mathematics

MDPI

Supplementary Material to

Potential-Growth Indicators Revisited: Higher Generality and Wider Merit of Indication

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Matlab functions *diopha21_1* and *diopha21_2* find all the solutions to the Diophantine systems of equations and inequalities (17), (21) and (18), (22) respectively for 9 and 13 integer-valued parameters. Below is the Matlab code for the more cumbrous *diopha21_2* (Colony *II*), the reduction to Colony *I* being obvious.

```
function [Pars, Lams] = diopha21_2
% finds all the solutions of the Diophantine system of equations&inequalities
% Colony 2 without excavation; hierarchy (6C) among parent Contributions
% reproductive-core 6x6 submatrix Lcor only (size L2 = 14x14)
% Output variables: Lams, vector of 2 λ1s extremal values over the set of solutions;
% Pars, 13x2 matrix of extremal parameters.
% @ Logofet 26.04.2021.
global ns % assign also in workspace
ns = 0;%number of solutions
Lcor=[0,    0,    0,    0,    0,    0;
   18/61,   0,    0,    0,    0,    0;
        0,  0,    0,    0,    0,    0;
   14/61,   0, 2/3,    0,    0,    0;
```

```
        0,    0,    0,    0,    0,    0;
   16/61,    0,    0,    0,    1,    0]; % reproductive-core 6x6 submatrix, without birth rates
  lam1min = 100; lam1max = 0;% unreal values to begin with
  a_min=0;b_min=0;c_min=0;d_min=0;e_min=0;f_min=0;g_min=0;h_min=0;k_min=0;l_min=0;m_min=0;n_min=0;o_min=0;
  a_max=0;b_max=0;c_max=0;d_max=0;e_max=0;f_max=0;g_max=0;h_max=0;k_max=0;l_max=0;m_max=0;n_max=0;o_max=0;
for a=122:178 a,% 122=207-(14+30+10+5+26),178=61*3-(1+1+1+1+1).
    for b=1:14% 1 due to 207-(178+30+10+5+26)<0, 14 when all the rhizomes go to v1.
        for c=1:29% 1 due to 207-(178+14+10+5+26)<0, 29=30-1.
            for d=1:9% 1 due to 207-(187+14+30+...)<0, 9=10-1.
                for e=1:4% similarly, 4= 5-1.
                    for f=1:24% similarly, 24= 26-(1+1).
                        for g=1:50% 1 due to 54-(29+9+4+24)<0, 50=54-(1+1+1+1).
                            for h=1:29% 1 due to 54-(50+9+4+24)<0, 29=30-1.
                                for k=1:9% 1 due to 54-(50+29+4+24)<0, 9=10-1.
                                    for l=1:4% similarly, 4=5-1.
                                        for m=1:24% similarly, 24=26-2.
                                            for n=1:6% 6 = 7-1.
                                                for o=1:6% 6 = 7-1. Hierarchy (6C): a+g+n ≤ c+h ≤ b ≤ d + k ≤ m + o
                                                    C1=a+g+n;C2=c+h;C3=f+m+o;C4=b;C5=d+k;C6=e+l;%for hierarchy (6C)
                                    if a+b+c+d+e+f)==207)&&((g+h+k+l+m)==54)&&((n+o)==7)&&(C1>=C2)&&(C2>=C3)&&(C3>=C4)&&(C4>=C5)&&(C5>=C6)
                                        ns = ns +1;% one more solution
                                        Lcor(1,1:8)=[a/61,b/8,0,c/6,d/6,0,e/1,f/6];
                                        Lcor(4,1:8)=[g/61,0,0,h/6,k/6,0,l/1,m/6];%'el'/1
                                        Lcor(7,1:8)=[n/61,0,0,0,0,0,0,o/6];% assigning the birth rates
                                        lam1 = max(eig(Lcor));
                                        if (lam1 < lam1min), lam1min = lam1;
                                            a_min = a;b_min = b;c_min = c;d_min = d;e_min = e;f_min = f;g_min = g;h_min = h;
```

```
                                    k_min = k;l_min = l;m_min = m;n_min = n;o_min = o; end
                                if (lam1 > lam1max), lam1max = lam1;
                                    a_max = a;b_max = b;c_max = c;d_max = d;e_max = e;f_max = f;g_max = g;h_max = h;
                                    k_max = k;l_max = l;m_max = m;n_max = n;o_max = o;end
                            end
                                        end
                                    end
                                end
                            end
                        end
                    end
                end
            end
        end
    end
end
ns %the number of solutions
Lams = [lam1min, lam1max];
Pars = [a_min b_min c_min d_min e_min f_min g_min h_min k_min l_min m_min n_min o_min]';
Pars = [Pars, [a_max b_max c_max d_max e_max f_max g_max h_max k_max l_max m_max n_max o_max]'];
```

Below is a Matlab expression for $R_{RT}(\boldsymbol{L}_{cor2})$ as a function of 13 parameters a, b, …, n, o:

```
max([a/61 + (9*b)/244,
a/61 + (9*b)/244 + (7*d)/183 + h/6 + k/9 - (a*h)/366 - (3*b*h)/488 + (c*g)/366 + (d*g)/549 - (a*k)/549 - (7*d*h)/1098 - (b*k)/244 + (7*c*k)/1098,
```

a/61 + (9*b)/244 + (7*d)/183 + (8*f)/183 + h/6 + k/9 + o/6 - (a*h)/366 - (3*b*h)/488 + (c*g)/366 + (d*g)/549 - (a*k)/549 - (7*d*h)/1098 - (b*k)/244 + (7*c*k)/1098 - (4*f*h)/549 - (a*o)/366 + (4*c*m)/549 - (3*b*o)/488 + (8*d*m)/1647 - (8*f*k)/1647 - (7*d*o)/1098 + (e*n)/61 + (8*e*o)/183 + (f*n)/366 - (h*o)/36 - (k*o)/54 + (a*h*o)/2196 + (b*h*o)/976 - (c*g*o)/2196 - (d*g*o)/3294 + (a*k*o)/3294 + (7*d*h*o)/6588 - (e*h*n)/366 + (b*k*o)/1464 - (4*e*h*o)/549 - (f*h*n)/2196 - (c*k*o)/94 + (c*l*n)/366 + (4*c*l*o)/549 + (c*m*n)/2196 + (d*l*n)/549 - (e*k*n)/549 + (8*d*l*o)/1647 + (d*m*n)/3294 - (8*e*k*o)/1647 - (f*k*n)/3294,
a/61 + (9*b)/244 + (7*d)/183 + h/6 + k/9 - (a*h)/366 - (3*b*h)/488 + (c*g)/366 + (d*g)/549 - (a*k)/549 - (7*d*h)/1098 - (b*k)/244 + (7*c*k)/1098 + (e*n)/61 - (e*h*n)/366 + (c*l*n)/366 + (d*l*n)/549 - (e*k*n)/549, a/61 + (9*b)/244 + h/6 - (a*h)/366 - (3*b*h)/488 + (c*g)/366,
a/61])