Appendix 3. Revised task and rubric by group 1

Read the following paragraph and answer the questions.

Suppose a continuous function on an arbitrary domain, such as a radical function like $y = 4\sqrt{x}$. We know that it could be easier to deal with polynomials such as quadratics or cubics than to deal with a radical function, and the method of approximation may be useful to model some situations. For example, an approximation of $y = 4\sqrt{x}$ can be $y = -x^2 + 4x + 1$ over any particular interval of the domain. Both functions have the same y-value, 4, at x = 1, and their function values are close to each other in the neighborhood of x = 1. Hence, both functions share similar properties. In addition, the radical function $y = 4\sqrt{x}$ can be approximated using a cubic function. It is known that the cubic function is even closer to the $y = 4\sqrt{x}$ and shares more similar properties than the quadratic function does.

A bus company was concerned about the profit margin from the timetable (from 6 am to 9 am) for a bus passing on route A three years ago. If the time interval between buses is too long, the number of passengers may decrease, resulting in a decrease in revenue. If the time interval is too short, the operating cost increases as the number of buses is dispatched, and thus revenue may decrease. Based on this, the profit function of the time interval was approximated by $f(t) = -(t-10)^2 + 300$ where t stands for the time interval in minutes. The company can maximize its profit to 300 (ten-thousand won) when t = 10, and so they have scheduled vehicles every 10 minutes for the last three years. (Let's assume that only the number of passengers and operating costs affect revenue.)

However, some staff suggests using the cubic function approximation method for a more accurate approximation than the quadratic function approximation. For example, if the time interval decreases from 10 minutes to −1 minute and becomes 9 minutes, a cubic function may produce an improved approximation.

This bus company accepted this employee's opinion and after investigating relevant factors, it is expected that when the time interval changes by $\boldsymbol{x}\ (\boldsymbol{-10 < x \leq 0})$ from 10 minutes during rush hours, the expected profit will change by $\mathbf{g}(\boldsymbol{x}) = \boldsymbol{\frac{9}{8}x^3 - \frac{9}{8}x^2 - 63x}$. When the time interval changes by $x\ (0 \leq x < 170)$ from 10 minutes, the expected profit will change by $h(x)$. For example, if the time interval between buses is reduced by −2 minutes, the expected profit of $g(-2) = \frac{9}{8} \times (-2)^3 - \frac{9}{8} \times (-2)^2 - 63 \times (-2) = 112.5$ ten-thousand won will change, so a profit of 300 + 112.5 = 421.5 ten-thousand won can be expected. In this case, the graph of the fucntion h(x) in the interval of $[0, 170)$ is a part of the graph of the cubic function.

[1-1] The following table shows a part of a chart that an employee of this company used to find $\boldsymbol{h(x)}$ on an interval [0,170). Find $h(x)$.

x	1	4
$h(x)$	80	140
$h'(x)$	80	

[1-2] For any constant $a\ (0 < a < 170)$ we have $h'(a) > 0$. When the time interval between vehicles has changed by a minutes from 10 minutes, what happens to the profit if the time interval gets greater than $(10 + a)$? Explain what happens, including its mathematical meaning.

[2-1] Construct a table of the increase and decrease in y-values for each of the $y = g(x)$ defined on the interval $(-10, 0)$ and $y = h(x)$ defined on the interval $\boldsymbol{[0, 170)}$. Using the table, graph the two functions $y = g(x)$ and $y = h(x)$. (Note: There is no need to find the second derivative.)

[2-2] Considering this particular bus line, find the range of time intervals between vehicles during rush hours that would generate more profit than the last three years.

[2-3] Building on your answer to question [2-2] find the time interval between vehicles during rush hours that will maximize profits.

	score	criteria in the revised rubric of [1-1]
[1−1]	0	A blank or meaningless statement
	1	$h(x) = ax^3 + bx^2 + cx + d$ is denoted
	2	A system of three equations is provided $a + b + c + d = 80 \quad 3a + 2b + c = 80 \quad 64a + 16b + 4c + d = 140$
	3	A system of four equations by substituting $h(0) = 0$ is provided
	4	$h(t)$ was not found correctly; a, b, c, d was obtained incorrectly due to calculation errors
	5	The system of equation is solved correctly without calculation errors; and $h(x) = -5x^3 + 10x^2 + 75x$ is provided

	score	criteria
[1−2]	0	A blank or unclear statement or incorrect descriptions
	2	The profit increases is mentioned but the mathematical meaning i.e., "when the time interval becomes larger than $(10 + a)$ minutes, the profit is greater than when it is $(10 + a)$ minutes" is not provided
	4	The profit increases is mentioned but the mathematical meaning i.e., "Since '$h'(a) > 0$, the function of h(x) increases in some interval containing $x = a$ and when the time interval becomes larger than $(10 + a)$ minutes, the profit is greater than when it is $(10 + a)$ minutes" is not provided

	score	criteria
[2−1]	0	A blank or unclear statement
	1	The derivative of $g(x)$, $g'(x) = \frac{27}{8}x^2 - \frac{9}{4}x - 63$ is provided
	2	A table shows the increase and decrease of $g(x)$, but the table has some errors
	3	A table shows the increase and decrease of $g(x)$ with no mathematical errors
	4	The graph of $g(x)$ is drawn correctly using the table

* $h(x)$ is scored in the same way with max. 4 points

	score	criteria
[2−2]	0	A blank or unclear statement
	1	$g(x) = \frac{9}{8}x(x-8)(x+7) > 0$ and the solution of $-7 < x < 0$ OR $h(x) = -5x(x+3)(x-5) > 0$ and the solution of $0 < x < 5$
	2	$g(x) = \frac{9}{8}x(x-8)(x+7) > 0$ and the solution of $-7 < x < 0$ AND $h(x) = -5x(x+3)(x-5) > 0$ and the solution of $0 < x < 5$
	3	For each inequality, the range of x is obtained correctly, but the range of the time interval expected to increase the profit is incorrect
	4	The description is accurate, i.e. "when the time interval is between 3 and 10 minutes or between 10 and 15 minutes, the profit increases."

	score	criteria
[2-3]	0	A blank or unclear statement
	1	The change in average revenue has a maximum value when $x = 3$ in the function of $h(x)$, but the description of the time interval is not clear
	2	The function of $h(x)$ has a maximum value when $x = 3$ When the time interval is 13 minutes, the average profit during the rush hour is maximum