

# Investigating the Performance of the Multi-Lobed Leaf-Shaped Oscillatory Obstacles in Micromixers Using Bulk Acoustic Waves (BAW): Mixing and Chemical Reaction

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## Governing equations:

For the microfluidic geometries, which comprise the flow of fluids, the equations of motion for a laminar regime (continuity and momentum) are defined as equations (1) and (2):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \quad (1)$$

$$\rho \frac{\partial v}{\partial t} + \rho(v \cdot \nabla)v = -\nabla p + \mu \nabla^2 v + \left(\mu_b + \frac{1}{3}\mu\right) \nabla(\nabla \cdot v) \quad (2)$$

where the parameter  $\rho$  is density,  $v$  is velocity,  $p$  is the pressure of the fluid,  $\mu$  is the dynamic viscosity, and  $\mu_b$  is the bulk viscosity of the fluid [1]. Additionally, the pressure in the fluid can be defined as a function of sound speed in liquid (equation 3) in which  $c_0^2$  is the speed of sound in the liquid [2].

$$p = c_0^2 \rho \quad (3)$$

Since it occurs in the mass transfer system, the equation for representing mass transport can be written as equation (4), where  $c_i$  is the concentration of species  $i$ ,  $J_i$  represents molecular mass flux, and  $R_i$  is the expression for reaction rate.

$$\frac{\partial c_i}{\partial t} + \nabla \cdot J_i + v \cdot \nabla c_i = R_i \quad (4)$$

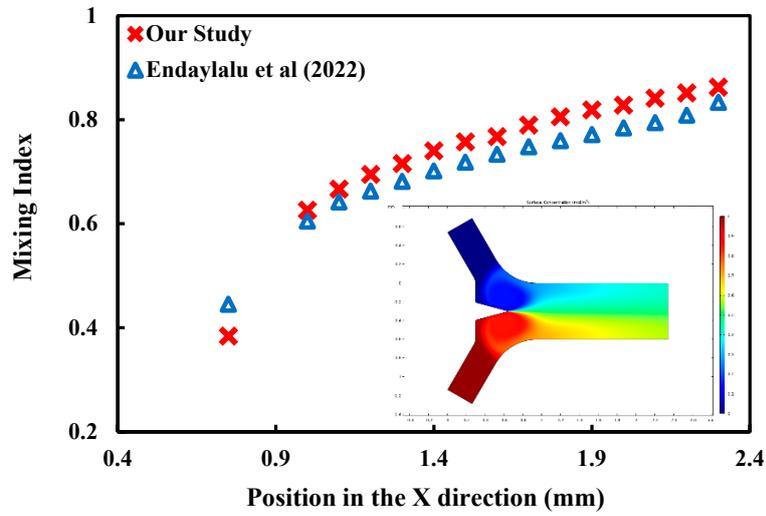
For the chemical reaction, we considered a bimolecular irreversible elementary chemical reaction with a variable reaction rate constant as shown in equations (5) and (6).



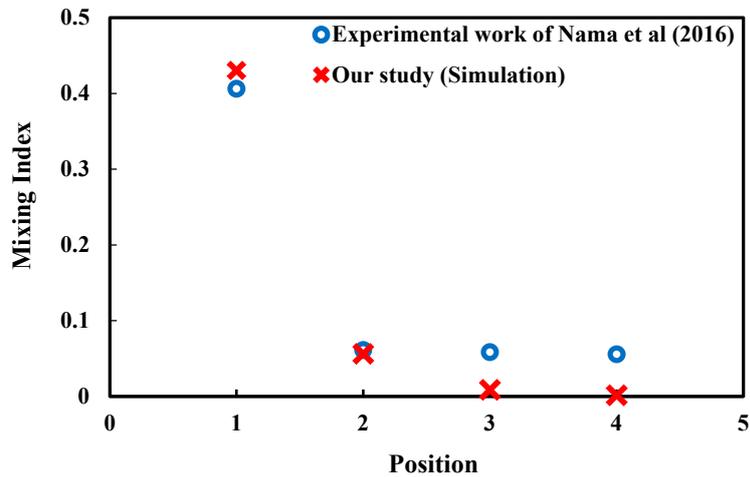
$$R_i = kC^a C^b \quad (6)$$

By considering appropriate boundary conditions and combining the above-mentioned equations, the movement of fluid in the microfluidic device can be solved numerically.

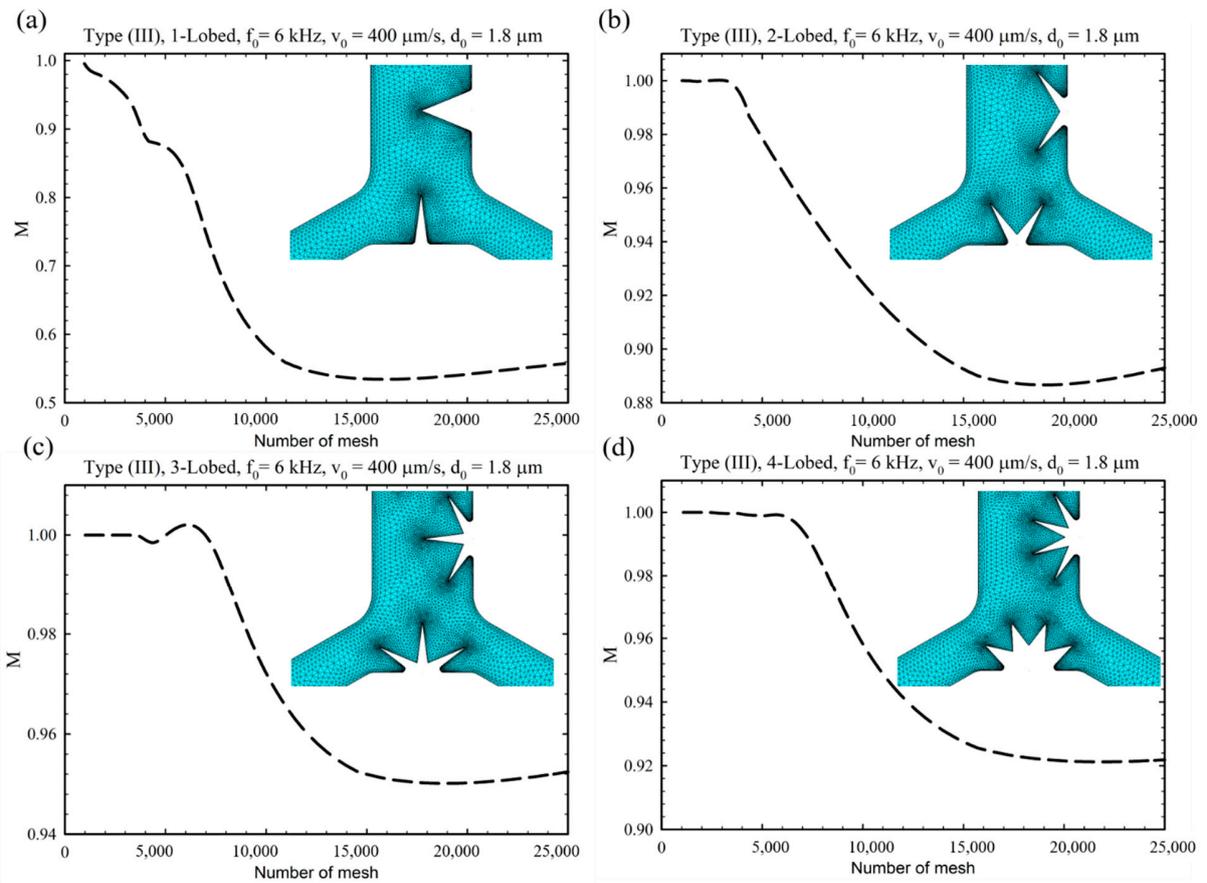
(a)



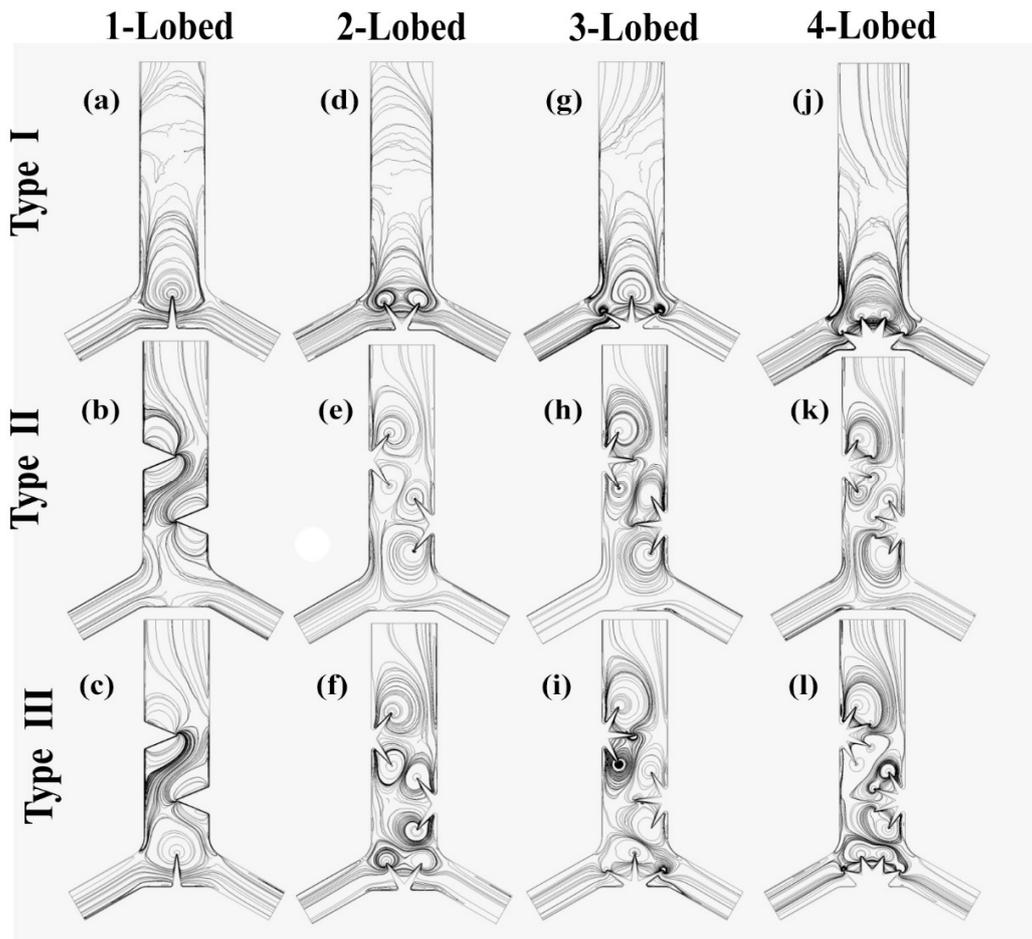
(b)



**Figure S1.** (a) Verification of our simulation vs the numerical work of Endaylalu and Tien (2022) [3] mean error  $\approx 5\%$  (inlet velocity =  $55.6 \mu\text{m/s}$ ) (MI = 1 represents perfect mixing). (b) The validation of our simulation results with the experimental works of Nama et. al at different length positions of microchannel ( $Q_{in} = 2 \mu\text{l/min}$ ), position 1:  $y = 1100 \mu\text{m}$ , position 2:  $y = 1700 \mu\text{m}$ , position 3:  $y = 2300 \mu\text{m}$ , and position 4:  $y = 2900 \mu\text{m}$  (MI = 0 represents perfect mixing and MI = 0.5 represents unmixed fluids) [4].

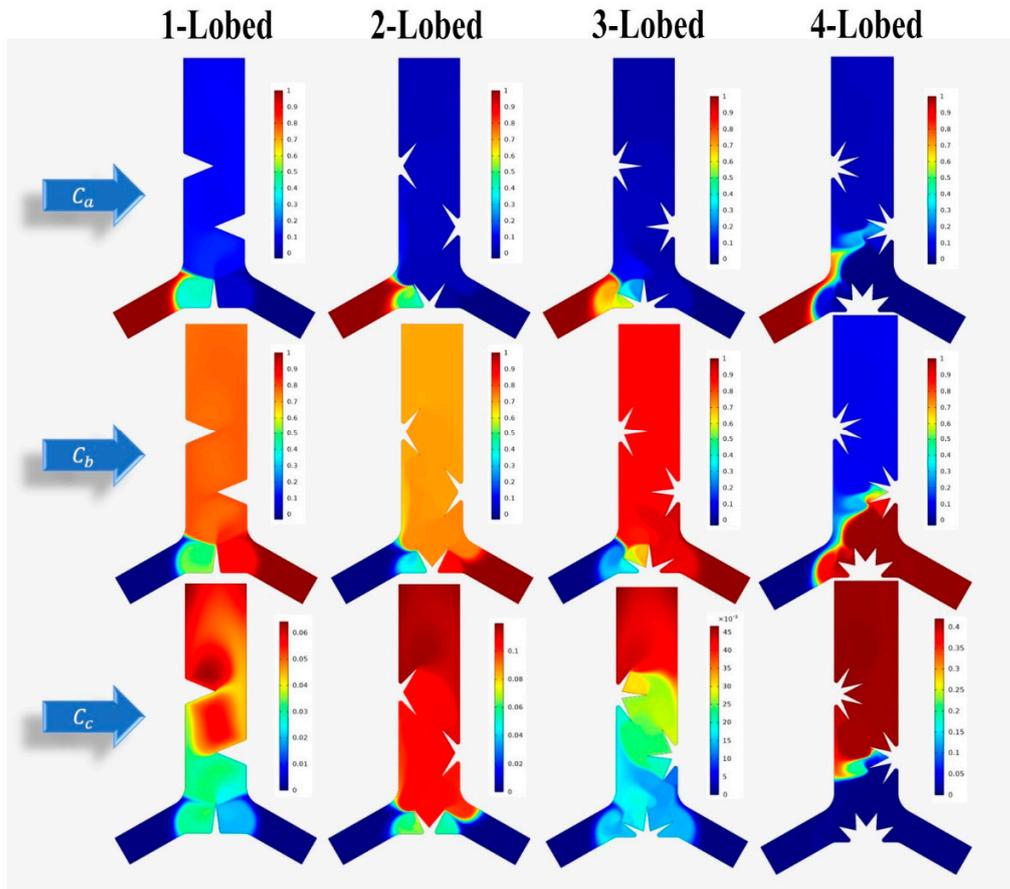


**Figure S2.** The graph for illustrating the mesh independent study. (a) 1-lobed, (b) 2-lobed, (c) 3-lobed, and (d) 4-lobed structures.

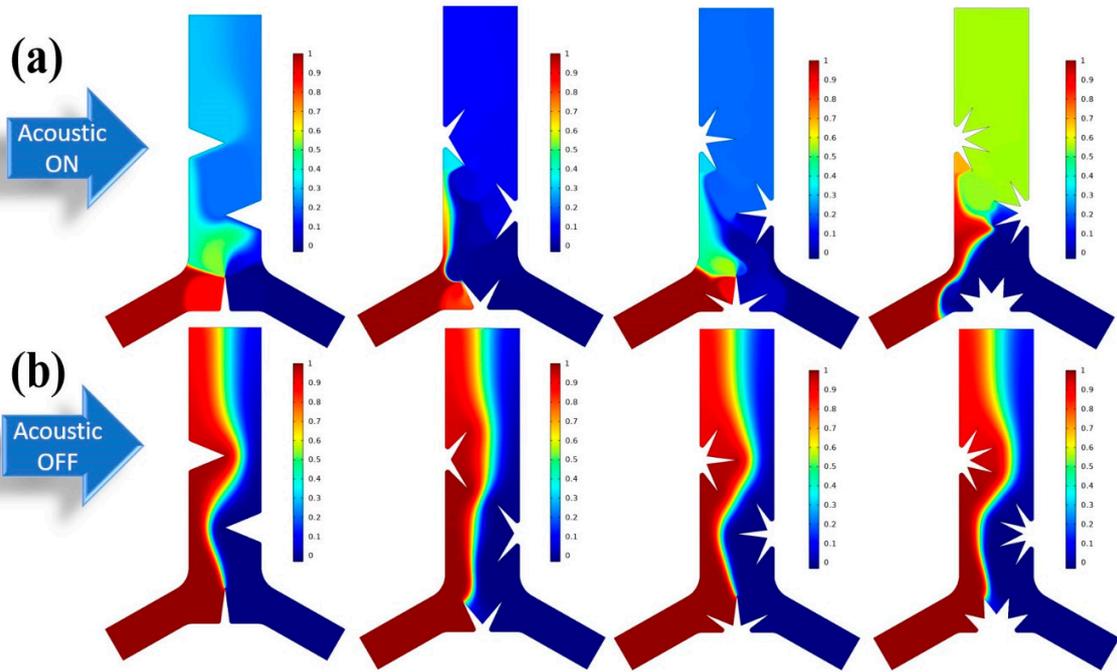


**Figure S3.** The illustration of generated vortices around oscillatory lobes for different structures and configurations. **(a,b,c)** 1-lobed structure in Type (I, II, and III) configurations, **(d,e,f)** 2-lobed structure in Type (I, II, and III) configurations, **(g,h,i)** 3-lobed structure in Type (I, II, and III) configurations, and **(j,k,l)** 4-lobed structure in Type (I, II, and III) configurations.

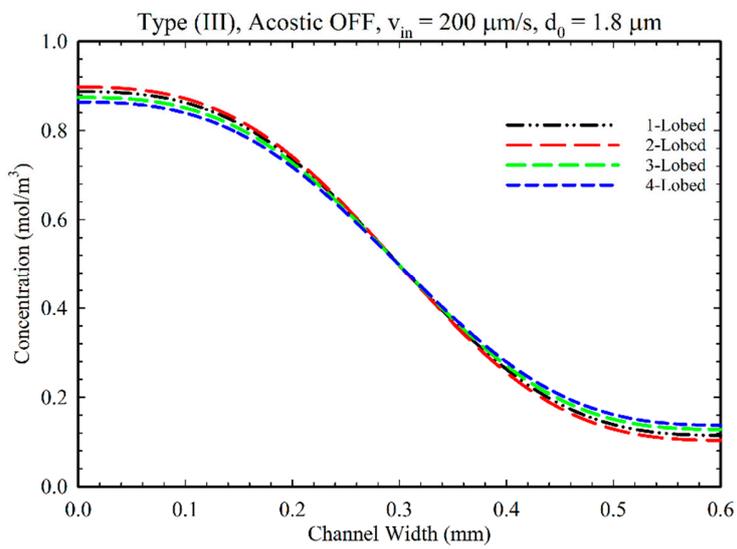
### Type III



**Figure S4.** The concentration distribution of reactants ( $C_a$  and  $C_b$ ) and products ( $C_c$ ) of chemical reaction through acoustic microchannel for different multi-lobed and configurations.



**Figure S5.** The distribution of solute concentration by considering (a) enabled and (b) disabled acoustic waves.



**Figure S6.** The concentration profile of solute along the microchannel width under disabled BAW conditions (Type III configuration, 1, 2, 3, 4-lobed structures, inlet velocity =  $200 \mu\text{m/s}$ )

## Reference

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