Supplementary Material

Likelihood Inference for the Marshall–Olkin Bivariate Weibull Model with Modified Singularity under Exact and Rounded Data

Hugo Brango Javier Ramirez-Montoya

This supplement reports the direct regular-parameter comparison on the retained first 1000 replications. No new Monte Carlo estimation run was substituted for the original experiment. The finite point estimates and RMSE/bias summaries were checked against the archived replication-level objects. The same samples underlie both methods. Uncertainty denominators differ because each method applies its own numerical curvature rule. The midpoint intervals condition on the selected pseudo-support; neither their nominal level nor interval-aware positive curvature establishes unconditional coverage.

Table S1: Bias, root mean squared error (RMSE), and conditional coverage of nominal 95% Wald intervals for the rate and shape parameters. I: interval-aware estimator; M: midpoint estimator. Each method uses the same 1000 simulated samples per scenario, with finite point estimates in all replications. Coverage is calculated only over the H replications with available standard errors; H is reported in parentheses.

p_0	r	Parameter	Bias I	Bias M	RMSE I	RMSE M	Cov. I (H)	Cov. M (H)
0.10	0.02	β	0.0082	0.0105	0.0555	0.0570	0.945 (293)	0.944 (1000)
0.10	0.02	λ_1	0.0063	0.2146	0.0831	0.2323	0.925 (293)	0.223 (1000)
0.10	0.02	λ_2	0.0055	0.1620	0.0775	0.1836	0.939 (293)	0.444 (1000)
0.10	0.02	λ_3	-0.0004	-0.2072	0.0518	0.2117	0.952 (293)	0.032 (1000)
0.10	0.05	β	0.0063	0.0102	0.0546	0.0548	0.955 (288)	0.949 (1000)
0.10	0.05	λ_1	-0.0011	0.1913	0.0846	0.2085	0.927 (288)	0.307 (1000)
0.10	0.05	λ_2	-0.0015	0.1255	0.0769	0.1483	0.931 (288)	0.647 (1000)
0.10	0.05	λ_3	0.0030	-0.1857	0.0535	0.1886	0.962 (288)	0.023 (1000)
0.10	0.10	β	0.0064	0.0122	0.0595	0.0589	0.925 (494)	0.930 (1000)
0.10	0.10	λ_1	-0.0009	0.1400	0.0872	0.1606	0.937 (494)	0.609 (1000)
0.10	0.10	λ_2	-0.0006	0.0618	0.0798	0.1001	0.929 (494)	0.885 (1000)
0.10	0.10	λ_3	0.0081	-0.1268	0.0607	0.1316	0.943 (494)	0.087 (1000)
0.10	0.20	β	0.0056	0.0104	0.0573	0.0551	0.950 (576)	0.957 (1000)
0.10	0.20	λ_1	-0.0093	0.0266	0.1133	0.0820	0.938 (576)	0.938 (1000)
0.10	0.20	λ_2	-0.0058	-0.0556	0.0964	0.0927	0.951 (576)	0.857 (1000)
0.10	0.20	λ_3	0.0186	-0.0155	0.0933	0.0450	0.941 (576)	0.920 (1000)
0.30	0.02	β	0.0072	0.0050	0.0607	0.0622	0.950 (605)	0.917 (1000)
0.30	0.02	λ_1	0.0058	0.8680	0.1067	0.8776	0.942 (605)	0.000 (1000)
0.30	0.02	λ_2	0.0047	0.6473	0.0959	0.6576	0.934 (605)	0.000 (1000)
0.30	0.02	λ_3	0.0047	-0.8393	0.1015	0.8410	0.952 (605)	0.000 (1000)
0.30	0.05	β	0.0069	0.0154	0.0589	0.0612	0.958 (550)	0.923 (1000)
0.30	0.05	λ_1	0.0002	0.8666	0.1045	0.8766	0.956 (550)	0.000 (1000)
0.30	0.05	λ_2	0.0049	0.6063	0.0945	0.6170	0.944 (550)	0.000 (1000)
0.30	0.05	λ_3	0.0108	-0.8184	0.1059	0.8197	0.949 (550)	0.000 (1000)
0.30	0.10	β	0.0079	0.0342	0.0617	0.0692	0.950 (676)	0.893 (1000)
0.30	0.10	λ_1	0.0020	0.7776	0.1096	0.7877	0.950 (676)	0.000 (1000)
0.30	0.10	λ_2	0.0000	0.4378	0.0991	0.4538	0.948 (676)	0.017 (1000)
0.30	0.10	λ_3	0.0008	-0.6995	0.1023	0.7024	0.954 (676)	0.000 (1000)
0.30	0.20	β	0.0033	0.0392	0.0599	0.0697	0.957 (819)	0.909 (1000)
0.30	0.20	λ_1	0.0031	0.5450	0.1161	0.5580	0.957 (819)	0.002 (1000)
0.30	0.20	λ_2	0.0031	0.1711	0.1014	0.2023	0.958 (819)	0.670 (1000)
0.30	0.20	λ_3	0.0062	-0.4447	0.1129	0.4521	0.944 (819)	0.001 (1000)
0.50	0.02	β	0.0068	-0.0007	0.0626	0.0647	0.946 (672)	0.898 (1000)
0.50	0.02	λ_1	0.0037	1.9418	0.1419	1.9560	0.942 (672)	0.000 (1000)
0.50	0.02	λ_2	0.0018	1.4690	0.1143	1.4802	0.945 (672)	0.000 (1000)
0.50	0.02	λ_3	0.0179	-1.8439	0.1872	1.8486	0.961 (672)	0.000 (1000)
0.50	0.05	β	0.0076	0.0103	0.0634	0.0660	0.947 (659)	0.881 (1000)

p_0	r	Parameter	Bias I	Bias M	RMSE I	RMSE M	Cov. I (H)	Cov. M (H)
0.50	0.05	λ_1	0.0102	1.9849	0.1425	2.0027	0.935 (659)	0.000 (1000)
0.50	0.05	λ_2	0.0057	1.4212	0.1192	1.4336	0.942 (659)	0.000 (1000)
0.50	0.05	λ_3	0.0261	-1.8192	0.1909	1.8228	0.953 (659)	0.000 (1000)
0.50	0.10	β	0.0042	0.0479	0.0607	0.0775	0.959 (729)	0.858 (1000)
0.50	0.10	λ_1	0.0068	1.9932	0.1422	2.0069	0.955 (729)	0.000 (1000)
0.50	0.10	λ_2	0.0009	1.0679	0.1263	1.0889	0.937 (729)	0.000 (1000)
0.50	0.10	λ_3	0.0071	-1.6200	0.1877	1.6252	0.957 (729)	0.000 (1000)
0.50	0.20	β	0.0068	0.0770	0.0625	0.0986	0.962 (837)	0.752 (1000)
0.50	0.20	λ_1	0.0094	1.6087	0.1531	1.6235	0.949 (837)	0.000 (1000)
0.50	0.20	λ_2	0.0019	0.5228	0.1283	0.5533	0.947 (837)	0.142 (1000)
0.50	0.20	λ_3	0.0185	-1.0970	0.2013	1.1092	0.938 (837)	0.000 (1000)

Reproducible numerical verification

The revision includes `R/01_revision_checks.R` for the targeted computational benchmark and `R/02_revision_tables.R` for the comparison tables. Run them from the folder containing this source. They use base R and the included frozen v2.4.2 engine. The new benchmark uses seed 19102026; the original Monte Carlo study uses its archived seeds and first-1000 reporting convention. The original experiment is not rerun by these revision scripts.

The file `results/rate_benchmark.csv` contains all 60 sample-level rate comparisons. The file `results/probability_benchmark.csv` contains all 72 probability cases, errors and elapsed times. `results/regular_parameter_comparison.csv` includes empirical SD, mean SE and denominators as well as bias, RMSE and coverage. `results/sessionInfo.txt` records the benchmark environment. The original simulation-host environment was not retained in these summaries and is not inferred from the revision host.