

# Supplementary Material: Silver(I) Recovery by Ion Flotation Process from Aqueous Solutions in Cells with Spargers

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Table S1. Calibration of the peristaltic feed.

Area of the cell = 361.00 cm <sup>2</sup>				
Pump Scale	Time to Feed 11.913 lts	lts/min	cm <sup>3</sup> /s	Jl cm/s
1.00	17.50	0.77	12.86	0.04
2.00	9.03	1.50	24.92	0.07
3.00	6.26	2.16	35.94	0.10
4.00	5.22	2.59	43.10	0.12
5.00	3.51	3.85	64.10	0.18
6.00	3.08	4.38	73.05	0.20
7.00	2.40	5.63	93.75	0.26
8.00	2.20	6.14	102.27	0.28
9.00	2.05	6.59	109.76	0.30
10.00	1.55	8.71	145.16	0.40

Table S2. Calibration of the peristaltic discharge pump (tails).

Area of the cell = 361.00 cm <sup>2</sup>				
Pump Scale	Time to Down- load 11.913 lts	lts/min	cm <sup>3</sup> /s	Jl cm/s
1.00	15.10	0.89	14.90	0.04
2.00	8.03	1.68	28.02	0.08
3.00	5.19	2.60	43.35	0.12
4.00	4.02	3.36	55.97	0.16
5.00	3.23	4.18	69.66	0.19
6.00	2.47	5.47	91.09	0.25
7.00	2.28	5.92	98.68	0.27
8.00	2.11	6.40	106.64	0.30
9.00	2.00	6.75	112.50	0.31
10.00	1.45	9.31	155.17	0.43

Table S3. Calculation of gas surface velocity J<sub>g</sub> (cm/s).

Area of the cell = 361 cm <sup>2</sup>		
J <sub>g</sub> cm/s	Q <sub>g</sub> cm <sup>3</sup> /s	Q <sub>g</sub> lts/min
0.1	36.1	2.166
0.3	108.3	6.498
0.5	180.5	10.83
0.8	288.8	17.328
1	361	21.66
1.3	469.3	28.158
1.5	541.5	32.49
1.8	649.8	38.988

2	722	43.32
2.3	830.3	49.818
2.5	902.5	54.15
2.8	1010.8	60.648
3	1083	64.98

### Bubble Diameter Estimation Using the Drift Flux Model

The concept of drag analysis was originally introduced by Wallis to relate phase flow velocities, trapped gas and physical properties [30–33].

From the entrainment analysis, the terminal bubble slip velocity is estimated using the relationship for two-phase systems. The main expression of the entrainment analysis is as follows [30–33]:

$$U_t = \frac{Jg}{\epsilon_g} \pm \frac{Jl}{(1-\epsilon_g)} \quad (1)$$

This equation is derived from the following analysis.

$$Qt = Qg \pm Ql \quad (2)$$

$$\frac{Qt}{V} = \frac{Qg}{Vg} \pm \frac{Ql}{Vl} \quad (3)$$

$$V = Ah; Vg = Ahg; Vl = Ah \quad (4)$$

$$\frac{Qt}{Ah} = \frac{Qg}{Ahg} \pm \frac{Ql}{Ahl} \quad (5)$$

$$\frac{Jt}{h} = \frac{Jg}{hg} \pm \frac{Jl}{hl} \quad (6)$$

$$Jt = \left( \frac{Jg}{hg} \pm \frac{Jl}{hl} \right) \times h \quad (7)$$

$$Jt = \frac{Jgh}{hg} \pm \frac{Jlh}{hl} = \frac{Jg}{hgh^{-1}} \pm \frac{Jl}{hlh^{-1}} \quad (8)$$

$$Jt = \frac{Jg}{h} \pm \frac{Jl}{h} \quad (9)$$

$$Jt = Ut; \frac{h}{h} = Eg; \frac{hl}{h} = El \quad (10)$$

Where El = Fraction of liquid = (1-Eg)

$$Ut = \frac{Jg}{Eg} \pm \frac{Jl}{(1-Eg)} \quad (11)$$

The Drift Flux estimation method cited in the literature follows the following procedure [30-33]:

1. Estimate a value of  $m$  in 3
2. Calculate the terminal velocity of a bubble Equation (1).
3. Calculate the Reynolds number ( $Re_s$ ) of the bubble cloud and the bubble diameter ( $D_b$ ) with expressions (12) and (13), the calculation is carried out by first estimating a value of 1.

From experimental observations it has been determined that  $m$  is close to 3 in all cases. Therefore, the carry-over analysis can be solved interactively from this procedure.

$$U_{sb} = \frac{J_g}{\epsilon_g (1 - \epsilon_g)^m} - \frac{J_g - J_l}{(1 - \epsilon_g)^m} \quad (12)$$

$$Re_s = \frac{D_b * U_s * \rho l * (1 - \epsilon_g)}{\mu l} \quad (13)$$

The bubble diameter is approximated by the ratio:

$$D_b = \left( \frac{18 * \mu l * U_t}{g * \Delta \rho} (1 + 0.15 Re_s)^{0.687} \right)^{0.5} \quad (14)$$

Where  $\rho l = 1 \text{ gr/cm}^3$ ;  $\mu l = 0.01 \text{ gr/cms}$ ;  $g = 981.6 \text{ cm/s}^2$

Once the bubble diameter ( $D_b$ ) is obtained, this value is substituted by the estimated value in the calculation of the Reynolds number ( $Re_s$ ), and is substituted until it is equal to the estimated value. Once equalized, the value of  $m$  is calculated with the following expression:

$$m = \left( 4.45 + 18 * \frac{D_b}{D_c} \right) (Re_b)^{-0.1} \quad 1 < Re_s < 200 \quad (15)$$

Where:

$$Re_b = \frac{D_b * U_t * \rho l}{\mu l} \quad (16)$$

The obtained value of  $m$  is substituted in the calculation of equation 12 where it was estimated in a value of 3. Once this value is substituted, the calculated value is adjusted again, interactively with the value of  $m$ , and so on until the estimated value is equal to the calculated value.