

Supplementary Materials: Modeling power consumption: A novel correlation for stirred media mills with variable bead filling ratios

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Table S1. Statistical analysis of Model 1.0a.

(Fitting based on the extracted data from Kwade & Schwedes¹.)

φ	Parameter	Estimate ^a	Std. Error	T-value	P-value	Significance
0.35	$K^{0.35}$	1.21×10^2	8.98×10^0	1.34×10^1	3.62×10^{-8}	Significant
	$\gamma^{0.35}$	-5.05×10^{-1}	1.13×10^{-2}	-4.48×10^{-1}	8.41×10^{-14}	Significant
0.65	$K^{0.65}$	2.14×10^2	3.15×10^1	6.81×10^0	2.91×10^{-5}	Significant
	$\gamma^{0.65}$	-5.09×10^{-1}	2.23×10^{-2}	-2.28×10^1	1.31×10^{-10}	Significant
0.90	$K^{0.90}$	4.27×10^3	1.26×10^3	3.38×10^0	8.11×10^{-3}	Significant
	$\gamma^{0.90}$	-7.84×10^{-1}	4.61×10^{-2}	-1.70×10^1	3.76×10^{-8}	Significant

^a Fitting region: $1.2 \times 10^2 < Re < 8.0 \times 10^3$, model extension: $8.0 \times 10^3 < Re < 3.5 \times 10^4$, constant: $Re > 3.5 \times 10^4$.

Table S2. Statistical analysis of Model 1.0b.

(Fitting based on the extracted data from Kwade & Schwedes¹.)

φ	Parameter	Estimate ^a	Std. Error	T-value	P-value	Significance
0.35	$K^{0.35}$	1.02×10^2	1.17×10^1	8.77×10^0	1.64×10^{-7}	Significant
	$\gamma^{0.35}$	-4.79×10^{-1}	1.68×10^{-2}	-2.84×10^1	4.00×10^{-15}	Significant
0.65	$K^{0.65}$	1.89×10^2	2.50×10^1	7.56×10^0	1.14×10^{-6}	Significant
	$\gamma^{0.65}$	-4.89×10^{-1}	1.96×10^{-2}	-2.49×10^1	3.22×10^{-14}	Significant
0.90	$K^{0.90}$	3.65×10^3	9.38×10^2	3.89×10^0	1.85×10^{-3}	Significant
	$\gamma^{0.90}$	-7.59×10^{-1}	3.97×10^{-2}	-1.91×10^1	6.71×10^{-11}	Significant

^a Fitting region: $1.2 \times 10^2 < Re < 3.5 \times 10^4$, no model extension, constant: $Re > 3.5 \times 10^4$.

Table S3. Statistical analysis of Model 1.1.

(Fitting based on the extracted data from Kwade & Schwedes¹.)

φ (%)	Parameter	Estimate	Std. Error	T-value	P-value	Significance
0.35	$K_1^{0.35}$	1.21×10^2	7.76×10^0	1.55×10^1	2.67×10^{-15}	Significant
	$K_2^{0.35}$	5.97×10^0	4.90×10^0	1.22×10^0	2.33×10^{-1}	Not significant

¹ Kwade, Arno, and Jörg Schwedes. "Wet grinding in stirred media mills." *Handbook of powder technology* 12 (2007): 251-382.

	$K_3^{0.35}$	1.82×10^1	2.46×10^1	7.40×10^{-1}	4.65×10^{-1}	Not significant
	$K_4^{0.35}$	6.93×10^{-1}	4.16×10^{-2}	1.66×10^1	4.70×10^{-16}	Significant
	$Y_1^{0.35}$	-5.05×10^{-1}	9.74×10^{-3}	-5.18×10^1	2.33×10^{-29}	Significant
	$Y_2^{0.35}$	-1.60×10^{-1}	8.50×10^{-2}	-1.88×10^0	7.08×10^{-2}	Not significant
	$Y_3^{0.35}$	-2.77×10^{-1}	1.21×10^{-1}	-2.29×10^0	3.01×10^{-2}	Significant
0.65	$K_1^{0.65}$	2.10×10^2	2.13×10^1	9.87×10^0	1.28×10^{-10}	Significant
	$K_2^{0.65}$	9.51×10^0	1.53×10^1	6.20×10^{-1}	5.40×10^{-1}	Not significant
	$K_3^{0.65}$	3.45×10^1	7.60×10^1	4.54×10^{-1}	6.54×10^{-1}	Not significant
	$K_4^{0.65}$	1.08×10^0	1.11×10^{-1}	9.70×10^0	1.86×10^{-10}	Significant
	$Y_1^{0.65}$	-5.06×10^{-1}	1.54×10^{-2}	-3.29×10^1	6.42×10^{-24}	Significant
	$Y_2^{0.65}$	-1.59×10^{-1}	1.66×10^{-1}	-9.55×10^{-1}	3.48×10^{-1}	Not significant
	$Y_3^{0.65}$	-2.84×10^{-1}	1.97×10^{-1}	-1.44×10^0	1.60×10^{-1}	Not significant
0.90	$K_1^{0.90}$	4.25×10^3	7.40×10^2	5.74×10^0	3.66×10^{-6}	Significant
	$K_2^{0.90}$	5.21×10^1	1.64×10^2	3.18×10^{-1}	7.53×10^{-1}	Not significant
	$K_3^{0.90}$	3.00×10^0	8.60×10^0	3.49×10^{-1}	7.30×10^{-1}	Not significant
	$K_4^{0.90}$	2.40×10^0	3.79×10^{-1}	6.32×10^0	7.82×10^{-7}	Significant
	$Y_1^{0.90}$	-7.83×10^{-1}	2.71×10^{-2}	-2.89×10^1	2.21×10^{-22}	Significant
	$Y_2^{0.90}$	-2.64×10^{-1}	3.32×10^{-1}	-7.97×10^{-1}	4.32×10^{-1}	Not significant
	$Y_3^{0.90}$	-4.14×10^{-3}	2.52×10^{-1}	-1.64×10^{-2}	9.87×10^{-1}	Not significant

Table S4. Statistical analysis of Model 1.2.

(Fitting based on the extracted data from Kwade & Schwedes ¹ .)						
φ (%)	Parameter	Estimate	Std. Error	T-value	P-value	Significance
0.35	$K^{0.35}$	1.18×10^2	1.02×10^1	1.15×10^1	4.98×10^{-11}	Significant
	$Y_1^{0.35}$	-5.01×10^{-1}	1.31×10^{-2}	-3.82×10^1	2.62×10^{-22}	Significant
	$Y_2^{0.35}$	-4.72×10^{-1}	1.09×10^{-2}	-4.34×10^1	1.41×10^{-23}	Significant
	$Y_3^{0.35}$	-4.45×10^{-1}	9.76×10^{-3}	-4.56×10^1	4.55×10^{-24}	Significant
0.65	$K^{0.65}$	2.10×10^2	2.63×10^1	7.98×10^0	6.09×10^{-8}	Significant
	$Y_1^{0.65}$	-5.06×10^{-1}	1.90×10^{-2}	-2.66×10^1	3.22×10^{-18}	Significant
	$Y_2^{0.65}$	-4.80×10^{-1}	1.62×10^{-2}	-2.97×10^1	2.98×10^{-19}	Significant
	$Y_3^{0.65}$	-4.47×10^{-1}	1.46×10^{-2}	-3.06×10^1	1.61×10^{-19}	Significant
0.90	$K^{0.90}$	3.00×10^2	N/A ^a	N/A ^a	N/A ^a	N/A ^a
	$Y_1^{0.90}$	-5.22×10^{-1}	N/A ^a	N/A ^a	N/A ^a	N/A ^a
	$Y_2^{0.90}$	-4.52×10^{-1}	N/A ^a	N/A ^a	N/A ^a	N/A ^a
	$Y_3^{0.90}$	-3.09×10^{-1}	N/A ^a	N/A ^a	N/A ^a	N/A ^a

^a N/A: Not applicable. Results are not displayed due to lack of fitting capability.

Table S5. Statistical analysis of Model 1.3.

(Fitting based on the extracted data from Kwade & Schwedes ¹ .)						
φ (%)	Parameter	Estimate	Std. Error	T-value	P-value	Significance
0.35	$K^{0.35}$	1.18×10^2	9.07×10^0	1.30×10^1	7.39×10^{-14}	Significant
	$K_4^{0.35}$	6.93×10^{-1}	5.00×10^{-2}	1.39×10^1	1.38×10^{-14}	Significant
	$Y_1^{0.35}$	-5.01×10^{-1}	1.16×10^{-2}	-4.31×10^1	1.53×10^{-28}	Significant
	$Y_2^{0.35}$	-4.72×10^{-1}	9.63×10^{-3}	-4.90×10^1	3.39×10^{-30}	Significant
	$Y_3^{0.35}$	-4.45×10^{-1}	8.65×10^{-3}	-5.15×10^1	7.77×10^{-31}	Significant

0.65	$K^{0.65}$	2.10×10^2	2.27×10^1	9.27×10^0	2.56×10^{-10}	Significant
	$K_4^{0.65}$	1.09×10^0	1.19×10^{-1}	9.17×10^0	3.27×10^{-10}	Significant
	$Y_1^{0.65}$	-5.06×10^{-1}	1.64×10^{-2}	-3.09×10^1	2.68×10^{-24}	Significant
	$Y_2^{0.65}$	-4.80×10^{-1}	1.39×10^{-2}	-3.45×10^1	1.04×10^{-25}	Significant
	$Y_3^{0.65}$	-4.47×10^{-1}	1.26×10^{-2}	-3.55×10^1	4.51×10^{-26}	Significant
0.90	$K^{0.90}$	4.14×10^3	7.71×10^2	5.37×10^0	8.25×10^{-6}	Significant
	$K_4^{0.90}$	2.40×10^0	4.08×10^{-1}	5.88×10^0	1.95×10^{-6}	Significant
	$Y_1^{0.90}$	-7.79×10^{-1}	2.90×10^{-2}	-2.69×10^1	1.54×10^{-22}	Significant
	$Y_2^{0.90}$	-7.30×10^{-1}	2.58×10^{-2}	-2.83×10^1	3.48×10^{-23}	Significant
	$Y_3^{0.90}$	-6.54×10^{-1}	2.16×10^{-2}	-3.03×10^1	4.67×10^{-24}	Significant

Table S6. Statistical analysis of Model 1.4.

(Fitting based on the extracted data from Kwade & Schwedes ¹ .)						
φ (%)	Parameter	Estimate	Std. Error	T-value	P-value	Significance
0.35	$a_1^{0.35}$	1.06×10^{-1}	3.13×10^{-1}	3.39×10^{-1}	7.37×10^{-1}	Not significant
	$a_2^{0.35}$	-1.26×10^{-1}	1.35×10^{-1}	-9.38×10^{-1}	3.56×10^{-1}	Not significant
	$a_3^{0.35}$	5.00×10^0	3.87×10^1	1.29×10^{-1}	8.98×10^{-1}	Not significant
	$a_4^{0.35}$	2.05×10^{-1}	8.52×10^{-1}	2.40×10^{-1}	8.12×10^{-1}	Not significant
	$a_5^{0.35}$	3.34×10^0	1.62×10^1	2.06×10^{-1}	8.38×10^{-1}	Not significant
0.65	$a_1^{0.65}$	7.91×10^{-1}	9.69×10^{-1}	8.16×10^{-1}	4.21×10^{-1}	Not significant
	$a_2^{0.65}$	-1.00×10^{-2}	8.19×10^{-2}	-1.22×10^{-1}	9.04×10^{-1}	Not significant
	$a_3^{0.65}$	2.37×10^0	4.05×10^1	5.86×10^{-2}	9.54×10^{-1}	Not significant
	$a_4^{0.65}$	2.34×10^{-1}	1.26×10^0	1.85×10^{-1}	8.54×10^{-1}	Not significant
	$a_5^{0.65}$	6.49×10^0	7.83×10^1	8.28×10^{-2}	9.35×10^{-1}	Not significant
0.90	$a_1^{0.90}$	8.36×10^{-1}	3.71×10^{-1}	2.25×10^0	3.18×10^{-2}	Significant
	$a_2^{0.90}$	-1.51×10^{-2}	4.05×10^{-2}	-3.73×10^{-1}	7.12×10^{-1}	Not significant
	$a_3^{0.90}$	2.88×10^0	3.66×10^1	7.87×10^{-2}	9.38×10^{-1}	Not significant
	$a_4^{0.90}$	2.82×10^{-1}	8.28×10^{-1}	3.41×10^{-1}	7.35×10^{-1}	Not significant
	$a_5^{0.90}$	9.92×10^0	8.38×10^1	1.18×10^{-1}	9.07×10^{-1}	Not significant

Table S7. Statistical analysis of Model 2.0.

(Fitting based on the extracted data from Kwade & Schwedes ¹ .)					
Parameter	Estimate	Std. Error	T-value	P-value	Significance
K	2.39×10^3	4.47×10^2	5.35×10^0	5.43×10^{-7}	Significant
Y	-6.44×10^{-1}	2.75×10^{-2}	-2.34×10^1	7.29×10^{-43}	Significant
X	3.20×10^0	2.05×10^{-1}	1.56×10^1	9.69×10^{-29}	Significant

Table S8. Statistical analysis of Model 2.1.

(Fitting based on the extracted data from Kwade & Schwedes ¹ .)					
Parameter	Estimate	Std. Error	T-value	P-value	Significance
K_1	3.85×10^3	7.10×10^2	5.42×10^0	4.66×10^{-7}	Significant
K_2	4.83×10^3	1.73×10^4	2.80×10^{-1}	7.80×10^{-1}	Not significant
K_3	5.88×10^2	1.92×10^3	3.05×10^{-1}	7.61×10^{-1}	Not significant
K_4	2.80×10^0	6.79×10^{-1}	4.13×10^0	7.99×10^{-5}	Significant

Y_1	-7.14×10^{-1}	2.78×10^{-2}	-2.57×10^1	2.55×10^{-44}	Significant
Y_2	-7.32×10^{-1}	3.83×10^{-1}	-1.91×10^0	5.90×10^{-2}	Not significant
Y_3	-4.66×10^{-1}	2.94×10^{-1}	-1.59×10^0	1.16×10^{-1}	Not significant
X_1	3.37×10^0	1.79×10^{-1}	1.88×10^1	1.15×10^{-33}	Significant
X_2	1.44×10^0	6.10×10^{-1}	2.36×10^0	2.03×10^{-2}	Significant
X_3	1.53×10^0	6.54×10^{-1}	2.34×10^0	2.12×10^{-2}	Significant
X_4	1.77×10^0	9.02×10^{-1}	1.97×10^0	5.23×10^{-2}	Not significant

Table S9. Statistical analysis of Model 2.2.

(Fitting based on the extracted data from Kwade & Schwedes ¹ .)					
Parameter	Estimate	Std. Error	T-value	P-value	Significance
K	4.03×10^3	7.69×10^2	5.24×10^0	9.20×10^{-7}	Significant
Y_1	-7.21×10^{-1}	2.87×10^{-2}	-2.51×10^1	3.03×10^{-44}	Significant
Y_2	-7.14×10^{-1}	3.01×10^{-2}	-2.37×10^1	3.71×10^{-42}	Significant
Y_3	-6.37×10^{-1}	2.55×10^{-2}	-2.50×10^1	4.79×10^{-44}	Significant
X_1	3.39×10^0	1.85×10^{-1}	1.83×10^1	3.39×10^{-33}	Significant
X_2	1.42×10^0	6.22×10^{-1}	2.29×10^0	2.42×10^{-2}	Significant
X_3	1.65×10^0	7.13×10^{-1}	2.31×10^0	2.28×10^{-2}	Significant
X_4	7.05×10^1	2.55×10^0	2.76×10^1	9.63×10^{-48}	Significant

Table S10. Statistical analysis of Model 2.3.

(Fitting based on the extracted data from Kwade & Schwedes ¹ .)					
Parameter	Estimate	Std. Error	T-value	P-value	Significance
K	4.03×10^3	7.58×10^2	5.32×10^0	6.88×10^{-7}	Significant
K_4	2.80×10^0	6.90×10^{-1}	4.06×10^0	9.96×10^{-5}	Significant
Y_1	-7.21×10^{-1}	2.83×10^{-2}	-2.55×10^1	1.74×10^{-44}	Significant
Y_2	-7.14×10^{-1}	2.97×10^{-2}	-2.41×10^1	1.91×10^{-42}	Significant
Y_3	-6.08×10^{-1}	3.02×10^{-2}	-2.01×10^1	3.48×10^{-36}	Significant
X_1	3.39×10^0	1.83×10^{-1}	1.85×10^1	1.77×10^{-33}	Significant
X_2	1.39×10^0	6.05×10^{-1}	2.31×10^0	2.33×10^{-2}	Significant
X_3	3.97×10^0	1.86×10^0	2.13×10^0	3.58×10^{-2}	Significant
X_4	1.77×10^0	9.16×10^{-1}	1.93×10^0	5.59×10^{-2}	Not significant

Table S11. Statistical analysis of Model 2.4.

(Fitting based on the extracted data from Kwade & Schwedes ¹ .)					
Parameter	Estimate	Std. Error	T-value	P-value	Significance
a_0	4.06×10^0	1.10×10^{-1}	3.69×10^1	1.15×10^{-59}	Significant
a_1	-1.20×10^0	5.03×10^{-2}	-2.38×10^1	9.39×10^{-43}	Significant
a_2	4.08×10^0	2.09×10^{-1}	1.95×10^1	1.04×10^{-35}	Significant
a_3	1.01×10^{-1}	5.66×10^{-3}	1.78×10^1	1.19×10^{-32}	Significant
a_4	4.12×10^0	2.92×10^{-1}	1.41×10^1	2.09×10^{-25}	Significant
a_5	-1.33×10^{-1}	3.01×10^{-2}	-4.41×10^0	2.59×10^{-5}	Significant

Table S12. Statistical analysis of Model 2.5.

(Fitting based on the extracted data from Kwade & Schwedes¹.)

Parameter	Estimate	Std. Error	T-value	P-value	Significance
a_0	5.41×10^0	4.30×10^{-1}	1.26×10^1	5.58×10^{-22}	Significant
a_1	-1.76×10^0	3.02×10^{-1}	-5.84×10^0	7.25×10^{-8}	Significant
a_2	1.89×10^1	1.68×10^0	1.12×10^1	4.10×10^{-19}	Significant
a_3	2.06×10^{-1}	6.98×10^{-2}	2.95×10^0	4.02×10^{-3}	Significant
a_4	7.53×10^1	8.86×10^0	8.50×10^0	2.68×10^{-13}	Significant
a_5	-1.41×10^0	3.17×10^{-1}	-4.43×10^0	2.51×10^{-5}	Significant
a_6	-6.44×10^{-3}	5.31×10^{-3}	-1.21×10^0	2.28×10^{-1}	Not significant
a_7	9.84×10^1	1.28×10^1	7.67×10^0	1.49×10^{-11}	Significant
a_8	1.01×10^{-1}	3.21×10^{-2}	3.15×10^0	2.18×10^{-3}	Significant
a_9	-7.79×10^{-1}	2.71×10^{-1}	-2.87×10^0	4.99×10^{-3}	Significant

Table S13. Statistical analysis of Model 2.6.

(Fitting based on the extracted data from Kwade & Schwedes¹.)

Parameter	Estimate	Std. Error	T-value	P-value	Significance
a_1	1.27×10^0	1.46×10^{-2}	8.72×10^1	7.01×10^{-97}	Significant
a_2	1.15×10^{-1}	2.91×10^{-3}	3.97×10^1	1.75×10^{-63}	Significant
a_3	2.85×10^{-1}	8.40×10^{-3}	3.40×10^1	4.31×10^{-57}	Significant
a_4	2.67×10^0	2.79×10^{-1}	9.58×10^0	7.71×10^{-16}	Significant

Table S14. Validation dataset statistical analysis of Model 2.4.

(Fitting based on the data from Heidari et al².)

Parameter	Estimate	Std. Error	T-value	P-value	Significance
a_0	10.7×10^0	3.73×10^0	2.87×10^0	2.82×10^{-2}	Significant
a_1	-5.64×10^0	2.00×10^0	-2.81×10^0	3.05×10^{-2}	Significant
a_2	-1.96×10^0	4.39×10^0	-4.46×10^{-1}	6.71×10^{-1}	Not significant
a_3	7.26×10^{-1}	2.69×10^{-1}	2.70×10^0	3.55×10^{-2}	Significant
a_4	-1.81×10^0	3.62×10^0	-4.99×10^{-1}	6.35×10^{-1}	Not significant
a_5	8.79×10^{-1}	1.21×10^0	7.28×10^{-1}	4.94×10^{-1}	Not significant

Table S15. Validation dataset statistical analysis of Model 2.6.

(Fitting based on the data from Heidari et al².)

Parameter	Estimate	Std. Error	T-value	P-value	Significance
a_1	3.47×10^5	1.66×10^1	2.084×10^4	3.14×10^{-32}	Significant
a_2	1.96×10^0	1.09×10^{-2}	1.78×10^2	1.07×10^{-15}	Significant
a_3	2.79×10^{-1}	1.03×10^{-2}	2.70×10^1	3.79×10^{-9}	Significant
a_4	1.46×10^0	1.74×10^{-1}	8.36×10^0	3.18×10^{-5}	Significant

² Heidari, H., Vargas, A., Macris, A., Muller, F., Pikhard, O., Guner, G., Clancy, D.J. and Bilgili, E., 2025. Control of temperature rise during wet stirred media milling. *Advanced Powder Technology*, 36(9), p.104981.