      | -0.9925 | -0.9487    | 0.6826   | 7.77E-04 |
|        | 10               | 0.05             | 0.878  | 0.7849     | -0.0122 | -4.9999    | 0.613    | 1.96E-03 |



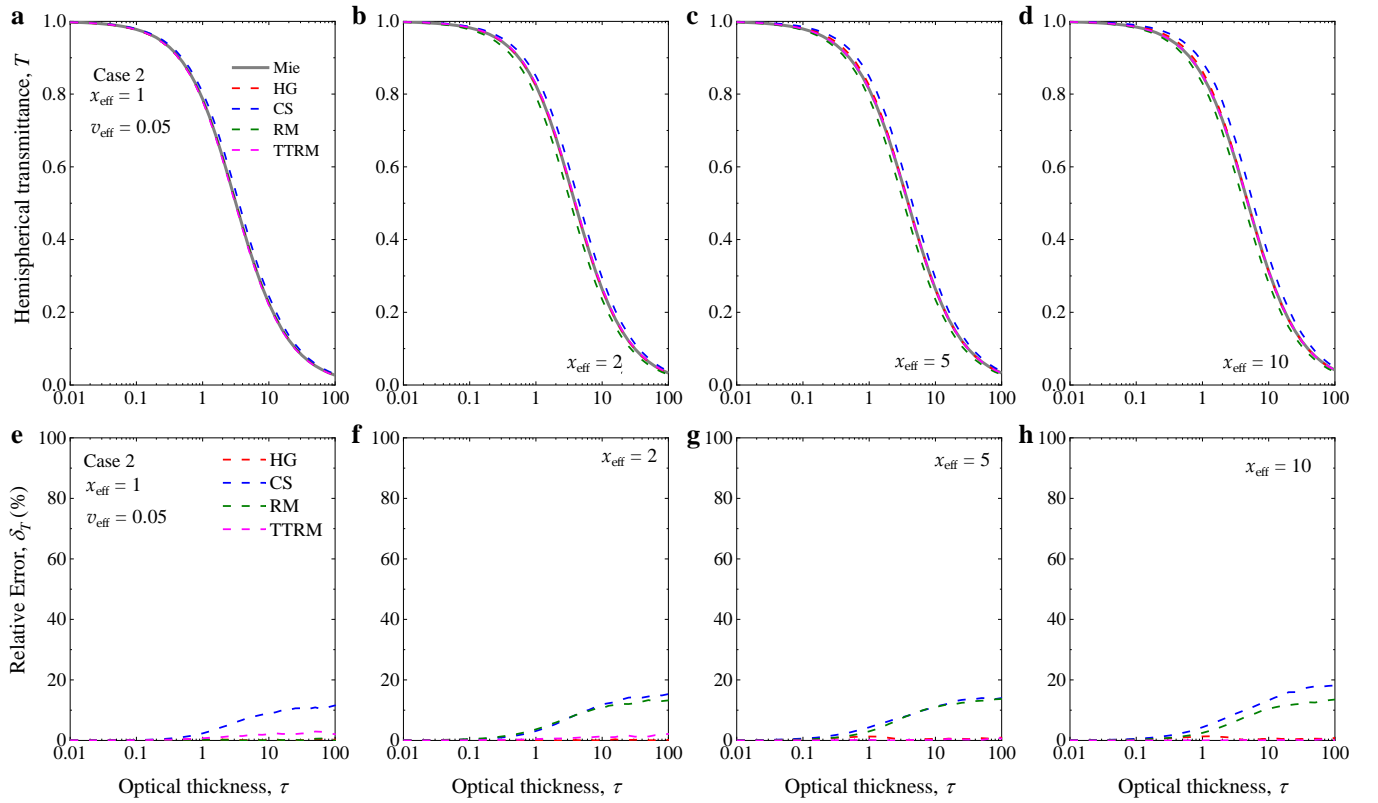
**Figure S1.** The hemispherical transmittance of the layers (Case 1,  $\text{SiO}_2$  particles,  $v_{\text{eff}} = 0$ ) versus the layer optical thickness for different phase function models, and the relative errors of the hemispherical transmittance caused by using four phase function empirical models.



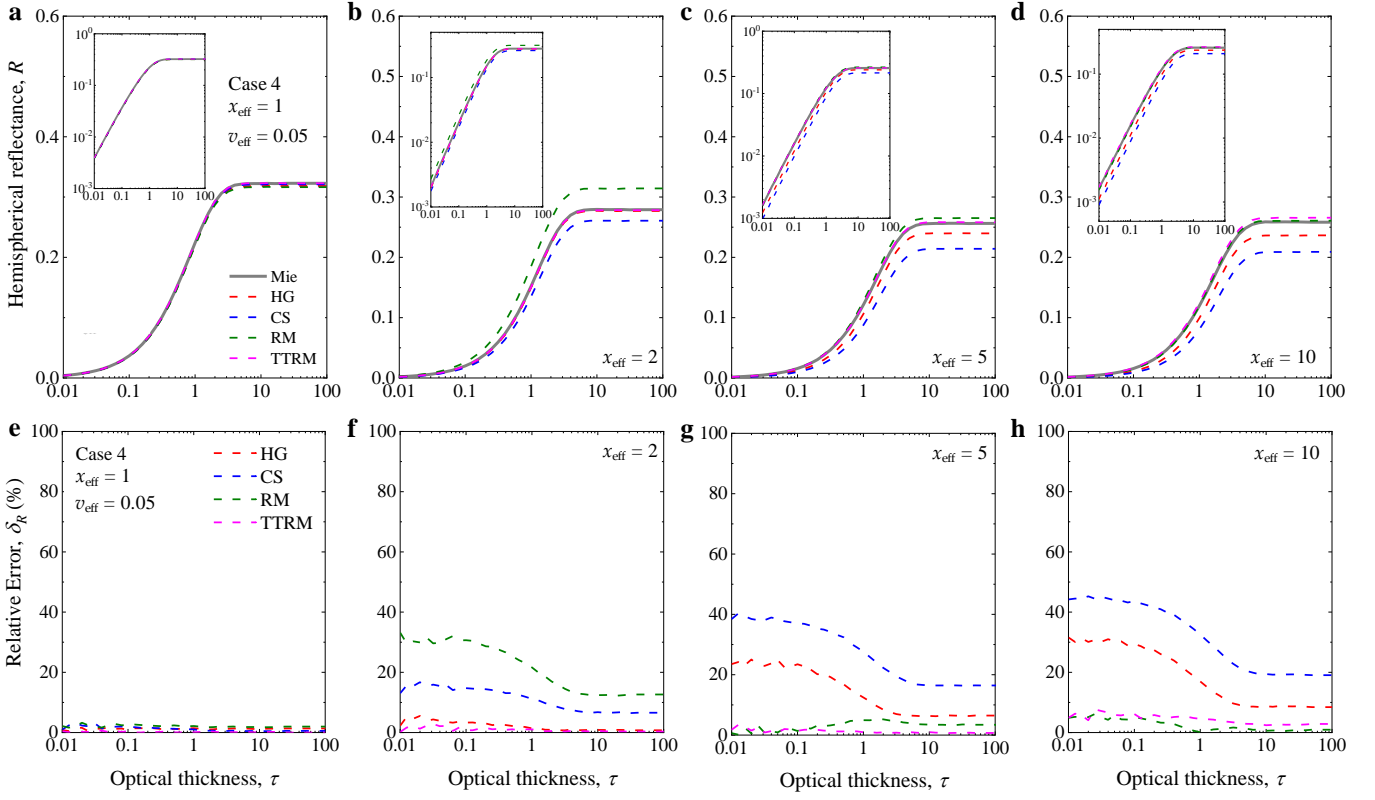
**Figure S2.** As in Figure S1, but for effective variance  $v_{\text{eff}} = 0.05$ .



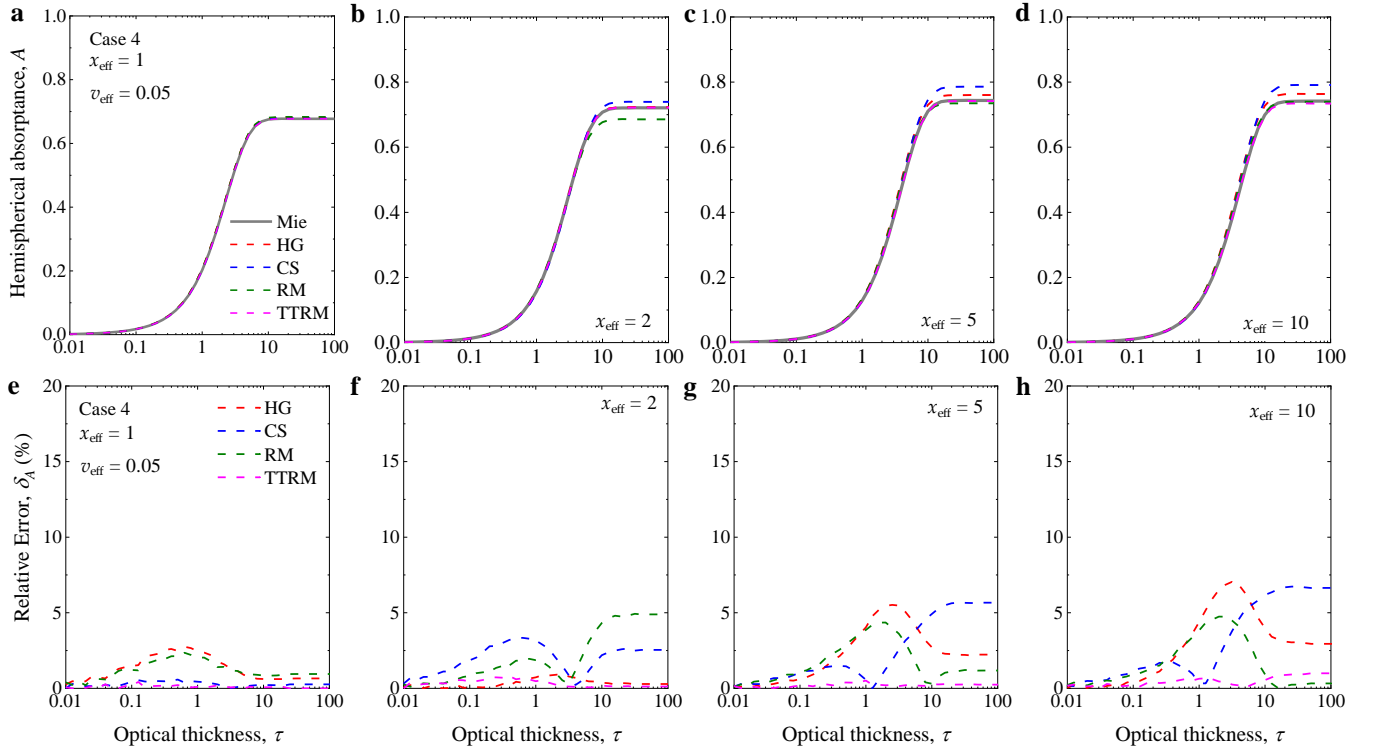
**Figure S3.** The hemispherical transmittance of the layers (Case 2,  $\text{TiO}_2$  particles,  $v_{\text{eff}} = 0$ ) versus the layer optical thickness for different phase function models, and the relative errors of the hemispherical transmittance caused by using four phase function empirical models.



**Figure S4.** As in Figure S3, but for effective variance  $v_{\text{eff}} = 0.05$ .



**Figure S5.** The hemispherical reflectance of the layers (Case 4, Au particles,  $v_{\text{eff}} = 0.05$ ) versus the layer optical thickness for different phase function models, and the relative errors of the hemispherical transmittance caused by using four phase function empirical models.



**Figure S6.** The total absorbance of the layers (Case 4, Au particles,  $v_{\text{eff}} = 0.05$ ) versus the layer optical thickness for different phase function models, and the relative errors of the absorbance caused by using four phase function empirical models.