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Supplementary Materials

Supplementary Materials for: Reference-Anchored Constrained Multi-Objective Optimization of High-Temperature Sodium Heat Pipes under Rated Power-Test Conditions

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Supplementary overview. This document extends the model, uncertainty, and method-audit information in Sections 2.3–2.4, 3.2, and 4.2–4.3 of the main manuscript. All supporting calculations use the final eight-constraint model with cooling-air mass flow fixed at 0.0125500 kg/s and five optimized structural variables. A dense ε-constraint reference identifies two unsupported segments in the prescribed scenario model. The comparison sets comprise the weighted-sum baseline, the 40-point single-seed original GA, and the 119-point proposed hybrid result. The guide-contact heat-leak fraction is evaluated within the prescribed 1–5% sensitivity bracket, with 2.5% as its representative midpoint.

S1. Fixed-Airflow Reformulation and Contact Heat-Leak-Fraction Bracket

Cooling airflow is an experimental boundary condition, not a heat-pipe manufacturing variable. Accordingly, every sampling and optimization vector contains exactly five structural variables: outer diameter, vapor-core diameter, evaporator length, condenser length, and wick porosity. The fixed airflow of 0.0125500327 kg/s is stored and propagated separately as a controlled condition.

The guide-contact heat-leak study covers contact off, contact on with zero heat leakage, and prescribed heat-leak fractions of 1%, 2.5%, and 5%. The manuscript uses 2.5% as the midpoint of this bracket. Table S1 summarizes the evaluated cases and complements the eighth-constraint definition in Section 2.3 of the main manuscript.

Table S1. Fixed-airflow guide-contact heat-leak-fraction bracket at 12 mm guide clearance.

Setting	ε points	NCG-associated	Contact-associated	Total	Max gap (K/W)
Contact off	51	3	0	3	7.542×10^{-4}
0%	51	4	0	4	6.485×10^{-4}
1%	51	4	3	7	6.485×10^{-4}
2.5%	51	4	7	11	6.485×10^{-4}
				15	
5%	51	4	11		1.192×10^{-3}

Contact-associated unsupported points occur at every nonzero member of the 1–5% heat-leak-fraction bracket (3, 7, and 11 points at 1%, 2.5%, and 5%). Thus, 2.5% represents the midpoint of this scenario family. Counts in the coarse 51-level screen describe bracket sensitivity, while the dense ε-constraint reference in Section S2 identifies the two unsupported segments used in the main comparison.

S2. High-Resolution Fixed-Airflow Verification

The verification study assigns four numerical roles: 121 weighted-sum levels form the supported-front baseline; the original single-seed GA (seed 6203) contributes 40 audited nondominated points as the unrefined global-search comparison; the proposed GA-globalized augmented-Tchebycheff SQP method returns 119 nondominated points as the hybrid result; and a 121-level ε-constraint calculation yields 120 unique reference points. Unsupported status is determined from the lower convex envelope of the dense ε-constraint reference. Table S2 summarizes the two segments, and Table S3 reports pointwise evidence for Segment 2.

Table S2. Two numerically verified unsupported segments of the prescribed scenario model.

Segment	Attribution	Mass (kg)	Points	Max gap (K/W)	NCG severity	Contact points
1	Bounded NCG transition	1.491726–1.572487	11	7.441×10^{-4}	1.214–1.350	0
2	Guide-contact induced	1.871299–1.992439	16	5.205×10^{-4}	0.754–0.840	16

Table S3. Pointwise evidence in the contact-induced segment.

Quantity	Range/count	Interpretation
Guide reaction	0.378–2.791 N	Positive at all 16 points
Contact heat leak	31.20–60.74 W	Activated with contact
NCG severity	0.754–0.840	Below upper cap

Segment 1 is entirely non-contacting and follows the bounded NCG transition. Segment 2 begins at contact onset; every one of its 16 points has positive guide reaction and contact-induced heat leakage. Its NCG severity remains below the upper cap, separating contact induction from the first mechanism.

S3. Front-Conditioned Perturbation Screen (±5% Inputs)

A simultaneous perturbation screen re-evaluated the ε-constraint reference designs for 300 random trials. Independent multipliers uniformly distributed over 0.95–1.05 were applied to the NCG reference resistance, guide-contact heat-leak coefficient, air-side heat-transfer coefficient, liquid conductivity, solid-wick conductivity, reference radial-resistance components, and reference vapor resistance. Feasibility and nondominance were recomputed for each trial. Table S4 summarizes the results and supports the uncertainty hierarchy in Section 2.4 of the main manuscript.

Table S4. Results of the front-conditioned perturbation screen under simultaneous ±5% input changes.

Trials	Band 1 persistence	Band 2 persistence	Both bands	Median max gap 1	Median max gap 2
300	100%	100%	100%	7.458×10^{-4}	5.162×10^{-4}

Both baseline mass bands contained unsupported points in all 300 trials. This front-conditioned screen measures local persistence of the reported gaps under simultaneous coefficient variation on the saved ε-constraint design set. It complements the experimental instrument-accuracy statement and the NCG model-form comparison reported below.

Instrument uncertainty and model-coefficient sensitivity are treated separately. The companion batch-test platform uses ±1.5 °C temperature-measurement accuracy and ±1.0% cooling-flow accuracy. These acquisition limits characterize the experimental reference state. The ±5% front-conditioned screen reported above instead probes local uncertainty in model coefficients and resistance scales; it does not propagate a full experimental posterior distribution through repeated global reoptimization.

S4. NCG Physical Basis and Model-Form Sensitivity

For a fixed retained amount of non-condensable gas, its mean concentration varies inversely with available condenser vapor volume. Increased concentration can expand gas blanketing and reduce effective condenser area. The model uses this inventory-volume relation only as a first-order severity surrogate:

$$\frac{C_{\mathrm{NCG}}}{C_{\mathrm{NCG,ref}}} \approx \frac{V_{c,\mathrm{ref}}}{V_c}, S_{\mathrm{NCG}} = \mathrm{clip}[\left(\frac{V_{c,\mathrm{ref}}}{V_c}\right)^{p}, S_{\min}, S_{\max}].$$

The baseline interval of 0.70–1.35 keeps the inverse-volume NCG surrogate within a response range consistent with the present experimental setup and operating conditions. The bounds define the scenario limits for this model. Alternative intervals, exponents, and response forms quantify how the predicted Pareto topology depends on that choice. Because NCG redistribution also depends on species inventory, temperature, diffusion, transients, and orientation, the inverse-volume relation is used as a first-order surrogate. Table S5 summarizes the model-form sensitivity and complements Sections 2.2 and 4.3 of the main manuscript.

Table S5. NCG bounds, exponent, and response-form sensitivity on the saved ε-constraint designs.

Scenario	Bounds	p	Form	Segments	Unsupported	Pre-contact max gap
baseline hard clip	0.70–1.35	1.0	Hard clip	2	27	7.441×10^{-4}
narrow bounds	0.80–1.20	1.0	Hard clip	3	30	5.692×10^{-4}
wide bounds	0.60–1.50	1.0	Hard clip	3	28	3.081×10^{-4}
weaker exponent	0.70–1.35	0.8	Hard clip	3	29	3.899×10^{-4}
stronger exponent	0.70–1.35	1.2	Hard clip	3	41	1.138×10^{-3}
smooth saturation	0.70–1.35	1.0	Smooth saturation	4	27	2.426×10^{-4}
unbounded power	–	1.0	Unbounded power	2	25	N/A

N/A denotes that the unbounded-power scenario produced no pre-contact unsupported point, so a pre-contact maximum gap is not defined. The baseline hard-clip model retains two segments, but alternative bounds, exponents, and smooth saturation produce three or four segments; the unbounded power response again produces two. Thus, the bounded-NCG attribution is a plausible interpretation within the prescribed scenario model, whereas the exact count, location, and allocation of NCG-associated unsupported points are model-form dependent and require experimental calibration.

S5. Weighted-Sum, Original GA, and Proposed Hybrid Coverage

Table S6. Retained method-comparison metrics relative to the 27-point ε-constraint reference.

Method	ND points	In two bands	Reference covered	Evaluations	Median distance
Weighted sum (121)	50	0	0/27	7185	0.0775
Original GA (seed 6203)	40	Not assessed	Not used as proof	55441	N/A
Proposed hybrid	119	27	23/27	2240950	0.0037
ε-constraint reference	120	27	27/27	48018	0

Table S6 shows that weighted-sum SQP placed no solution inside either unsupported mass band. The original GA contains 40 audited nondominated points, spans both single-objective endpoints to numerical tolerance, and provides the unrefined global-search comparison. The proposed hybrid method contains 119 feasible nondominated points and recovers 23 of the 27 ε-constraint-confirmed unsupported reference points (7/11 in Segment 1 and 16/16 in Segment 2).

The reported comparison contains two GA-based result sets: the 40-point single-seed original GA and the 119-point proposed hybrid method. The former quantifies direct GA coverage, while the latter is the manuscript's final optimization result. Table S6 reports both sets against the common ε-constraint reference used in Section 4.2 of the main manuscript.

The uncertainty hierarchy used in the revised manuscript is therefore: instrument accuracy for the experimental anchor, deterministic solver tolerances for numerical

convergence, simultaneous coefficient perturbations for local model sensitivity, and alternative NCG response forms for model-form sensitivity.

S6. Engineering Interpretation and Calibration Scope

Engineering use. The optimized geometries provide candidates for prototype manufacture and rated power testing. System-specific values for the guide-contact heat-leak fraction, clearance, hot modulus, allowable guide reaction, leakage law, NCG inventory, exponent, and saturation bounds are calibrated against the final mounting and operating configuration before production selection.