

Supplemental Material

Tables S1 - S4 displays the results obtained in the same way as with Table 1 except that we only considered $\beta_0 = (0.5, 0.5, \dots, 0)^\top$ and $\beta_0 = (0.5, 0.7, 0, \dots, 0)^\top$ and the bandwidth selection methods (a), (b), (c) and (d) were used, respectively. It is apparent that they gave similar conclusions as above and again indicate that the proposed variable selection approach seems to work well. Furthermore, they suggest that the joint selection of the tuning parameter λ_n and the bandwidth can be expected to yield larger TP and smaller FP and MMSE.

Table S1. Simulation results based on the bandwidth selection method (a).

p	Method	$n = 300$			$n = 500$		
		$\beta = (0.5, 0.5, 0, \dots, 0)$					
		TP	FP	MMSE(SD)	TP	FP	MMSE(SD)
10	LASSO	1.96	1.56	0.180(0.073)	2.00	1.65	0.152(0.060)
	SCAD	1.87	1.35	0.164(0.097)	1.98	1.39	0.131(0.068)
	MCP	1.85	1.23	0.165(0.100)	1.97	1.21	0.127(0.069)
	BAR	1.70	0.57	0.174(0.092)	1.86	0.43	0.138(0.071)
30	LASSO	1.91	4.76	0.257(0.128)	1.99	4.26	0.218(0.307)
	SCAD	1.82	4.31	0.280(0.237)	1.98	3.92	0.131(0.068)
	MCP	1.80	4.11	0.347(0.306)	1.97	3.44	0.127(0.069)
	BAR	1.68	1.52	0.245(0.155)	1.86	1.29	0.138(0.071)
50	LASSO	1.96	7.85	0.267(0.114)	1.97	6.41	0.232(0.100)
	SCAD	1.83	7.21	0.309(0.214)	1.93	6.72	0.249(0.239)
	MCP	1.81	6.98	0.437(0.324)	1.90	6.10	0.314(0.321)
	BAR	1.66	2.05	0.266(0.151)	1.82	2.28	0.231(0.161)
100	LASSO	1.89	16.22	0.505(0.992)	1.97	13.60	0.330(0.160)
	SCAD	1.72	15.61	0.876(2.562)	1.88	13.36	0.309(0.292)
	MCP	1.64	14.66	1.341(2.733)	1.85	12.05	0.455(0.455)
	BAR	1.51	4.98	0.641(1.569)	1.76	4.17	0.293(0.178)
$\beta = (0.5, 0.7, 0, \dots, 0)$							
10	LASSO	1.96	1.21	0.172(0.068)	1.99	1.63	0.237(0.104)
	SCAD	1.95	1.33	0.234(0.121)	1.96	1.26	0.210(0.179)
	MCP	1.95	1.26	0.230(0.120)	1.96	1.08	0.208(0.185)
	BAR	1.87	0.54	0.244(0.106)	1.95	0.46	0.207(0.115)
30	LASSO	1.96	3.87	0.306(0.104)	2.00	4.27	0.305(0.300)
	SCAD	1.84	4.48	0.350(0.268)	1.98	3.80	0.301(0.614)
	MCP	1.84	4.33	0.419(0.344)	1.97	3.33	0.324(0.653)
	BAR	1.72	1.58	0.321(0.191)	1.94	1.29	0.276(0.465)
50	LASSO	1.98	7.80	0.444(0.641)	2.00	7.54	0.335(0.183)
	SCAD	1.92	7.21	0.572(1.479)	1.96	6.72	0.350(0.528)
	MCP	1.90	6.90	0.692(1.551)	1.97	6.24	0.430(0.650)
	BAR	1.79	2.44	0.487(1.064)	1.87	2.19	0.312(0.284)
100	LASSO	1.95	16.36	0.617(0.972)	1.99	14.02	0.349(0.124)
	SCAD	1.83	15.49	0.982(2.344)	1.93	13.29	0.366(0.257)
	MCP	1.78	14.72	1.504(2.800)	1.93	11.69	0.499(0.447)
	BAR	1.62	4.73	0.710(1.480)	1.88	3.88	0.338(0.172)

Table S2. Simulation results based on the bandwidth selection method (b).

p	Method	$n = 300$			$n = 500$		
		$\beta = (0.5, 0.5, 0, \dots, 0)$					
		TP	FP	MMSE(SD)	TP	FP	MMSE(SD)
10	LASSO	1.96	1.21	0.172(0.068)	1.99	1.35	0.159(0.058)
	SCAD	1.89	1.12	0.146(0.074)	1.96	1.18	0.132(0.070)
	MCP	1.89	1.02	0.141(0.074)	1.95	0.98	0.128(0.073)
	BAR	1.80	0.42	0.152(0.073)	1.86	0.27	0.138(0.067)
30	LASSO	1.96	3.80	0.215(0.081)	1.98	3.67	0.186(0.074)
	SCAD	1.88	3.37	0.198(0.107)	1.97	3.37	0.156(0.093)
	MCP	1.85	3.20	0.227(0.140)	1.94	2.94	0.167(0.121)
	BAR	1.74	1.14	0.192(0.105)	1.89	1.03	0.115(0.091)
50	LASSO	1.97	6.41	0.232(0.100)	2.00	6.44	0.224(0.088)
	SCAD	1.88	6.15	0.237(0.184)	1.90	5.96	0.258(0.131)
	MCP	1.86	5.57	0.307(0.301)	1.89	5.18	0.244(0.196)
	BAR	1.74	1.63	0.214(0.132)	1.85	1.64	0.192(0.121)
100	LASSO	1.97	13.60	0.330(0.160)	1.98	11.63	0.241(0.118)
	SCAD	1.84	12.87	0.415(0.366)	1.90	10.88	0.219(0.137)
	MCP	1.77	12.09	0.690(0.628)	1.88	9.60	0.295(0.227)
	BAR	1.61	3.67	0.357(0.232)	1.84	3.04	0.230(0.155)
$\beta = (0.5, 0.7, 0, \dots, 0)$							
10	LASSO	1.97	1.22	0.263(0.083)	1.99	1.32	0.240(0.069)
	SCAD	1.95	1.10	0.212(0.090)	1.97	1.08	0.195(0.076)
	MCP	1.94	1.08	0.211(0.089)	1.96	0.93	0.197(0.088)
	BAR	1.88	0.36	0.228(0.093)	1.94	0.35	0.206(0.079)
30	LASSO	1.96	3.87	0.306(0.104)	2.00	3.65	0.271(0.087)
	SCAD	1.89	3.45	0.260(0.123)	1.99	3.43	0.221(0.100)
	MCP	1.89	3.21	0.284(0.149)	1.98	2.94	0.233(0.118)
	BAR	1.82	1.19	0.258(0.113)	1.95	1.07	0.224(0.095)
50	LASSO	1.98	6.27	0.321(0.107)	2.00	6.23	0.307(0.090)
	SCAD	1.96	5.66	0.286(0.189)	1.99	5.80	0.262(0.125)
	MCP	1.95	5.74	0.356(0.279)	1.98	5.19	0.306(0.173)
	BAR	1.87	1.74	0.274(0.128)	1.88	1.78	0.265(0.122)
100	LASSO	1.98	13.25	0.411(0.166)	1.98	11.67	0.330(0.127)
	SCAD	1.84	12.53	0.462(0.405)	1.93	11.19	0.285(0.166)
	MCP	1.81	11.78	0.724(0.690)	1.93	10.09	0.378(0.265)
	BAR	1.71	3.35	0.403(0.236)	1.88	3.12	0.298(0.179)

Table S3. Simulation results based on the bandwidth selection method (c).

p	Method	$n = 300$			$n = 500$		
		$\beta = (0.5, 0.5, 0, \dots, 0)$					
		TP	FP	MMSE(SD)	TP	FP	MMSE(SD)
10	LASSO	1.95	1.32	0.176(0.072)	2.00	1.48	0.159(0.061)
	SCAD	1.89	1.16	0.149(0.077)	1.94	1.20	0.135(0.068)
	MCP	1.89	1.08	0.146(0.078)	1.93	1.08	0.132(0.074)
	BAR	1.77	0.40	0.158(0.079)	1.91	0.63	0.133(0.063)
30	LASSO	1.95	4.05	0.225(0.089)	1.98	3.78	0.188(0.075)
	SCAD	1.82	3.62	0.215(0.129)	1.97	3.47	0.158(0.096)
	MCP	1.81	3.38	0.248(0.156)	1.96	3.37	0.286(1.148)
	BAR	1.70	1.26	0.205(0.114)	1.89	1.48	0.164(0.096)
50	LASSO	1.96	6.67	0.243(0.110)	2.00	6.71	0.223(0.075)
	SCAD	1.88	6.46	0.256(0.201)	1.89	6.39	0.206(0.114)
	MCP	1.85	6.04	0.341(0.316)	1.89	5.49	0.252(0.174)
	BAR	1.71	1.82	0.232(0.151)	1.84	1.89	0.201(0.116)
100	LASSO	1.96	14.20	0.354(0.201)	1.96	12.46	0.252(0.127)
	SCAD	1.80	13.53	0.478(0.428)	1.88	11.64	0.242(0.167)
	MCP	1.75	12.95	0.801(0.739)	1.86	10.49	0.354(0.463)
	BAR	1.58	3.92	0.394(0.276)	1.81	4.67	0.279(0.186)
$\beta = (0.5, 0.7, 0, \dots, 0)$							
10	LASSO	1.97	1.34	0.266(0.086)	1.99	1.32	0.240(0.069)
	SCAD	1.94	1.13	0.218(0.098)	1.97	1.08	0.195(0.076)
	MCP	1.94	1.13	0.214(0.094)	1.96	0.93	0.197(0.088)
	BAR	1.87	0.41	0.234(0.099)	1.94	0.35	0.206(0.079)
30	LASSO	1.95	4.05	0.306(0.104)	2.00	3.65	0.271(0.087)
	SCAD	1.88	3.58	0.278(0.145)	1.99	3.43	0.221(0.100)
	MCP	1.87	3.37	0.305(0.169)	1.98	2.94	0.233(0.118)
	BAR	1.82	1.16	0.265(0.120)	1.95	1.07	0.224(0.095)
50	LASSO	1.98	6.68	0.331(0.117)	2.00	6.23	0.307(0.090)
	SCAD	1.96	6.04	0.311(0.210)	1.99	5.80	0.262(0.125)
	MCP	1.95	5.74	0.387(0.297)	1.98	5.19	0.306(0.173)
	BAR	1.86	1.80	0.286(0.143)	1.88	1.78	0.265(0.122)
100	LASSO	1.97	14.20	0.442(0.218)	1.98	11.67	0.330(0.127)
	SCAD	1.83	13.51	0.535(0.494)	1.93	11.19	0.285(0.166)
	MCP	1.79	12.63	0.828(0.778)	1.93	10.09	0.378(0.265)
	BAR	1.70	3.54	0.435(0.281)	1.88	3.12	0.298(0.179)

Table S4. Simulation results based on the bandwidth selection method (d).

p	Method	$n = 300$			$n = 500$		
		$\beta = (0.5, 0.5, 0, \dots, 0)$					
		TP	FP	MMSE(SD)	TP	FP	MMSE(SD)
10	LASSO	1.97	1.39	0.185(0.072)	2.00	1.60	0.153(0.061)
	SCAD	1.88	1.17	0.164(0.080)	1.99	1.38	0.128(0.073)
	MCP	1.86	1.10	0.162(0.084)	1.94	1.11	0.125(0.069)
	BAR	1.89	0.49	0.242(0.090)	1.93	0.45	0.199(0.082)
30	LASSO	1.93	4.46	0.257(0.115)	1.99	4.16	0.247(0.631)
	SCAD	1.80	4.08	0.274(0.192)	1.97	3.65	0.264(1.094)
	MCP	1.80	3.89	0.322(0.254)	1.96	3.37	0.286(1.148)
	BAR	1.73	1.57	0.321(0.396)	1.95	1.08	0.303(0.856)
50	LASSO	1.95	7.17	0.255(0.097)	1.99	6.94	0.228(0.086)
	SCAD	1.87	6.70	0.277(0.177)	1.94	6.48	0.224(0.157)
	MCP	1.83	6.33	0.378(0.287)	1.91	5.73	0.285(0.254)
	BAR	1.82	2.07	0.373(0.578)	1.89	2.60	0.221(0.124)
100	LASSO	1.92	15.23	0.384(0.224)	1.98	13.05	0.251(0.127)
	SCAD	1.78	14.48	0.556(0.692)	1.89	12.07	0.263(0.288)
	MCP	1.68	13.66	0.952(1.070)	1.86	10.49	0.354(0.463)
	BAR	1.53	4.30	0.454(0.346)	1.81	4.27	0.272(0.138)
$\beta = (0.5, 0.7, 0, \dots, 0)$							
10	LASSO	1.98	1.41	0.277(0.086)	2.00	1.49	0.233(0.078)
	SCAD	1.94	1.19	0.232(0.093)	1.98	1.27	0.188(0.086)
	MCP	1.94	1.18	0.227(0.094)	1.98	1.08	0.188(0.085)
	BAR	1.89	0.49	0.242(0.090)	1.93	0.45	0.199(0.082)
30	LASSO	1.93	4.61	0.342(0.145)	2.00	4.10	0.323(0.580)
	SCAD	1.86	4.30	0.339(0.219)	1.99	3.62	0.324(1.024)
	MCP	1.84	4.11	0.407(0.299)	1.99	3.20	0.342(1.071)
	BAR	1.73	1.57	0.321(0.396)	1.95	1.08	0.303(0.856)
50	LASSO	1.98	7.21	0.358(0.123)	2.00	6.73	0.333(0.229)
	SCAD	1.91	6.61	0.429(0.808)	1.98	6.35	0.340(0.614)
	MCP	1.92	6.38	0.526(0.913)	1.99	3.20	0.342(1.071)
	BAR	1.82	2.07	0.373(0.578)	1.88	2.06	0.315(0.388)
100	LASSO	1.96	15.21	0.498(0.334)	1.99	12.18	0.327(0.118)
	SCAD	1.83	14.27	0.713(0.872)	1.96	11.89	0.314(0.277)
	MCP	1.80	13.52	1.106(1.296)	1.96	10.55	0.413(0.447)
	BAR	1.60	4.27	0.554(0.416)	1.91	3.05	0.293(0.153)