

Supplementary Materials

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Mathematical analysis

In the remainder of this supplementary material, we provide the comprehensive mathematical analysis that was performed. Due to the extensiveness of the paper, the individual tables of mathematical analysis have been moved to these supplementary materials, but the reference to these tables is included in the paper. Only the first tables of mathematical analysis (always Monday) were left in the paper for specific examples of the current state, for national tickets, and for international tickets.

Current state

Table S1 shows the mathematical analysis of the current system on Tuesday, Wednesday and Thursday.

Table S1. Mathematical analysis of the current system on Tuesday, Wednesday and Thursday.

Interval	6.00 AM – 8.00 AM	9.00 AM – 1.00 PM	2.00 PM – 3.00 PM	4.00 PM – 6.00 PM	7.00 PM – 9.00 PM	10.00 PM – 5.00 AM
Number of service lines	4	5	5	4	3	2
Σ tickets	1,940	4,029	2,424	2,395	615	765
λ (customer/hour)	107.78	134.30	202.00	133.06	34.17	15.94
μ (customer/hour)	41.16	41.16	41.16	41.16	41.16	41.16
E(S) (-)	2.59	3.26	4.91	3.23	0.71	0.39
R (%)	64.69	65.11	98.15	80.81	23.65	19.35
E(L) (-)	0.35	0.57	50.72	0.61	0.01	0.02
E(W) (sec)	11.83	15.25	903.88	16.54	1.46	3.41

Table S2 analyses this interval during two January days (two Fridays) as part of personal observation.

Table S2. Mathematical analysis of the current system on Friday.

Interval	6.00 AM – 8.00 AM	9.00 AM – 1.00 PM	2.00 PM – 5.00 PM	6.00 PM – 7.00 PM	7.00 PM – 8.00 PM	8.00 PM – 9.00 PM	10.00 PM – 5.00 AM
Number of service lines	4	5	5	4	3	3	2
Σ tickets	652	1,807	1,624	305	125	149	269
λ (customer/hour)	108.67	180.70	203	152.50	62.50	37.25	16.81
μ (customer/hour)	41.16	41.16	41.16	41.16	41.16	41.16	41.16
E(S) (-)	2.64	4.39	4.93	3.71	1.52	0.90	0.41
R (%)	65.96	87.80	98.64	92.63	50.62	30.13	20.41
E(L) (-)	0.37	5.14	70.04	0.84	0.25	0.03	0.02
E(W) (sec)	12.20	102.42	1,242.11	19.87	14.37	2.95	3.81

Table S3 shows the mathematical analysis of the current system on Saturday.

Table S3. Mathematical analysis of the current system on Saturday.

Interval	6.00 AM – 7.00 AM	7.00 AM – 8.00 AM	9.00 AM – 12.00 AM	1.00 PM – 3.00 PM	4.00 PM – 6.00 PM	7.00 PM – 9.00 PM	10.00 PM – 5.00 AM
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Number of service lines	3	4	5	5	4	3	2
Σ tickets	126	456	1,192	953	682	277	240
λ (customer/hour)	63	114	149	159	114	46	15
μ (customer/hour)	41.16	41.16	41.16	41.16	41.16	41.16	41.16
E(S) (-)	1.53	2.77	3.62	3.86	2.77	1.12	0.36
R (%)	51.02	69.24	72.40	77.26	69.24	37.25	18.22
E(L) (-)	0.26	0.42	1.09	1.71	0.42	0.07	0.01
E(W) (sec)	14.75	13.17	26.43	38.65	13.17	5.53	3.03

Table S4 shows the mathematical analysis of the current system on Sunday.

Table S4. Mathematical analysis of the current system on Sunday.

Interval	6.00 AM – 7.00 AM	7.00 AM – 8.00 AM	9.00 AM – 12.00 AM	1.00 PM – 5.00 PM	6.00 PM – 7.00 PM	7.00 PM – 9.00 PM	10.00 PM – 5.00 AM
Number of service lines	3	4	5	5	4	3	2
Σ tickets	55	274	1,030	1,699	310	375	84
λ (customer/hour)	27.5	68.5	128.75	169.9	155	62.5	5.25
μ (customer/hour)	41.16	41.16	41.16	41.16	41.16	41.16	41.16
E(S) (-)	0.668	1.664	3.128	4.128	3.766	1.518	0.128
R (%)	22.22	41.61	62.56	82.56	94.14	50.62	6.38
E(L) (-)	0.009	0.102	0.450	2.861	0.873	0.250	0.001
E(W) (sec)	1.226	5.362	12.570	60.610	20.284	14.374	0.036

National tickets

Table S5 analyses the situation where 11,824 customers entered the system during this interval.

Table S5. Mathematical analysis of the proposed system on Tuesday, Wednesday and Thursday.

Interval	6.00 AM – 7.00 AM	8.00 AM – 9.00 AM	9.00 AM – 1.00 PM	2.00 PM – 3.00 PM	4.00 PM – 7.00 PM	8.00 PM – 5.00 AM
Number of service lines	2	3	3	3	2	1
Σ tickets	1,299	515	3,874	2,320	2,546	1,270
λ (customer/hour)	30.93	85.83	129.13	193.33	106.08	21.17
μ (customer/hour)	88.25	88.25	88.25	88.25	88.25	88.25
E(S) (-)	0.35	0.97	1.46	2.19	1.20	0.24
R (%)	17.52	32.42	48.78	73.01	60.10	23.98
E(L) (-)	0.01	0.04	0.21	1.45	0.68	0.08
E(W) (sec)	1.29	1.71	5.94	27.07	23.07	12.87

Table S6 shows the mathematical analysis of the proposed system on Friday.

Table S6. Mathematical analysis of the proposed system on Friday.

Interval	6.00 AM – 7.00 AM	8.00 AM – 9.00 AM	9.00 AM – 1.00 PM	2.00 PM – 3.00 PM	4.00 PM – 7.00 PM	8.00 PM – 5.00 AM
Number of service lines	2	3	3	3	2	1
Σ tickets	420	223	1,771	1,023	1,282	211
λ (customer/hour)	105	111.5	177.1	255.75	160.25	10.55
μ (customer/hour)	88.25	88.25	88.25	88.25	88.25	88.25
E(S) (-)	1.19	1.26	1.94	2.90	1.82	0.12
R (%)	59.49	42.12	64.63	96.60	90.76	11.90

E(L) (-)	0.65	0.12	0.76	26.61	8.49	0.02
E(W) (sec)	22.35	3.75	16.00	374.58	190,76	5.51

Table S7 shows enough proposed service lines, while a reserve is maintained in case of a higher random entry of customers into the system.

Table S7. Mathematical analysis of the proposed system (Saturday).

Interval	6.00 AM – 7.00 AM	8.00 AM – 9.00 AM	9.00 AM – 1.00 PM	2.00 PM – 3.00 PM	4.00 PM – 7.00 PM	8.00 PM – 5.00 AM
Number of service lines	1	3	3	3	2	1
Σ tickets	342	229	1,470	624	796	392
λ (customer/hour)	85.5	114.5	147	156	99.5	19.6
μ (customer/hour)	88.25	88.25	88.25	88.25	88.25	88.25
E(S) (-)	0.97	1.30	1.67	1.77	1.12	0.22
R (%)	56.66	43.25	55.52	58.92	55.95	21.59
E(L) (-)	0.74	0.13	0.37	0.49	0.51	0.06
E(W) (sec)	53.32	4.07	9.15	11.29	18.59	11.23

Table S8 shows the mathematical analysis of the proposed system on Sunday.

Table S8. Mathematical analysis of the proposed system (Sunday).

Interval	6.00 AM – 7.00 AM	8.00 AM – 9.00 AM	9.00 AM – 1.00 PM	2.00 PM – 3.00 PM	4.00 PM – 7.00 PM	8.00 PM – 5.00 AM
Number of service lines	1	3	3	3	2	1
Σ tickets	176	107	1,242	734	1,134	274
λ (customer/hour)	44	53.5	124.2	183.5	141.75	13.7
μ (customer/hour)	88.25	88.25	88.25	88.25	88.25	88.25
E(S) (-)	0.50	0.61	1.41	2.08	1.61	0.16
R (%)	49.86	20.21	46.91	69.31	80.31	15.52
E(L) (-)	0.50	0.01	0.18	1.09	2.92	0.03
E(W) (sec)	40.56	0.43	5.25	21.37	74.12	7.50

International tickets

On Tuesday, Wednesday and Thursday, 344 customers entered the system with a request to issue an international travel ticket, as shown in Table S9.

Table S9. Mathematical analysis of the proposed system for international travel tickets (Tuesday, Wednesday and Thursday).

Interval	5.00 AM – 6.00 AM	6.00 AM – 8.00 AM	9.00 AM – 3.00 PM	4.00 PM – 7.00 PM	8.00 PM – 4.00 AM
Number of service lines	1	1	2	1	1
Σ tickets	5	27	214	56	42
λ (customer/hour)	1.67	3.00	10.19	4.67	1.56
μ (customer/hour)	13.1	13.1	13.1	13.1	13.1
E(S) (-)	0.13	0.23	0.78	0.35	0.11
R (%)	12.67	22.90	38.89	35.11	11.45
E(L) (-)	0.02	0.07	0.14	0.19	0.01
E(W) (sec)	39.88	81.63	48.98	148.72	35.54

A mathematical analysis of the proposed system for international Friday tickets is shown in Table S10.

Table 10. Mathematical analysis of the proposed system for international travel tickets (Friday).

Interval	5.00 AM – 6.00 AM	6.00 AM – 8.00 AM	9.00 AM – 3.00 PM	4.00 PM – 7.00 PM	8.00 PM – 4.00 AM
Number of service lines	1	1	2	1	1
Σ tickets	2	9	84	17	6
λ (customer/hour)	1.00	1.50	6.00	2.13	0.33
μ (customer/hour)	13.1	13.1	13.1	13.1	13.1
E(S) (-)	0.08	0.11	0.46	0.16	0.03
R (%)	0.08	0.11	0.23	0.16	0.03
E(L) (-)	0.01	0.01	0.03	0.03	0.00
E(W) (sec)	22.71	35.54	15.21	53.21	7.10

Table S11 shows the mathematical analysis of the proposed system for international Saturday tickets. Most requests are estimated in the interval from 9.00 AM to 3.00 PM, when a total of 51 customers entered the system during the two Saturdays as part of the observation. The use of the system also includes a possible reserve for a higher intensity of customer entry.

Table S11. Mathematical analysis of the proposed system for international travel tickets (Saturday).

Interval	5.00 AM – 6.00 AM	6.00 AM – 7.00 AM	8.00 AM – 9.00 AM	9.00 AM – 3.00 PM	4.00 PM – 7.00 PM	8.00 PM – 4.00 AM
Number of service lines	1	1	1	2	1	1
Σ tickets	2	1	10	51	5	4
λ (customer/hour)	1	0.25	5	3.64	0.625	0.22
μ (customer/hour)	13.1	13.1	13.1	13.1	13.1	13.1
E(S) (-)	0.08	0.02	0.38	0.28	0.05	0.02
R (%)	7.63	1.91	38.17	13.89	4.77	1.68
E(L) (-)	0.01	0.00	0.24	0.01	0.00	0.00
E(W) (sec)	22.71	5.35	169.64	5.41	13.77	4.69

A mathematical analysis of the proposed system for international Sunday tickets is shown in Table S12.

Table S12. Mathematical analysis of the proposed system for international travel tickets (Sunday).

Interval	5.00 AM – 6.00 AM	6.00 AM – 7.00 AM	8.00 AM – 9.00 AM	9.00 AM – 3.00 PM	4.00 PM – 7.00 PM	8.00 PM – 4.00 AM
Number of service lines	1	1	1	2	1	1
Σ tickets	1	7	1	71	71	9
λ (customer/hour)	0.5	1.75	0.5	5.17	8.88	0.5
μ (customer/hour)	13.1	13.1	13.1	13.1	13.1	13.1
E(S) (-)	0.04	0.13	0.04	0.39	0.68	0.04
R (%)	3.82	13.36	3.82	19.35	67.71	3.82
E(L) (-)	0.00	0.02	0.00	0.02	1.42	0.00
E(W) (sec)	10.91	42.37	10.91	10.70	576.25	10.91