Online Supplement to

"Multivariate Structural Equation Modeling Techniques for Estimating Reliability, Measurement Error, and Subscale Viability When Using Both Composite and Subscale Scores in Practice"

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Contents

1. Data preparation

Examples used in this supplement represent responses to Extraversion domain and facet scales from the Big Five Inventory (BFI-2; Soto & John, 2017) on two occasions. We used the *lavaan* package in R to perform multivariate structural equation model analyses and the *semTools* package to obtain Monte-Carlo confidence intervals for key parameters. Item numbers are the same as those used in the BFI-2.

```
library(lavaan)
library(semTools)
bfi2_ext <- read.csv("bfi2_ext_389.csv")
```

2. Variance Partitioning and Scale Viability for simplified essential tau-equivalent (S-ETE) multivariate designs

To obtain the results of variance partitioning and scale viability indices for S-ETE multivariate designs, the following formula code should be used as a model argument for the *lavaan* function. Within the formula code for each design, the number of occasions and items can be adjusted for alternative designs.

2.1 $p^{\bullet} \times I^{\circ} \times O^{\bullet}$ S-ETE multivariate design

The following code is used for variance partitioning and derivation of indices to access subscale added value for the simplified essential tau-equivalent (S-ETE) multivariate design.

```
model<-'

## Person factors
F1 =~ 1*T1_1+1*T1_16+1*T1_31+1*T1_46
      +1*T2_1+1*T2_16+1*T2_31+1*T2_46
F2 =~ 1*T1_6+1*T1_21+1*T1_36+1*T1_51
      +1*T2_6+1*T2_21+1*T2_36+1*T2_51
F3 =~ 1*T1_11+1*T1_26+1*T1_41+1*T1_56
      +1*T2_11+1*T2_26+1*T2_41+1*T2_56

## Occasion factors
OCC1_F1 =~ 1*T1_1+1*T1_16+1*T1_31+1*T1_46
OCC2_F1 =~ 1*T2_1+1*T2_16+1*T2_31+1*T2_46
OCC1_F2 =~ 1*T1_6+1*T1_21+1*T1_36+1*T1_51
OCC2_F2 =~ 1*T2_6+1*T2_21+1*T2_36+1*T2_51
OCC1_F3 =~ 1*T1_11+1*T1_26+1*T1_41+1*T1_56
OCC2_F3 =~ 1*T2_11+1*T2_26+1*T2_41+1*T2_56

## Item factors
ITEM1 =~ 1*T1_1 + 1*T2_1
```

```
ITEM2 =~ 1*T1_16 + 1*T2_16
ITEM3 =~ 1*T1_31 + 1*T2_31
ITEM4 =~ 1*T1_46 + 1*T2_46
ITEM5 =~ 1*T1_6 + 1*T2_6
ITEM6 =~ 1*T1_21 + 1*T2_21
ITEM7 =~ 1*T1_36 + 1*T2_36
ITEM8 =~ 1*T1_51 + 1*T2_51
ITEM9 =~ 1*T1_11 + 1*T2_11
ITEM10 =~ 1*T1_26 + 1*T2_26
ITEM11 =~ 1*T1_41 + 1*T2_41
ITEM12 =~ 1*T1_56 + 1*T2_56

## Person variances
F1~~F1_V*F1
F2~~F2_V*F2
F3~~F3_V*F3

## Person covariances

F1 ~~ COV_12*F2
F1 ~~ COV_13*F3
F2 ~~ COV_23*F3

## Occasion variances
OCC1_F1~~OC_F1*OCC1_F1
OCC2_F1~~OC_F1*OCC2_F1
OCC1_F2~~OC_F2*OCC1_F2
OCC2_F2~~OC_F2*OCC2_F2
OCC1_F3~~OC_F3*OCC1_F3
OCC2_F3~~OC_F3*OCC2_F3

## Occasion covariances
OCC1_F1~~OC_12*OCC1_F2
OCC1_F1~~OC_13*OCC1_F3
OCC1_F3~~OC_23*OCC1_F2
OCC2_F1~~OC_12*OCC2_F2
OCC2_F1~~OC_13*OCC2_F3
OCC2_F3~~OC_23*OCC2_F2

## Item variances
ITEM1 ~~ S_pi1*ITEM1
ITEM2 ~~ S_pi1*ITEM2
ITEM3 ~~ S_pi1*ITEM3
ITEM4 ~~ S_pi1*ITEM4
ITEM5 ~~ S_pi2*ITEM5
ITEM6 ~~ S_pi2*ITEM6
ITEM7 ~~ S_pi2*ITEM7
ITEM8 ~~ S_pi2*ITEM8
ITEM9 ~~ S_pi3*ITEM9
ITEM10 ~~ S_pi3*ITEM10
```

```
ITEM11 ~~ S_pi3*ITEM11
ITEM12 ~~ S_pi3*ITEM12

## Uniquenesses
T1_1~~e_var1*T1_1
T1_16~~e_var1*T1_16
T1_31~~e_var1*T1_31
T1_46~~e_var1*T1_46
T1_6~~e_var2*T1_6
T1_21~~e_var2*T1_21
T1_36~~e_var2*T1_36
T1_51~~e_var2*T1_51
T1_11~~e_var3*T1_11
T1_26~~e_var3*T1_26
T1_41~~e_var3*T1_41
T1_56~~e_var3*T1_56
T2_1~~e_var1*T2_1
T2_16~~e_var1*T2_16
T2_31~~e_var1*T2_31
T2_46~~e_var1*T2_46
T2_6~~e_var2*T2_6
T2_21~~e_var2*T2_21
T2_36~~e_var2*T2_36
T2_51~~e_var2*T2_51
T2_11~~e_var3*T2_11
T2_26~~e_var3*T2_26
T2_41~~e_var3*T2_41
T2_56~~e_var3*T2_56'

cal_pio <- '
# Numbers of occasions and items again can be adjusted for use in prophecy
# formulas. The example that follows is based on one occasion and four items
# per subscale.
occ := 1
item := 4

## Variance components and partitioning at the composite level
p_VAR:=((COV_12 +COV_13 +COV_23)*2+F1_V+F2_V+F3_V)/9
pi_VAR:=(S_pi1 +S_pi2 + S_pi3)/9
po_VAR:=((OC_12 +OC_13 +OC_23)*2+OC_F1+OC_F2+OC_F3)/9
pio_VAR:=(e_var1 +e_var2 +e_var3)/9
GSUM:=p_VAR + po_VAR/occ + pi_VAR/item+ pio_VAR/(item*occ)
G_COEF:=p_VAR/GSUM
SFE:=(pi_VAR/item)/GSUM
TE:=(po_VAR/occ)/GSUM
RRE:=(pio_VAR/item*occ)/GSUM
TTE:=SFE+TE+RRE

## Variance components and partitioning at the subscale level
SOC_G:=(F1_V)/(F1_V+S_pi1/item+OC_F1/occ+e_var1/(item*occ))
```

```
SOC_SFE:=(S_pi1/item)/(F1_V+S_pi1/item+OC_F1/occ+e_var1/(item*occ))
SOC_TE:=(OC_F1/occ)/(F1_V+S_pi1/item+OC_F1/occ+e_var1/(item*occ))
SOC_RRE:=(e_var1/(item*occ))/(F1_V+S_pi1/item+OC_F1/occ+e_var1/(item*occ))
SOC_TTE:=(OC_F1+S_pi1/4+e_var1/4)/(F1_V+S_pi1/item+OC_F1/occ+e_var1/(item*occ)
)
ASS_G:=(F2_V)/(F2_V+S_pi2/item+OC_F2/occ+e_var2/(item*occ))
ASS_SFE:=(S_pi2/item)/(F2_V+S_pi2/item+OC_F2/occ+e_var2/(item*occ))
ASS_TE:=(OC_F2/occ)/(F2_V+S_pi2/item+OC_F2/occ+e_var2/(item*occ))
ASS_RRE:=(e_var2/(item*occ))/(F2_V+S_pi2/item+OC_F2/occ+e_var2/(item*occ))
ASS_TTE:=(OC_F2+S_pi2/4+e_var2/4)/(F2_V+S_pi2/item+OC_F2/occ+e_var2/(item*occ)
)
ENE_G:=(F3_V)/(F3_V+S_pi3/item+OC_F3/occ+e_var3/(item*occ))
ENE_SFE:=(S_pi3/4)/(F3_V+S_pi3/item+OC_F3/occ+e_var3/(item*occ))
ENE_TE:=(OC_F3)/(F3_V+S_pi3/item+OC_F3/occ+e_var3/(item*occ))
ENE_RRE:=(e_var3/4)/(F3_V+S_pi3/item+OC_F3/occ+e_var3/(item*occ))
ENE_TTE:=(OC_F3+S_pi3/4+e_var3/4)/(F3_V+S_pi3/item+OC_F3/occ+e_var3/(item*occ)
)

## Disattenuated and observed correlations
dis_cor_ASS_SOC := COV_12/sqrt(F1_V*F2_V)
dis_cor_ENE_SOC := COV_13/sqrt(F1_V*F3_V)
dis_cor_ASS_ENE := COV_23/sqrt(F2_V*F3_V)
obs_cor_ASS_SOC := dis_cor_ASS_SOC*sqrt(SOC_G*ASS_G)
obs_cor_ENE_SOC := dis_cor_ENE_SOC*sqrt(SOC_G*ENE_G)
obs_cor_ASS_ENE := dis_cor_ASS_ENE*sqrt(ASS_G*ENE_G)

## Indices for assessing subscale added value
SOC_PRMSE := (F1_V+COV_12+COV_13)^2/(F1_V*GSUM*9)
ASS_PRMSE := (F2_V+COV_12+COV_23)^2/(F2_V*GSUM*9)
ENE_PRMSE := (F3_V+COV_13+COV_23)^2/(F3_V*GSUM*9)
SOC_v_ar := SOC_G/SOC_PRMSE
ASS_v_ar := ASS_G/SOC_PRMSE
ENE_v_ar := ENE_G/ENE_PRMSE '

## Results for the analyses described above and related confidence intervals
Results <- lavaan(model = c(model,cal_pio), orthogonal = "TRUE",
                  data = bfi2_ext, estimator= "MLM")
monte<-monteCarloCI(Results, level=0.95)
monte

##                      est ci.lower ci.upper
## G_COEF             0.816    0.770    0.855
## SFE                0.081    0.067    0.099
## TE                 0.052    0.019    0.087
## RRE                0.050    0.043    0.059
## TTE                0.184    0.145    0.230
## SOC_G              0.746    0.691    0.790
## SOC_SFE            0.150    0.117    0.189
## SOC_TE             0.035    0.014    0.058
## SOC_RRE            0.069    0.058    0.083
```

```
## SOC_TTE          0.254    0.210    0.309
## ASS_G            0.676    0.613    0.729
## ASS_SFE          0.148    0.119    0.181
## ASS_TE           0.060    0.023    0.096
## ASS_RRE          0.117    0.099    0.139
## ASS_TTE          0.324    0.271    0.387
## ENE_G            0.652    0.584    0.709
## ENE_SFE          0.195    0.158    0.239
## ENE_TE           0.026   -0.007    0.059
## ENE_RRE          0.128    0.107    0.152
## ENE_TTE          0.348    0.291    0.416

## Disattenuated and observed correlations
## dis_cor_ASS_SOC  0.745    0.659    0.825
## dis_cor_ENE_SOC  0.683    0.593    0.768
## dis_cor_ASS_ENE  0.580    0.464    0.687
## obs_cor_ASS_SOC  0.528    0.452    0.597
## obs_cor_ENE_SOC  0.476    0.398    0.547
## obs_cor_ASS_ENE  0.385    0.297    0.465

# PRMSEs and VARs
parameterEstimates(Results)
##                    est
## SOC_PRMSE        0.707
## ASS_PRMSE        0.628
## ENE_PRMSE        0.574
## SOC_v_ar         1.054
## ASS_v_ar         1.077
## ENE_v_ar         1.135
```

2.2 Restricted $p^{\bullet} \times I^{\circ}$ S-ETE multivariate design

```
cal_pi <- '
# Numbers of occasions and items again can be adjusted for use in prophecy
# formulas. The example that follows is based on one occasion and four items
# per subscale.
item := 4
occ := 1

## Partitioning at the composite level
GSUM:=(COV_12 + COV_13 + COV_23)*2 + F1_V + F2_V + F3_V +
((OC_12 + OC_13 + OC_23)*2 + OC_F1 + OC_F2 + OC_F3)/occ +
(S_pi1 +S_pi2 + S_pi3)/item +
(e_var1 + e_var2 + e_var3)/(item*occ)
G_COEF:= ((COV_12 + COV_13 + COV_23)*2 + F1_V + F2_V + F3_V +
((OC_12 + OC_13 + OC_23)*2 + OC_F1 + OC_F2 + OC_F3)/occ)/GSUM
terror:=((S_pi1 +S_pi2 + S_pi3)/item +
(e_var1 + e_var2 + e_var3)/(item*occ))/GSUM

## Partitioning at the subscale level
SOC_sum:= F1_V+S_pi1/item+OC_F1/occ+e_var1/(item*occ)
```

```
SOC_G:=(F1_V+OC_F1/occ)/SOC_sum
SOC_TTE:=(S_pi1/item+e_var1/(item*occ))/SOC_sum
ASS_sum:= F2_V+S_pi2/item+OC_F2/occ+e_var2/(item*occ)
ASS_G:=(F2_V+OC_F2/occ)/ASS_sum
ASS_TTE:=(S_pi2/item+e_var2/(item*occ))/ASS_sum
ENE_sum:= F3_V+S_pi3/item+OC_F3/occ+e_var3/(item*occ)
ENE_G:=(F3_V+OC_F3/occ)/ENE_sum
ENE_TTE:=(S_pi3/item+e_var3/(item*occ))/ENE_sum


## Indices for assessing subscale added value
SOC_PRMSE :=(F1_V+COV_12+COV_13 + (OC_F1+OC_12+OC_13)/occ)^2 /((F1_V+OC_F1/occ
)*GSUM)
ASS_PRMSE :=(F2_V+COV_12+COV_23 + (OC_F2+OC_12+OC_23)/occ)^2 /((F2_V+OC_F2/occ
)*GSUM)
ENE_PRMSE :=(F3_V+COV_13+COV_23 + (OC_F3+OC_13+OC_23)/occ)^2 /((F3_V+OC_F3/occ
)*GSUM)
SOC_v_ar := SOC_G/SOC_PRMSE
ASS_v_ar := ASS_G/ASS_PRMSE
ENE_v_ar := ENE_G/ENE_PRMSE '

## Results for the analyses described above and related confidence intervals
Results <- lavaan(model = c(model,cal_pi), orthogonal = "TRUE",
                  data = bfi2_ext, estimator= "MLM")
monte<-monteCarloCI(Results, level=0.95)
monte

##                est ci.lower ci.upper
## G_COEF       0.868    0.844    0.887
## terror       0.132    0.113    0.156
## SOC_sum      0.924    0.815    1.033
## SOC_G        0.781    0.734    0.819
## SOC_TTE      0.219    0.181    0.266
## ASS_sum      0.695    0.613    0.777
## ASS_G        0.735    0.688    0.774
## ASS_TTE      0.265    0.226    0.312
## ENE_sum      0.615    0.539    0.690
## ENE_G        0.677    0.618    0.725
## ENE_TTE      0.323    0.275    0.382

# PRMSEs and VARs
parameterEstimates(Results)
## SOC_PRMSE    0.756
## ASS_PRMSE    0.666
## ENE_PRMSE    0.622
## SOC_v_ar     1.033
## ASS_v_ar     1.104
## ENE_v_ar     1.090
```

2.3 Restricted $p^{\bullet} \times O^{\bullet}$ S-ETE multivariate design

```
cal_po<-'

# Numbers of occasions and items again can be adjusted for use in prophecy
# formulas. The example that follows is based on one occasion and four items
# per subscale.
item := 4
occ := 1

## Partitioning at the composite level
GSUM:=(COV_12 + COV_13 + COV_23)*2 + F1_V + F2_V + F3_V +
((OC_12 + OC_13 + OC_23)*2 + OC_F1 + OC_F2 + OC_F3)/occ +
(S_pi1 +S_pi2 + S_pi3)/item +
(e_var1 + e_var2 + e_var3)/(item*occ)
G_COEF:= ((COV_12 + COV_13 + COV_23)*2 + F1_V + F2_V + F3_V +
(S_pi1 +S_pi2 + S_pi3)/item)/GSUM
TTE:=(((OC_12 + OC_13 + OC_23)*2 + OC_F1 + OC_F2 + OC_F3)/occ +
(e_var1 + e_var2 + e_var3)/(item*occ))/GSUM

## Partitioning at the subscale level
SOC_sum:= F1_V+S_pi1/item+OC_F1/occ+e_var1/(item*occ)
SOC_G:=(F1_V+S_pi1/item)/SOC_sum
SOC_TTE:=(OC_F1/occ+e_var1/(item*occ))/SOC_sum
ASS_sum:= F2_V+S_pi2/item+OC_F2/occ+e_var2/(item*occ)
ASS_G:=(F2_V+S_pi2/item)/ASS_sum
ASS_TTE:=(OC_F2/occ+e_var2/(item*occ))/ASS_sum
ENE_sum:= F3_V+S_pi3/item+OC_F3/occ+e_var3/(item*occ)
ENE_G:=(F3_V+S_pi3/item)/ENE_sum
ENE_TTE:=(OC_F3/occ+e_var3/(item*occ))/ENE_sum


## Indices for assessing subscale added value
SOC_PRMSE :=(F1_V+COV_12+COV_13 + S_pi1/item)^2 /((F1_V+S_pi1/item)*GSUM)
ASS_PRMSE :=(F2_V+COV_12+COV_23 + S_pi2/item)^2 /((F2_V+S_pi2/item)*GSUM)
ENE_PRMSE :=(F3_V+COV_13+COV_23 + S_pi3/item)^2 /((F3_V+S_pi3/item)*GSUM)
SOC_v_ar := SOC_G/SOC_PRMSE
ASS_v_ar := ASS_G/ASS_PRMSE
ENE_v_ar := ENE_G/ENE_PRMSE
'
## Results for the analyses described above and related confidence intervals
Results <- lavaan(model = c(model,cal_po), orthogonal = "TRUE",
                  data = bfi2_ext, estimator= "MLM")
monte<-monteCarloCI(Results, level=0.95)
monte

##                  est ci.lower ci.upper
## G_COEF         0.897    0.859    0.932
## TTE            0.103    0.068    0.141
## SOC_G          0.895    0.868    0.919
```

```
## SOC_TTE      0.105    0.081    0.132
## ASS_G        0.823    0.780    0.862
## ASS_TTE      0.177    0.138    0.220
## ENE_G        0.847    0.806    0.883
## ENE_TTE      0.153    0.117    0.194

# PRMSEs and VARs
parameterEstimates(Results)
## SOC_PRMSE    0.705
## ASS_PRMSE    0.612
## ENE_PRMSE    0.553
## SOC_v_ar     1.270
## ASS_v_ar     1.346
## ENE_v_ar     1.531
```

3. Variance Partitioning and Scale Viability for congeneric (CON) multivariate designs

To obtain the results of variance partitioning and scale viability indices for the preceding bifactor model, the following formula code should be used as a model argument for the *lavaan* function. Within the formula code for each design, the number of occasions and items can be adjusted for alternative designs.

3.1 $p^{\bullet} \times I^{\circ} \times O^{\bullet}$ CON multivariate design

The following code is used for variance partitioning and derivation of indices to access subscale added value for the congeneric (CON) multivariate design.

```
model <- "

## Person factors
F1 =~ NA*T1_1+dlt1*T1_1+dlt2*T1_16+dlt3*T1_31+dlt4*T1_46
               +dlt1*T2_1+dlt2*T2_16+dlt3*T2_31+dlt4*T2_46
F2 =~ NA*T1_6+dlt5*T1_6+dlt6*T1_21+dlt7*T1_36+dlt8*T1_51
               +dlt5*T2_6+dlt6*T2_21+dlt7*T2_36+dlt8*T2_51
F3 =~ NA*T1_11+dlt9*T1_11+dlt10*T1_26+dlt11*T1_41+dlt12*T1_56
                 +dlt9*T2_11+dlt10*T2_26+dlt11*T2_41+dlt12*T2_56


## Occasion factors
OCC1_F1 =~ NA*T1_1+sgm1*T1_1+sgm2*T1_16+sgm3*T1_31+sgm4*T1_46
OCC2_F1 =~ NA*T2_1+sgm1*T2_1+sgm2*T2_16+sgm3*T2_31+sgm4*T2_46
OCC1_F2 =~ NA*T1_6+sgm5*T1_6+sgm6*T1_21+sgm7*T1_36+sgm8*T1_51
OCC2_F2 =~ NA*T2_6+sgm5*T2_6+sgm6*T2_21+sgm7*T2_36+sgm8*T2_51
OCC1_F3 =~ NA*T1_11+sgm9*T1_11+sgm10*T1_26+sgm11*T1_41+sgm12*T1_56
OCC2_F3 =~ NA*T2_11+sgm9*T2_11+sgm10*T2_26+sgm11*T2_41+sgm12*T2_56
```

```
## Item factors
ITEM1 =~ 1*T1_1 + 1*T2_1
ITEM2 =~ 1*T1_16 + 1*T2_16
ITEM3 =~ 1*T1_31 + 1*T2_31
ITEM4 =~ 1*T1_46 + 1*T2_46
ITEM5 =~ 1*T1_6 + 1*T2_6
ITEM6 =~ 1*T1_21 + 1*T2_21
ITEM7 =~ 1*T1_36 + 1*T2_36
ITEM8 =~ 1*T1_51 + 1*T2_51
ITEM9 =~ 1*T1_11 + 1*T2_11
ITEM10 =~ 1*T1_26 + 1*T2_26
ITEM11 =~ 1*T1_41 + 1*T2_41
ITEM12 =~ 1*T1_56 + 1*T2_56

## Person variances
F1~~1*F1
F2~~1*F2
F3~~1*F3

## Person covariances
F1 ~~ COV_12*F2
F1 ~~ COV_13*F3
F2 ~~ COV_23*F3

## Occasion variances
OCC1_F1~~1*OCC1_F1
OCC1_F2~~1*OCC1_F2
OCC1_F3~~1*OCC1_F3
OCC2_F1~~1*OCC2_F1
OCC2_F2~~1*OCC2_F2
OCC2_F3~~1*OCC2_F3

## Occasion covariances
OCC1_F1~~OC_12*OCC1_F2
OCC1_F1~~OC_13*OCC1_F3
OCC1_F3~~OC_23*OCC1_F2
OCC2_F1~~OC_12*OCC2_F2
OCC2_F1~~OC_13*OCC2_F3
OCC2_F3~~OC_23*OCC2_F2


## Item variances
ITEM1 ~~ pi1_var*ITEM1
ITEM2 ~~ pi2_var*ITEM2
ITEM3 ~~ pi3_var*ITEM3
ITEM4 ~~ pi4_var*ITEM4
ITEM5 ~~ pi5_var*ITEM5
ITEM6 ~~ pi6_var*ITEM6
ITEM7 ~~ pi7_var*ITEM7
ITEM8 ~~ pi8_var*ITEM8
ITEM9 ~~ pi9_var*ITEM9
ITEM10 ~~ pi10_var*ITEM10
```

```
ITEM11 ~~ pi11_var*ITEM11
ITEM12 ~~ pi12_var*ITEM12


## Uniquenesses
T1_1~~e_var1*T1_1
T1_16~~e_var2*T1_16
T1_31~~e_var3*T1_31
T1_46~~e_var4*T1_46
T1_6~~e_var5*T1_6
T1_21~~e_var6*T1_21
T1_36~~e_var7*T1_36
T1_51~~e_var8*T1_51
T1_11~~e_var9*T1_11
T1_26~~e_var10*T1_26
T1_41~~e_var11*T1_41
T1_56~~e_var12*T1_56
T2_1~~e_var1*T2_1
T2_16~~e_var2*T2_16
T2_31~~e_var3*T2_31
T2_46~~e_var4*T2_46
T2_6~~e_var5*T2_6
T2_21~~e_var6*T2_21
T2_36~~e_var7*T2_36
T2_51~~e_var8*T2_51
T2_11~~e_var9*T2_11
T2_26~~e_var10*T2_26
T2_41~~e_var11*T2_41
T2_56~~e_var12*T2_56
"

cal_pio <-'
## Variance components at the subscale level
p_var1:= ((dlt1+dlt2+dlt3+dlt4)^2)/16
pi_var1:= (pi1_var+pi2_var+pi3_var+pi4_var)/4
po_var1:= ((sgm1+sgm2+sgm3+sgm4)^2)/16
pio_var1:= (e_var1+e_var2+e_var3+e_var4)/4
p_var2:= ((dlt5+dlt6+dlt7+dlt8)^2)/16
pi_var2:= (pi5_var+pi6_var+pi7_var+pi8_var)/4
po_var2:=((sgm5+sgm6+sgm7+sgm8)^2)/16
pio_var2:= (e_var5+e_var6+e_var7+e_var8)/4
p_var3:= ((dlt9+dlt10+dlt11+dlt12)^2)/16
pi_var3:= (pi9_var+pi10_var+pi11_var+pi12_var)/4
po_var3:= ((sgm9+sgm10+sgm11+sgm12)^2)/16
pio_var3:= (e_var9+e_var10+e_var11+e_var12)/4

# Numbers of occasions and items again can be adjusted for use in prophecy
# formulas. The example that follows is based on one occasion and four items
# per subscale.
```

```
occ := 1
item := 4

## Variance components and partitioning at the composite level
p_var :=((COV_12*(sqrt(p_var1*p_var2))+COV_13*(sqrt(p_var1*p_var3))+COV_23*(sq
rt(p_var3*p_var2)))*2+p_var1+p_var2+p_var3)/9
pi_var :=(pi_var1+pi_var2+pi_var3)/9
po_var :=((OC_12*(sqrt(po_var1*po_var2))+OC_13*(sqrt(po_var1*po_var3))+OC_23*(
sqrt(po_var3*po_var2)))*2+po_var1+po_var2+po_var3)/9
pio_var := (pio_var1 + pio_var2 + pio_var3)/9
GSUM:=p_var + po_var/occ + pi_var/item+ pio_var/(item*occ)
G_COEF:=p_var/GSUM
SFE:=(pi_var/item)/GSUM
TE:=(po_var/occ)/GSUM
RRE:=(pio_var/(item*occ))/GSUM
TTE:=SFE+TE+RRE

## Partitioning at the subscale level
gsum_1:=p_var1+pi_var1/item+po_var1/occ+pio_var1/(item*occ)
SOC_G:=p_var1/gsum_1
SOC_SFE:=(pi_var1/item)/gsum_1
SOC_TE:=(po_var1/occ)/gsum_1
SOC_RRE:=(pio_var1/(item*occ))/gsum_1
SOC_TTE:=ETE_SFE+SOC_TE+SOC_RRE
gsum_2:=(p_var2+pi_var2/item+po_var2/occ+pio_var2/(item*occ))
ASS_G:=p_var2/gsum_2
ASS_SFE:=(pi_var2/item)/gsum_2
ASS_TE:=(po_var2/occ)/gsum_2
ASS_RRE:=(pio_var2/(item*occ))/gsum_2
ASS_TTE:=sfe_2+te_2+rre_2
gsum_3:=(p_var3+pi_var3/item+po_var3/occ+pio_var3/(item*occ))
ENE_G:=p_var3/gsum_3
ENE_SFE:=(pi_var3/item)/gsum_3
ENE_TE:=(po_var3/occ)/gsum_3
ENE_RRE:=(pio_var3/(item*occ))/gsum_3
ENE_TTE:=sfe_3+te_3+rre_3

## Disattenuated and observed correlations
dis_cor_ASS_SOC := COV_12
dis_cor_ENE_SOC := COV_13
dis_cor_ASS_ENE := COV_23
obs_cor_ASS_SOC := dis_cor_ASS_SOC*sqrt(SOC_G * ASS_G)
obs_cor_ENE_SOC := dis_cor_ENE_SOC*sqrt(SOC_G * ENE_G)
obs_cor_ASS_ENE := dis_cor_ASS_ENE*sqrt(ASS_G * ENE_G)'

## Indices for assessing subscale added value
SOC_PRMSE := (F1_V+sqrt(p_var1*p_var2)*COV_12+sqrt(p_var1*p_var3)*COV_13)^2/
(F1_V*GSUM*9)
ASS_PRMSE := (F2_V+ sqrt(p_var1*p_var2)* sqrt(p_var2*p_var3)*COV_12+COV_23)^2/
(F2_V*GSUM*9)
ENE_PRMSE := (F3_V+ sqrt(p_var1*p_var3)*COV_13+ sqrt(p_var2*p_var3)*COV_23)^2/
```

```
(F3_V*GSUM*9)
SOC_v_ar := SOC_G/SOC_PRMSE
ASS_v_ar := ASS_G/ASS_PRMSE
ENE_v_ar := ENE_G/ENE_PRMSE '

## Variance components and partitioning at the item level
item_name=paste0("I_",substr(colnames(bfi2_ext)[1:12], 4,5))
cal_pio_item<-c('n_i:=1', # adjust n_i value
  c(paste0(item_name,rep(c('v_p :=dlt','v_po:=sgm'),each=12),1:12,'^2')),
              c(paste0(item_name,'v_pi:=pi',1:12,'_var')),
              c(paste0(item_name,'v_pio:=e_var',1:12)),
              c(paste0(item_name,'rel_e:=',paste0(item_name,'v_p+',item_name
,'v_po+',item_name,'v_pi/n_i+',item_name,'v_pio/n_i'))),
c(paste0(item_name,'Gcoef:=',paste0(item_name,'v_p/',paste0(item_name,'rel_e')
))),
c(paste0(item_name,'SFE:=',paste0('(pi',1:12,'_var)/n_i/',paste0(item_name,'re
l_e')))),
c(paste0(item_name,'TE:=',paste0(item_name,'v_po/',paste0(item_name,'rel_e')))
) ,
c(paste0(item_name,'RRE:=',paste0("(",item_name,'v_pio)/n_i/',paste0(item_name
,'rel_e')))) ,
c(paste0(item_name,'totalE:=',paste0('(',item_name,'rel_e-',item_name,'v_p)/',
paste0(item_name,'rel_e')))))


## Results for the analyses described above and related confidence intervals
Results <- lavaan(model = c(model, cal_pio, cal_pio_item), orthogonal = "TRUE"
, data =bfi2_ext, estimator= "MLM")
set.seed(123987)
monte<-monteCarloCI(Results, level=0.95)
monte

##                   est ci.lower ci.upper
## G_COEF          0.819    0.773    0.855
## SFE             0.080    0.066    0.096
## TE              0.050    0.023    0.088
## RRE             0.051    0.044    0.060
## TTE             0.181    0.145    0.227
## SOC_G           0.729    0.670    0.777
## SOC_SFE         0.164    0.128    0.206
## SOC_TE          0.032    0.013    0.060
## SOC_RRE         0.075    0.062    0.090
## SOC_TTE         0.271    0.223    0.330
## ASS_G           0.699    0.640    0.748
## ASS_SFE         0.129    0.103    0.157
## ASS_TE          0.058    0.028    0.099
## ASS_RRE         0.115    0.097    0.135
## ASS_TTE         0.301    0.252    0.360
## ENE_G           0.659    0.596    0.712
## ENE_SFE         0.180    0.148    0.216
```

```
## ENE_TE          0.041    0.015    0.078
## ENE_RRE         0.120    0.101    0.141
## ENE_TTE         0.341    0.288    0.404
## Disattenuated and observed correlations
## dis_cor_ASS_SOC  0.680    0.600    0.760
## dis_cor_ENE_SOC  0.761    0.693    0.828
## dis_cor_ASS_ENE  0.516    0.413    0.617
## obs_cor_ASS_SOC  0.485    0.413    0.557
## obs_cor_ENE_SOC  0.528    0.465    0.589
## obs_cor_ASS_ENE  0.350    0.272    0.428

# Item level results
## I_1Gcoef    0.688    0.605    0.765
## I_16Gcoef   0.280    0.185    0.385
## I_31Gcoef   0.172    0.100    0.259
## I_46Gcoef   0.670    0.594    0.741
## I_6Gcoef    0.346    0.253    0.443
## I_21Gcoef   0.694    0.599    0.782
## I_36Gcoef   0.176    0.113    0.248
## I_51Gcoef   0.411    0.317    0.509
## I_11Gcoef   0.200    0.126    0.286
## I_26Gcoef   0.102    0.047    0.175
## I_41Gcoef   0.653    0.567    0.733
## I_56Gcoef   0.613    0.526    0.694
## I_1SFE      0.148    0.079    0.219
## I_16SFE     0.519    0.413    0.616
## I_31SFE     0.601    0.504    0.684
## I_46SFE     0.172    0.103    0.242
## I_6SFE      0.390    0.297    0.478
## I_21SFE     0.085    0.004    0.166
## I_36SFE     0.433    0.347    0.513
## I_51SFE     0.261    0.173    0.347
## I_11SFE     0.463    0.358    0.556
## I_26SFE     0.653    0.568    0.726
## I_41SFE     0.094    0.024    0.163
## I_56SFE     0.131    0.061    0.202
## I_1TE       0.041    0.015    0.079
## I_16TE      0.012    0.002    0.028
## I_31TE      0.012    0.002    0.030
## I_46TE      0.015    0.002    0.042
## I_6TE       0.063    0.029    0.110
## I_21TE      0.055    0.017    0.114
## I_36TE      0.017    0.001    0.049
## I_51TE      0.009    0.000    0.034
## I_11TE      0.008    0.000    0.043
## I_26TE      0.000    0.000    0.010
## I_41TE      0.079    0.029    0.153
## I_56TE      0.049    0.018    0.093
## I_1RRE      0.122    0.085    0.162
## I_16RRE     0.190    0.151    0.229
```

```
## I_31RRE     0.215    0.171    0.261
## I_46RRE     0.142    0.110    0.177
## I_6RRE      0.202    0.149    0.255
## I_21RRE     0.165    0.121    0.211
## I_36RRE     0.373    0.302    0.446
## I_51RRE     0.319    0.240    0.395
## I_11RRE     0.328    0.253    0.403
## I_26RRE     0.245    0.189    0.299
## I_41RRE     0.174    0.119    0.229
## I_56RRE     0.207    0.158    0.259
## I_1totalE   0.312    0.235    0.395
## I_16totalE  0.720    0.615    0.815
## I_31totalE  0.828    0.741    0.900
## I_46totalE  0.330    0.259    0.406
## I_6totalE   0.654    0.557    0.747
## I_21totalE  0.306    0.218    0.401
## I_36totalE  0.824    0.752    0.887
## I_51totalE  0.589    0.491    0.683
## I_11totalE  0.800    0.714    0.874
## I_26totalE  0.898    0.825    0.953
## I_41totalE  0.347    0.267    0.433
## I_56totalE  0.387    0.306    0.474

# PRMSEs and VARs
parameterEstimates(Results)
##                      est
## SOC_PRMSE          0.719
## ASS_PRMSE          0.574
## ENE_PRMSE          0.598
## SOC_v_ar           1.014
## ASS_v_ar           1.218
## ENE_v_ar           1.101
```

3.2 Restricted $p^{\bullet} \times I^{\circ}$ CON multivariate design

```
cal_pi <-'
# Numbers of occasions and items again can be adjusted for use in prophecy
# formulas. The example that follows is based on one occasion and four items
# per subscale.
occ := 1
item := 4

## Variance components at the subscale level
p_var1:= ((dlt1+dlt2+dlt3+dlt4)^2)/16
pi_var1:= (pi1_var+pi2_var+pi3_var+pi4_var)/4
po_var1:= ((sgm1+sgm2+sgm3+sgm4)^2)/16
pio_var1:= (e_var1+e_var2+e_var3+e_var4)/4
```

```
p_var2:= ((dlt5+dlt6+dlt7+dlt8)^2)/16
pi_var2:= (pi5_var+pi6_var+pi7_var+pi8_var)/4
po_var2:=((sgm5+sgm6+sgm7+sgm8)^2)/16
pio_var2:= (e_var5+e_var6+e_var7+e_var8)/4

p_var3:= ((dlt9+dlt10+dlt11+dlt12)^2)/16
pi_var3:= (pi9_var+pi10_var+pi11_var+pi12_var)/4
po_var3:= ((sgm9+sgm10+sgm11+sgm12)^2)/16
pio_var3:= (e_var9+e_var10+e_var11+e_var12)/4

## Variance components at the composite level
p_var :=(COV_12*(sqrt(p_var1*p_var2))+COV_13*(sqrt(p_var1*p_var3))+COV_23*(sqr
t(p_var3*p_var2)))*2+p_var1+p_var2+p_var3
pi_var :=pi_var1+pi_var2+pi_var3
po_var :=(OC_12*(sqrt(po_var1*po_var2))+OC_13*(sqrt(po_var1*po_var3))+OC_23*(s
qrt(po_var3*po_var2)))*2+po_var1+po_var2+po_var3
pio_var := pio_var1 + pio_var2 + pio_var3

## Partitioning at the subscale level
gsum_1:=p_var1+pi_var1/item+po_var1/occ+pio_var1/(item*occ)
SOC_G:=(p_var1+po_var1/occ)/gsum_1
SOC_TTE:=(pi_var1/item+pio_var1/(item*occ))/gsum_1
gsum_2:=p_var2+pi_var2/item+po_var2/occ+pio_var2/(item*occ)
ASS_G:=(p_var2+po_var2/occ)/gsum_2
ASS_TTE:=(pi_var2/item+pio_var2/(item*occ))/gsum_2
gsum_3:=p_var3+pi_var3/item+po_var3/occ+pio_var3/(item*occ)
ENE_G:=(p_var3+po_var3/occ)/gsum_3
ENE_TTE:=(pi_var3/item+pio_var3/(item*occ))/gsum_3

## Partitioning at the composite level
GSUM:=p_var + po_var/occ + pi_var/item+ pio_var/(item*occ)
G_COEF:= (p_var+po_var/occ)/GSUM
TTE:=(pi_var/item+pio_var/(item*occ))/GSUM

## Indices for assessing subscale added value
SOC_PRMSE :=(p_var1+COV_12*(sqrt(p_var1*p_var2))+COV_13*(sqrt(p_var1*p_var3))+
po_var1/occ+OC_12*(sqrt(po_var1*po_var2))+OC_13*(sqrt(po_var1*po_var3)))^2
/((p_var1+po_var1/occ)*GSUM)
ASS_PRMSE :=(p_var2+COV_12*(sqrt(p_var1*p_var2))+COV_23*(sqrt(p_var2*p_var3))+
po_var2/occ+OC_12*(sqrt(po_var1*po_var2))+OC_23*(sqrt(po_var2*po_var3)))^2
/((p_var2+po_var2/occ)*GSUM)
ENE_PRMSE :=(p_var3+COV_13*(sqrt(p_var1*p_var3))+COV_23*(sqrt(p_var2*p_var3))+
po_var3/occ+OC_13*(sqrt(po_var1*po_var3))+OC_23*(sqrt(po_var2*po_var3)))^2
/((p_var3+po_var3/occ)*GSUM)
SOC_v_ar := SOC_G/SOC_PRMSE
ASS_v_ar := ASS_G/ASS_PRMSE
ENE_v_ar := ENE_G/ENE_PRMSE '
```

```
## Results for the analyses described above and related confidence intervals
Results <- lavaan(model = c(model, cal_pi), orthogonal = "TRUE", data =bfi2_ex
t, estimator= "MLM")
set.seed(123987)
monte<-monteCarloCI(Results, level=0.95)
monte
##              est ci.lower ci.upper
## SOC_G      0.761    0.710    0.805
## SOC_TTE    0.239    0.195    0.290
## ASS_G      0.757    0.716    0.793
## ASS_TTE    0.243    0.207    0.284
## ENE_G      0.700    0.651    0.743
## ENE_TTE    0.300    0.257    0.349
## G_COEF     0.869    0.846    0.889
## error      0.131    0.111    0.154

# PRMSEs and VARs
parameterEstimates(Results)
## SOC_PRMSE  0.765
## ASS_PRMSE  0.608
## ENE_PRMSE  0.631
## SOC_v_ar   0.995
## ASS_v_ar   1.245
## ENE_v_ar   1.110
```

3.3 Restricted $p^{\bullet} \times O^{\bullet}$ CON multivariate design

```
cal_po<-'
# Numbers of occasions and items again can be adjusted for use in prophecy
# formulas. The example that follows is based on one occasion and four items
# per subscale.
occ := 1
item := 4

## Variance components at the subscale level
p_var1:= ((dlt1+dlt2+dlt3+dlt4)^2)/16
pi_var1:= (pi1_var+pi2_var+pi3_var+pi4_var)/4
po_var1:= ((sgm1+sgm2+sgm3+sgm4)^2)/16
pio_var1:= (e_var1+e_var2+e_var3+e_var4)/4

p_var2:= ((dlt5+dlt6+dlt7+dlt8)^2)/16
pi_var2:= (pi5_var+pi6_var+pi7_var+pi8_var)/4
po_var2:=((sgm5+sgm6+sgm7+sgm8)^2)/16
pio_var2:= (e_var5+e_var6+e_var7+e_var8)/4

p_var3:= ((dlt9+dlt10+dlt11+dlt12)^2)/16
pi_var3:= (pi9_var+pi10_var+pi11_var+pi12_var)/4
po_var3:= ((sgm9+sgm10+sgm11+sgm12)^2)/16
```

```
pio_var3:= (e_var9+e_var10+e_var11+e_var12)/4

## Variance components at the composite level
p_var :=(COV_12*(sqrt(p_var1*p_var2))+COV_13*(sqrt(p_var1*p_var3))+COV_23*(sqr
t(p_var3*p_var2)))*2+p_var1+p_var2+p_var3
pi_var :=pi_var1+pi_var2+pi_var3
po_var :=(OC_12*(sqrt(po_var1*po_var2))+OC_13*(sqrt(po_var1*po_var3))+OC_23*(s
qrt(po_var3*po_var2)))*2+po_var1+po_var2+po_var3
pio_var := pio_var1 + pio_var2 + pio_var3

## Partitioning at the subscale level
gsum_1:=p_var1+pi_var1/item+po_var1/occ+pio_var1/(item*occ)
SOC_G:=(p_var1+pi_var1/item)/gsum_1
SOC_TTE:=(po_var1/occ+pio_var1/(item*occ))/gsum_1
gsum_2:=p_var2+pi_var2/item+po_var2/occ+pio_var2/(item*occ)
ASS_G:=(p_var2+pi_var2/item)/gsum_2
ASS_TTE:=(po_var2/occ+pio_var2/(item*occ))/gsum_2
gsum_3:=p_var3+pi_var3/item+po_var3/occ+pio_var3/(item*occ)
ENE_G:=(p_var3+pi_var3/item)/gsum_3
ENE_TTE:=(po_var3/occ+pio_var3/(item*occ))/gsum_3

## Partitioning at the composite level
GSUM:=p_var + po_var/occ + pi_var/item+ pio_var/(item*occ)
G_COEF:= (p_var+pi_var/item)/GSUM
TTE:=(po_var/occ+pio_var/(item*occ))/GSUM

## Indices for assessing subscale added value
SOC_PRMSE :=(p_var1+COV_12*(sqrt(p_var1*p_var2))+COV_13*(sqrt(p_var1*p_var3))+
pi_var1/item)^2/((p_var1+pi_var1/item)*GSUM)
ASS_PRMSE :=(p_var2+COV_12*(sqrt(p_var1*p_var2))+COV_23*(sqrt(p_var2*p_var3))+
pi_var2/item)^2/((p_var2+pi_var2/item)*GSUM)
ENE_PRMSE :=(p_var3+COV_13*(sqrt(p_var1*p_var3))+COV_23*(sqrt(p_var2*p_var3))+
pi_var3/item)^2/((p_var3+pi_var3/item)*GSUM)
SOC_v_ar := gcoef_1/SOC_PRMSE
ASS_v_ar := gcoef_2/ASS_PRMSE
ENE_v_ar := gcoef_3/ENE_PRMSE '

## Results for the analyses described above and related confidence intervals
Results <- lavaan(model = c(model, cal_po), orthogonal = "TRUE", data =bfi2_ex
t, estimator= "MLM")
set.seed(123987)
monte<-monteCarloCI(Results, level=0.95)
monte
##              est ci.lower ci.upper
## SOC_G      0.893    0.862    0.916
## SOC_TTE    0.107    0.084    0.138
## ASS_G      0.827    0.783    0.863
## ASS_TTE    0.173    0.137    0.217
## ENE_G      0.839    0.795    0.873
```

```
## ENE_TTE   0.161    0.127    0.205
## G_COEF    0.899    0.859    0.928
## TTE       0.101    0.072    0.141

# PRMSEs and VARs
parameterEstimates(Results)
## SOC_PRMSE 0.713
## ASS_PRMSE 0.568
## ENE_PRMSE 0.577
## SOC_v_ar  1.253
## ASS_v_ar  1.458
## ENE_v_ar  1.454
```

Abbreviations used throughout the supplement

BFI-2: Big Five Inventory Form 2
ASS: Assertiveness
ENE: Energy Level
SOC: Sociability
Open circle ($^{\circ}$): Nested facet
Closed circle ($^{\bullet}$): Crossed facet
G: G coefficient
SFE: Specific-factor error
TE: Transient error
RRE: Random-response error
TTE: Total error
PMRSE: Proportional Reductions in Mean Squared Error
VAR or V_AR: Value-Added Ratio
dis_cor: Disattenuated correlation coefficient
obs_cor: Observed correlation coefficient

References

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Rosseel, Y. (2012). *lavaan*: An R Package for Structural Equation Modeling. *Journal of Statistical Software*, *48*(2), 1-36. https://doi: 10.18637/jss.v048.i02.

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