Supplementary Materials: Evolutionary Dynamics of Gig Economy Labor Strategies under Technology, Policy and Market Influence

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1. Overview

In this demonstration of payoff generation and analysis, we explore market influences on firm and laborer strategies in different firm settings. We discretize firms by size and skill set into four classifications : small low-skill firms (*i.e., family owned restaurant business*), large low-skill firms (*i.e., Uber*), small high-skill firms (*i.e., early stage technology startup*) and large high-skill firms (*i.e., Microsoft*). For each of the four firm categories, we generate payoffs that reflect the incentives for each labor strategy in a bear or bull market. We recast gig-employee trade-offs as employment incentives or deterrents for each strategy. For the firm, the strategy set consists of hiring either a gig worker or an employee. For the laborer, the strategy set consists of participating in the labor market as either a gig worker or an employee. To generate each strategy payoff for the firm, we consider factors such as operational revenue, cost of labor and other hiring considerations such as worker reliability, cost of talent acquisition and labor flexibility. For laborer payoffs, we factor in compensation, bonuses and utility gained from alternative engagements outside the work contract. Firm and laborer payoffs are represented in a payoff bi-matrix.

2. Payoff Generation

In this next section, we generate utility payoffs for firms and laborers across firm category and market condition. In this demonstration, we evaluate four categories of firms; the categories are defined as *Small Low-Skill Firm*, *Small High-Skill Firm*, *Large Low-Skill Firm*, and *Large High-Skill Firm*. For each firm category, a pair of payoff matrices are generated to represent the firm in bear and bull market conditions.

2.1. GameStates, Contracts and Payoff Coefficients

A firm's operation consists of numerous processes or tasks that must be effectuated within a time interval in order for the firm to maintain operation and stay competitive. We simplify the myriad of possible processes by representing these tasks with four contract categories; the categories are *Short-Term Low-Skill Contract*, *Short-Term High-Skill Contract*, *Long-Term Low-Skill Contract*, and *Long-Term High-Skill Contract*. Indeed, each firm's operational need translates into a discrete distribution involving these four contract categories; this is the firm's labor demand.

To fulfill each individual contract, a firm can choose to hire either a gig worker or an employee. Conversely, a laborer can participate as either a gig worker or an employee in competition of a contract. Accordingly, payoffs are assigned to each firm and laborer strategy to model the efficacy of the strategy.

A firm's labor demand distribution is determined by market condition (bear or bull), firm size (small or large), and firm skill-set (low or high skill). The eight combinations of the aforementioned three constituents (market condition, firm size and firm skill-set) constitute the eight discrete GameStates; a GameState represents one of the four firm categories in one of the two market conditions. For example, the GameState *Small Low Bear* denotes a small low-skill firm in a bear market.

In addition to discrete contract distributions, each GameState will have four coefficients for flexibility, reliability, talent retention and potential alternatives. These coefficients represent the importance of additional employment incentives for firms and laborers beyond compensation or cost of labor. Each payoff coefficient instantiates the respective weight or importance of flexibility, reliability, talent retention and potential alternatives in each GameState.

The flexibility payoff coefficient weighs the importance of flexible labor for the firm. The reliability payoff coefficient accounts for the significance of labor quality and worker reliability for the firm. The talent retention payoff coefficient represents the firm's cost of obtaining and retaining labor talent. Finally, the potential alternatives coefficient denotes the laborer's potential utility from participating in alternative activities outside the contract, serving as a proxy for laborer flexibility. In sum, payoff coefficients are additional employment considerations for firms and laborers.

For each GameState constituent (Firm size (small or large) denoted by F_{size}, Firm skill set (high or low-skill) denoted by F_{skill}, and market denoted by M (bear or bull)), we assume a contributing weight for each payoff coefficient. For each GameState, i.e., Small Low Bear, payoff coefficients are calculated by taking the summation of applicable weights. To illustrate an example, the potential alternatives coefficient Π_P is determined by firm size, firm skill and market condition. In the *SmallBearLow* Gamestate, the potential alternatives coefficient is a combination of potential alternatives weights assigned by $F_{size} : Small$, $F_{skill} : Low$ and $M : Bear$.

$$\Pi_P(F_{size}, F_{skill}, M) = \Pi_P(F_{size} : Small) + \Pi_P(F_{skill} : Low) + \Pi_P(M : Bear)$$

GameState constituents also influence how a firm partitions a labor budget across different types of contracts. We apply a similar operation to determine the proportion of high and low-skill contracts and the proportion of long and short contracts.

2.1.1. GameState Constituent Assumptions

In this section, we specify the contract proportions and payoff weights for each GameState constituent. $\gamma_{(S)}$ denotes the weighted fraction of short contracts, $\gamma_{(L)}$ denotes the weighted fraction of long contracts, Π_F denotes flexibility, Π_R denotes reliability, Π_T denotes talent retention, Π_P denotes potential alternatives, $\mu_{(Lo)}$ denotes the fraction of low skill contracts and $\mu_{(Hi)}$ denotes the fraction of high skill contracts.

M **Bear: (**$\gamma_{(S)}$**: +**$\frac{3}{10}$**,** $\gamma_{(L)}$**: +**$\frac{2}{10}$**,** Π_F**:** $+1$**,** Π_R**:** $+5$**,** Π_T**:** 0**,** Π_P**:** 0**)**

We assume that bear market conditions incentivize firms to become more risk averse in their long term strategies and therefore spend more conservatively on long term projects and risky innovation. Simultaneously, we reason that firms will focus resources on flexible short term strategies that allow for quick adaptability to unfavorable developments. Accordingly, we assign a bear market influence of $+\frac{3}{10}$ and $+\frac{2}{10}$ for a firm's demand for short and long term contracts respectively.

For payoff coefficients, we assign a weight of +1 for *flexibility* to account for fluctuating business needs and strategy pivots from immediate market stressors and a weight of +5 for *reliability* by reason of a lower threshold for and higher cost of error in a bear market. Weights for *talent retention* and *potential alternatives* are unaffected by bear market conditions.

M **Bull: (**$\gamma_{(S)}$**: +**$\frac{2}{10}$**,** $\gamma_{(L)}$**: +**$\frac{3}{10}$**,** Π_F**:** 0**,** Π_R**:** 0**,** Π_T**:** $+7$**,** Π_P**:** $+5$**)**

We expect that bull market conditions will incentivize firms to become more risk seeking in their long term strategies and therefore spend more aggressively on risk-seeking innovation bets and long term strategies. As a result of optimistic market conditions, firms can plan ahead with more foresight and integrate short term requirements into longer term programs. Accordingly, we assign a bull market weight of $+\frac{2}{10}$ and $+\frac{3}{10}$ for a firm's demand for short and long term contracts respectively.

A bull market can serve as a proxy for low unemployment rates. For payoff coefficients, we assign a weight of +7 for *talent retention* as unemployment rates are low, and there are more opportunities for workers to pursue, thereby increasing the cost of talent acquisition and retention for firms. Further, we assign a weight of +5 for *potential alternatives* as laborers

have access to pursue a broader range of alternative engagements in a bull market. Weights for *flexibility* and *reliability* are unaffected by bull market conditions.

F_{size} **Small: (**$\gamma_{(S)}$**: +**$\frac{4}{10}$**,** $\gamma_{(L)}$**: +**$\frac{1}{10}$**,** Π_F**:** $+10$**,** Π_R**:** $+2$**,** Π_T**:** 0**,** Π_P**:** 0**)**

We conjecture that small firms behave more dynamically in the short term due to increased pivots as they develop product-market fit and build out their operations. Resultantly, we expect small firms to focus on short term strategy in order to accommodate changing business requirements and concentrate resources on immediate operational needs. Accordingly, we assign a small firm influence of $+\frac{4}{10}$ and $+\frac{1}{10}$ for a firm's demand for short and long term contracts respectively.

For payoff coefficients, we assign a weight of +10 for *flexibility* by logic of increased agility, fluctuating business needs and strategy pivots attributable to small businesses. A weight of +2 is designated to *reliability* because each individual's contribution and responsibility is more substantial in a smaller team, thereby increasing the impact of error for each individual. Weights for *talent retention* and *potential alternatives* are unaffected by small firm size.

F_{size} **Large: (**$\gamma_{(S)}$**: +**$\frac{1}{10}$**,** $\gamma_{(L)}$**: +**$\frac{4}{10}$**,** Π_F**:** 0**,** Π_R**:** 0**,** Π_T**:** 0**,** Π_P**:** 0**)**

We reason that large firms behave less dynamically in the short term as a result of established, sustainable business models and a lower likelihood of pivoting at size. Accordingly, we assign a large firm influence of $+\frac{1}{10}$ and $+\frac{4}{10}$ for a firm's demand for short and long term contracts respectively.

For payoff coefficients, weights for *flexibility*, *reliability*, *talent retention* and *potential alternatives* are unaffected by large firm size.

F_{skill} **Low: (**$\mu_{(Hi)}$**:** $\frac{2}{10}$**,** $\mu_{(Lo)}$**:** $\frac{8}{10}$**,** Π_F**:** 0**,** Π_R**:** 0**,** Π_T**:** 0**,** Π_P**:** 0**)**

We presume that low-skill firms maintain an operational demand distribution split between 20 percent high skill and 80 percent low skill contracts. We expect that a firm's required skill set does not impact the demand distribution of short and long term tasks.

For payoff coefficients, weights for *flexibility*, *reliability*, *talent retention* and *potential alternatives* are unaffected by large firm size.

F_{skill} **High: (**$\mu_{(Hi)}$**:** $\frac{8}{10}$**,** $\mu_{(Lo)}$**:** $\frac{2}{10}$**,** Π_F**:** 0**,** Π_R**:** $+10$**,** Π_T**:** $+3$**,** Π_P**:** $+5$ **)**

We presume that high-skill firms maintain an operational demand distribution split between 80 percent high skill and 20 percent low skill contracts. We assume that a firm's required skill set does not impact the demand distribution of short and long term tasks.

For payoff coefficients, weights for *flexibility*, *reliability*, *talent retention* and *potential alternatives* are unaffected by large firm size.

2.1.2. Compounded GameState Constituent Assumptions

$Firm_{size}$ **Large** *and* $Firm_{skill}$ **High: (**Π_F**:** $+5$**,** Π_R**:** -3**)**

Large high-skill firms such as Microsoft are consistently pursuing a breadth of projects ranging across industries. We apply a $flexibility$ weight of +5 to account for the variety of skills required to accommodate this wide-ranging horizon of programs and projects. Although high-skill labor warrants an increased sensitivity to labor reliability, we assign large high skill firms a *reliability* weight of -3 as large team size reduces the average impact of error for each worker.

$Firm_{size}$ **Large** *and* M **Bull: (**Π_T**:** -5**)**

For large firms in a bull market, we assign *talent retention* a weight of -5. We reason that large companies can leverage corporate brand names to attract a larger and more consistent pool of applicants.

$Firm_{size}$ **Small** *and* *$Firm_{Skill}$* **Low: (Π_F: -7)**

For small low-skill firms, we assign a $flexibility$ weight of -7. We posit that most small low-skill firms (i.e., family owned restaurants) operate static business models and experience marginal business innovation, thereby decreasing the operational demand for flexible skills.

2.1.3. GameState Contract Demand and Payoff Coefficients

To model operational demand, we first designate an annual labor spend to each firm category by firm size. We model payoffs in units of utility. Large and small firms are assigned $100M$ and $2M$ annual labor budgets respectively. Annual labor spend is represented with $\xi_{(Large,Small)}$. High-skill and low-skill labor, respectively cost $100K$ and $30K$ annually regardless of worker type (gig or employee). Labor cost by skill-set is denoted with $\Psi_{(Hi,Lo)}$. Short contracts span 2 weeks and long contracts span 26 week (half year) intervals.

To calculate the firm's requirement of a specific contract in a GameState, we first allocate a fraction of the annual labor spend to the type of task; each contract category captures a fraction of the firm's annual labor spend. This apportionment is determined by partitioning the labor spend according to the fraction of short or long contracts and the fraction of low or high-skill requirements. These proportions are calculated according to coefficients specified in the contract type ($ProportionLength : \gamma_{(S,L)}$) ($ProportionSkill : \mu_{(Hi,Lo)}$). Finally, we divide the partitioned budget by the cost of labor and normalize the contract count to reflect an annual interval; $\chi_{(S,L)}$ equates to 52 weeks divided by the contract duration (short or long) in weeks.

$$ContractDemand_* = (\frac{(\xi_{(Large,Small)})(\gamma_{(S,L)})(\mu_{(Hi,Lo)})}{\Psi_{(Hi,Lo)}})(\chi_{(S,L)}) \quad (1)$$

Example with Short Low Skill Contract for Small Low Bear

$$Demand_{shortlowskill} = (\frac{(\xi_{(Small)})(\gamma_{(S)})(\mu_{(Lo)})}{\Psi_{(Lo)}})(\chi_{(S)}) = (\frac{(2000000)(\frac{7}{10})(\frac{8}{10})}{30000})(\frac{52}{2}) \approx 970 \quad (2)$$

With this example calculation, we see that a small low-skill firm will have an annual demand for 970 short term low skill contracts. We select the $\gamma_{(S)}$, $\chi_{(S)}$ $\mu_{(Lo)}$, $\xi_{(Small)}$ and $\Psi_{(Lo)}$ coefficients in this calculation, because the contract we are calculating for is *short-term* and *low-skill* and the firm size is *small*.

Determining payoff coefficients ($\Pi_F, \Pi_R, \Pi_T, \Pi_P$) involves the summation of respective *flexibility*, *reliability*, *talent retention* and *potential alternatives* weights in the three constituent states that comprise the GameState. We generate contract distributions and payoff coefficients for the eight GameState with a Jupyter Notebook script, see Tables S1 and S2.

Table S1: GameState Contract Demand Distribution

$GameState \backslash Contract$	Short Low Skill	Short High Skill	Long Low Skill	Long High Skill
Small Low Bear	970.0	72.0	32.0	2.0
Small Low Bull	832.0	62.0	42.0	3.0
Large Low Bear	27733.0	2080.0	3200.0	240.0
Large Low Bull	20800.0	1560.0	3733.0	280.0
Small High Bear	242.0	291.0	8.0	9.0
Small High Bull	208.0	249.0	10.0	12.0
Large High Bear	6933.0	8320.0	800.0	960.0
Large High Bull	5200.0	6240.0	933.0	1120.0

Table S2: GameState Payoff Coefficients

$_{GameState}\backslash^{Coefficient}$	Flexibility	Reliability	Talent Retention	Potential Alternatives
Small Low Bear	4	7	0	0
Small Low Bull	3	2	7	5
Large Low Bear	1	5	0	0
Large Low Bull	0	0	2	5
Small High Bear	11	17	3	5
Small High Bull	10	12	10	10
Large High Bear	6	12	3	5
Large High Bull	5	7	5	10

In this section, we introduce the structure of payoff matrices and methods for payoff generation. Each payoff bi-matrix models the payoff for firm and laborer strategy pairs. We generate 5 payoff matrices for each GameState. The first 4 matrices model the 4 contract types in the GameState setting. The fifth matrix incorporates the GameState's contract demand distribution and models the comprehensive GameState with the weighted summation of respective contract payoffs; in other words, we represent the firm payoff with an aggregate of contract payoffs.

In this first 4x4 matrix (see Table S3), we see that the laborer has 4 strategies; the laborer can participate as a low or high skill gig worker, or a low or high skill employee when competing for a single contract. Similarly, the firm has 4 complementary strategies when hiring for each contract. The highlighted cells in the matrix represent matching strategies. The un-highlighted cells denote a strategy mismatch. In a matched strategy pair, the worker is hired. Conversely, a mismatched strategy pair indicates that no worker is hired, but utility is expended to execute the strategy; mismatched strategy pairs are assigned a marginal negative payoff to reflect this expended utility. Column headers represent firm strategies and row headers represent laborer strategies. Subscript f denotes the firm payoff and subscript l denotes the laborer payoff.

Table S3: 4x4 Payoff Matrix: Matching Strategy Pairs

$_{Laborer}\backslash^{Firm}$	Gig Lowskill	Gig Highskill	Employee Lowskill	Employee Highskill
Gig Lowskill	$[a_l, a_f]$	$[e_l, e_f]$	$[i_l, i_f]$	$[m_l, m_f]$
Gig Highskill	$[b_l, b_f]$	$[f_l, f_f]$	$[j_l, j_f]$	$[n_l, n_f]$
Employee Lowskill	$[c_l, c_f]$	$[g_l, g_f]$	$[k_l, k_f]$	$[o_l, o_f]$
Employee Highskill	$[d_l, d_f]$	$[h_l, h_f]$	$[l_l, l_f]$	$[p_l, p_f]$

To simplify the game, we first compress the matrix into a 4x2 matrix (see Table S4) and thereby eliminate a handful of the mismatched strategies. Later, we will compress the matrix into a 2x2 matrix to further reduce the dimensions of the evolutionary model.

Table S4: 4x2 Payoff Matrix: Matching Strategy Pairs

$_{Laborer}\backslash^{Firm}$	Gig	Employee
Gig Lowskill	$[a_l, a_f]$	$[e_l, e_f]$
Gig Highskill	$[b_l, b_f]$	$[f_l, f_f]$
Employee Lowskill	$[c_l, c_f]$	$[g_l, g_f]$
Employee Highskill	$[d_l, d_f]$	$[h_l, h_f]$

Only high skill laborers can participate in high-skill contracts. Accordingly, cells $[a_l, a_f]$ and $[g_l, g_f]$ become a mismatch for two of our four contract categories: short and long high-skill contracts. In the matrices for these two contract types, the only matching strategies are cells $[b_l, b_f]$ and $[h_l, h_f]$, see Table S5.

Table S5: 4x2 Payoff Matrix: Matching Strategy Pairs for High-Skill Contracts

$Laborer \backslash Firm$	Gig	Employee
Gig Lowskill	$[a_l, a_f]$	$[e_l, e_f]$
Gig Highskill	$[b_l, b_f]$	$[f_l, f_f]$
Employee Lowskill	$[c_l, c_f]$	$[g_l, g_f]$
Employee Highskill	$[d_l, d_f]$	$[h_l, h_f]$

2.2. Firm Strategy Payoffs

2.2.1. Matching Strategies

To calculate the firm payoff for matching strategies, we apply a general equation that incorporates our GameState assumptions and payoff coefficients.

$$Payoff_* = \frac{\phi(1 - \frac{\beta}{\lambda} - \frac{\Pi_R}{\delta_R + \Theta} - \frac{\Pi_T}{\delta_T} + \frac{\Pi_F}{\delta_F})}{\chi} \tag{3}$$

Compensation for high skill and low skill contracts respectively costs $100K$ and $30K$ annually regardless of worker type (gig or employee). *Operational Revenue* ϕ, signifying revenue generated from labor, is 8x the compensation for high skill work and 4x for low skill work; we refer to this factor as the *Revenue Multiplier* λ. We assume employees receive 40 percent additional compensation in the form of benefits and bonuses; accordingly, the *Benefits Multiplier* β is 1.4 for employee and 1 for gig strategies. The first component of firm payoff debits the bonus-adjusted-compensation from operational revenue. GameState payoff coefficients, denoted with Π, incorporate additional hiring considerations that impact a strategy's overall payoff. Discount values, denoted with δ, are applied to each payoff coefficient to account for gig worker and employee disparities. Larger discount values further reduce the importance of the payoff coefficient. Subscripts R, F and T are used to indicate discounts and coefficients for *reliability*, *flexibility* and *talent retention* respectively. *Reliability* Π_R (the importance of reliability expressed through the cost of expected error) and *talent retention* Π_T (the cost of labor acquisition and retention) detract from the payoff while labor *flexibility* Π_F is an asset. Hiring considerations denoted by payoff coefficients directly impact operational revenue. Finally, we standardize the annual payoff to measure for the interval of the contract, 2 or 26 weeks depending on the contract length; χ equates to 52 weeks divided by the contract duration in weeks.

Gig and employee strategies are assigned a δ_R of 8 and 40 respectively, indicating that employees are 7.5x more reliable than gig workers. Further, talent retention is more expensive for employees than for gig workers. We assign gig and employee strategies a δ_T of 200 and 50 respectively, indicating that employees are 4x more expensive for firms to acquire and retain. Since gig arrangements enable on-demand labor, gig workers provide more flexibility to the firm. Therefore, gig and employee strategies are assigned a discount δ_F of 8 and 60 respectively, indicating gig workers provide 6.25x more labor flexibility than employees. We introduce an *OverSkill* Θ discount of +5 if a high skill worker engages in a low skill contract; this overskill mismatch implies a reduced cost of expected error when a high skill worker takes on low skill tasks. Below, we break down the specific firm payoffs for each potential matching strategy, see Table S6.

Table S6: 4x2 Payoff Matrix: Matching Strategy Pairs for Firms

$Laborer \backslash Firm$	Gig	Employee
Gig Lowskill	$[a_l, a_f]$	$[e_l, e_f]$
Gig Highskill	$[b_l, b_f]$	$[f_l, f_f]$
Employee Lowskill	$[c_l, c_f]$	$[g_l, g_f]$
Employee Highskill	$[d_l, d_f]$	$[h_l, h_f]$

$$a_f = \frac{\phi(1 - \frac{1}{\lambda} - \frac{\Pi_R}{8} - \frac{\Pi_T}{200} + \frac{\Pi_F}{8})}{\chi} \tag{4}$$

$$b_f = \frac{\phi(1 - \frac{1}{\lambda} - \frac{\Pi_R}{8+\Theta} - \frac{\Pi_T}{200} + \frac{\Pi_F}{8})}{\chi} \tag{5}$$

$$g_f = \frac{\phi(1 - \frac{1.4}{\lambda} - \frac{\Pi_R}{40} - \frac{\Pi_T}{50} + \frac{\Pi_F}{60})}{\chi} \tag{6}$$

$$h_f = \frac{\phi(1 - \frac{1.4}{\lambda} - \frac{\Pi_R}{40+\Theta} - \frac{\Pi_T}{50} + \frac{\Pi_F}{60})}{\chi} \tag{7}$$

2.2.2. Mismatched Strategies

For mismatching strategies, we assign a negative payoff to reflect the firm's utility expenditure in pursuing the strategy of hiring a category of worker but failing to hire. The assigned payoff is the negative value of one fiftieth of the contract's operational revenue over the contract duration. As low skill laborers can not work high skill contracts, a_l and g_l become mismatching strategies and assigned the mismatched strategy payoff. For low skill contracts, strategies highlighted in pink are mismatching strategies. For high skill contracts, strategies highlighted in pink and yellow are mismatching strategies, see Table S7.

Table S7: 4x2 Payoff Matrix: Mismatching Strategy Pairs for Firms

$Laborer \backslash Firm$	Gig	Employee
Gig Lowskill	$[a_l, a_f]$	$[e_l, e_f]$
Gig Highskill	$[b_l, b_f]$	$[f_l, f_f]$
Employee Lowskill	$[c_l, c_f]$	$[g_l, g_f]$
Employee Highskill	$[d_l, d_f]$	$[h_l, h_f]$

$$e_f = f_f = c_f = d_f = a_f = g_f = -\frac{\phi}{50\chi} \tag{8}$$

2.3. Laborer Strategy Payoffs

2.3.1. Matching Strategies

To calculate the laborer payoffs for matching strategies, we apply the following general equation.

$$Payoff_* = \frac{\Psi(\beta\epsilon + \frac{\Pi_P}{\delta_P})}{\chi} \tag{9}$$

Compensation Ψ for high skill and low skill contracts respectively costs $100K$ and $30K$ annually regardless of worker type (gig or employee). In a bull market, we assume that workers experience a 5 percent rate of involuntary attrition, translating into an *employment stability* coefficient, denoted with ϵ, of 0.95. In a bear market, we assign a value of 0.7 to ϵ, implying a 30 percent rate of involuntary attrition. In addition to receiving compensation for labor, laborers receive a payoff from potential alternative engagements outside of their primary contracts. Π_P and δ_P denote the potential alternatives payoff coefficient and discount respectively. As gig arrangements champion flexibility and self governance, we assume that gig workers have the opportunity to take part in 5x the potential alternative engagements compared to employees. In our model, this translates to a δ_P of 5 and 25 for gig and employee strategies respectively.

We adjust parameters in specific scenarios to account for additional phenomena. Employees competing for high skill contract during a bull market will receive 3x additional benefits, an optimistic gratuity, to account for compounding stock options or carried interest bonus. During a bear market, low skill workers, especially those with short term work arrangements, are more adversely affected as industries discharge commodity skill laborers. We subtract 8 from Π_P if a low skill worker competes for a gig contract during a bear market. As unemployment rates hike during a bear market, low skill firms begin to hire high skill workers. Accordingly, high skill workers gain access to a broader set of alternative work options during a bear market. Since a gig worker has increased flexibility, high skill gig workers benefit the most from this increased opportunity. To account for this, we add 15 to Π_P for high skill gig strategy payoffs during a bear market. Below, we break down the specific laborer payoffs for each potential matching strategy, see Table S8.

Table S8: 4x2 Payoff Matrix: Matching Strategy Pairs for Laborers

$Laborer \backslash Firm$	Gig	Employee
Gig Lowskill	[a_l , a_f]	[e_l, e_f]
Gig Highskill	[b_l , b_f]	[f_l, f_f]
Employee Lowskill	[c_l, c_f]	[g_l , g_f]
Employee Highskill	[d_l, d_f]	[h_l ,h_f]

$$a_l = \frac{\Psi(\epsilon + \frac{\Pi_P}{5})}{\chi} \tag{10}$$

$$b_l = \frac{\Psi(\epsilon + \frac{\Pi_P}{5})}{\chi} \tag{11}$$

$$g_l = \frac{\Psi(1.4\epsilon + \frac{\Pi_P}{25})}{\chi} \tag{12}$$

$$h_l = \frac{\Psi(1.4\epsilon + \frac{\Pi_P}{25})}{\chi} \tag{13}$$

2.3.2. Mismatched Strategies

For mismatching strategies, we assign a negative payoff to reflect the laborer's utility expenditure in pursuing an employment strategy in competition of a contract but failing to get hired. The assigned payoff is the negative value of one fiftieth of the contract's

compensation over the contract duration. As low skill laborers can not work high skill contracts, a_l and g_l become mismatching strategies and assigned the mismatched strategy payoff. For low skill contracts, strategies highlighted in pink are mismatched strategies. For high skill contracts, strategies highlighted in pink and yellow are mismatched strategies, see Table S9.

Table S9: 4x2 Payoff Matrix: Mismatching Strategy Pairs for Laborers

$Laborer \backslash^{Firm}$	Gig	Employee
Gig Lowskill	[a_l , a_f]	[e_l , e_f]
Gig Highskill	[b_l, b_f]	[f_l , f_f]
Employee Lowskill	[c_l , c_f]	[g_l , g_f]
Employee Highskill	[d_l , d_f]	[h_l , h_f]

$$e_l = f_l = c_l = d_l = a_l = g_l = -\frac{\Psi}{50\chi} \tag{14}$$

2.4. GameState Aggregate: Weighted Payoff

The fifth matrix incorporates the GameState's contract distribution and models the payoff for the GameState; while the first four matrices represent payoffs for each contract, the fifth matrix represents the payoff for an entire firm in a market setting. Since the firm consists of a distribution of contracts, we calculate the GameState payoff by taking the weighted summation of contract payoffs. Below, GS denotes the GameState payoff. α represents the firm demand for each contract type in a single GameState. We previously calculated these values when generating our GameState contract demand distributions. Subscript 1,2,3 and 4 respectively denote short low skill, short high skill, long low skill and long high skill contracts.

$$GS = \alpha_1 \begin{pmatrix} [a_l, a_f] & [e_f, e_l] \\ [b_l, b_f] & [f_l, f_f] \\ [c_l, c_f] & [g_l, g_f] \\ [d_l, d_f] & [h_l, h_f] \end{pmatrix}_1 + \alpha_2 \begin{pmatrix} [a_l, a_f] & [e_f, e_l] \\ [b_l, b_f] & [f_l, f_f] \\ [c_l, c_f] & [g_l, g_f] \\ [d_l, d_f] & [h_l, h_f] \end{pmatrix}_2 + \alpha_3 \begin{pmatrix} [a_l, a_f] & [e_f, e_l] \\ [b_l, b_f] & [f_l, f_f] \\ [c_l, c_f] & [g_l, g_f] \\ [d_l, d_f] & [h_l, h_f] \end{pmatrix}_3 + \alpha_4 \begin{pmatrix} [a_l, a_f] & [e_f, e_l] \\ [b_l, b_f] & [f_l, f_f] \\ [c_l, c_f] & [g_l, g_f] \\ [d_l, d_f] & [h_l, h_f] \end{pmatrix}_4 \tag{15}$$

2.5. Payoff Matrices

We generate payoff matrices for the eight GameStates with a Jupyter Notebook script that implements all of our assumptions.

We reduce GameState payoff matrices from 4x2 to 2x2 bi-matrices to further truncate model dimensions, see Tables S10 and S11. We combine high and low skill strategy payoffs for gig and employee strategies respectively. Therefore in the 2x2 bi-matrix, the 2x2 gig strategy accounts for high and low skill gig strategies and the 2x2 employee strategy accounts for high and low skill employee strategies. We implement this matrix reduction process with a Jupyter Notebook script and generate 8 GameState payoff matrices, see Tables S12-19.

Table S10: 4x4 Payoff Matrix

$_{Laborer}\backslash^{Firm}$	Gig	Employee
Gig Lowskill	$[a_l, a_f]$	$[e_l, e_f]$
Gig Highskill	$[b_l, b_f]$	$[f_l, f_f]$
Employee Lowskill	$[c_l, c_f]$	$[g_l, g_f]$
Employee Highskill	$[d_l, d_f]$	$[h_l, h_f]$

Table S11: 2x2 Payoff Matrix

$_{Laborer}\backslash^{Firm}$	Gig	Employee
Gig	$[a_l + b_l, a_f + b_f]$	$[e_l + f_l, e_f + f_f]$
Employee	$[c_l + d_l, c_f + d_f]$	$[g_l + h_l, g_f + h_f]$

3. Payoff Matrices

3.1. Small Low Bear

Table S12: 2x2 Payoff Matrix for GameState in Setting Small Low Bear

$_{Laborer}\backslash^{Firm}$	Gig	Employee
Gig	[1066048.0, 8397092.0]	[-78764.0, -375840.0]
Employee	[-78764.0, -375840.0]	[3494832.0, 9154366.0]

3.2. Small Low Bull

Table S13: 2x2 Payoff Matrix for GameState in Setting Small Low Bull

$_{Laborer}\backslash^{Firm}$	Gig	Employee
Gig	[6950788.0, 14231744.0]	[-78896.0, -378148.0]
Employee	[-78896.0, -378148.0]	[6558500.0, 8587992.0]

3.3. Large Low Bear

Table S14: 2x2 Payoff Matrix for GameState in Setting Large Low Bear

$_{Laborer}\backslash^{Firm}$	Gig	Employee
Gig	[57566264.0, 293689368.0]	[-3991878.0, -19181272.0]
Employee	[-3991878.0, -19181272.0]	[175951620.0, 462572232.0]

3.4. Large Low Bull

Table S15: 2x2 Payoff Matrix for GameState in Setting Large Low Bull

$_{Laborer}\backslash^{Firm}$	Gig	Employee
Gig	[350581940.0, 608754400.0]	[-3993720.0, -19185200.0]
Employee	[-3993720.0, -19185200.0]	[331969900.0, 512758880.0]

3.5. Small High Bear

Table S16: 2x2 Payoff Matrix for GameState in Setting Small High Bear

$_{Laborer}\backslash^{Firm}$	Gig	Employee
Gig	[8062392.0, 2386627.0]	[-78164.0, -565658.0]
Employee	[-78164.0, -565658.0]	[2762366.0, 7506322.0]

3.6. Small High Bull

Table S17: 2x2 Payoff Matrix for GameState in Setting Small High Bull

$_{Laborer}\backslash^{Firm}$	Gig	Employee
Gig	[6864878.0, 9219843.0]	[-77416.0, -560542.0]
Employee	[-77416.0, -560542.0]	[8452547.0, 6917537.0]

3.7. Large High Bear

Table S18: 2x2 Payoff Matrix for GameState in Setting Large High Bear

$_{Laborer}\backslash^{Firm}$	Gig	Employee
Gig	[410392908.0, 101342039.0]	[-3983558.0, -28789272.0]
Employee	[-3983558.0, -28789272.0]	[139995466.0, 413857629.0]

3.8. Large High Bull

Table S19: 2x2 Payoff Matrix for GameState in Setting Large High Bull

$_{Laborer}\backslash^{Firm}$	Gig	Employee
Gig	[352366500.0, 474091576.0]	[-3987480.0, -28791200.0]
Employee	[-3987480.0, -28791200.0]	[433984460.0, 467401118.0]

4. Market Influences on Firm and Laborer Gig Preference

4.1. Assumptions and Observations.

In our data generated model, we assume that the system exists by oscillating in the trapping zone. The system's oscillatory behavior reflects observable fluctuations in gig economy strategies across market cycles. For small firms, we assign an $\omega = 0.00000001$ that is 50x larger than that for large firms, $\omega = 0.0000000002$; since small firms employ a smaller work force, labor composition is more notably impacted by each individual labor decision, implying a faster rate of system evolution. Across our four firm categories, we observe two patterns in oscillatory behavior marked by firm skill set. While low-skill firms and laborers demonstrate a matching oscillatory behavior (when firm preference for gig work increases, laborer preference for gig strategies also increases), high skill firms and laborers exhibit a mismatching oscillatory behavior. Further, the position of the attractor arc is higher on the y_1 axis for low-skill firms, implying that these firms maintain a higher proportion of gig workers. In this section, we explore why skill set bifurcates our firm cohort into two categories of oscillatory behaviors.

Our theoretical firm categories can be mapped to empirical examples. For instance, a small family owned restaurant business can be understood as a small low-skill firm. A large low-skill business embodies ride share companies like Uber and Lyft. An early stage technology startup characterizes a small high-skill firm. Enterprises like Microsoft or Google can be presumed to be large high-skill firms.

4.2. Firm-Level Discussion

Market Influence on Low-skill Firms and Laborers

During a bear market, low-skill firms and their laborers increase their preference for the employee strategy, see Figures S1 and S2. Several studies find that low-skill laborers are the most adversely affected during a bear market, and they often constitute the majority of layoffs [1–3]. Workers with commodity skills have the fewest options for alternative engagements during this recessionary period. Indeed, the emergence of economic contraction entails a decreased demand for consumer goods and services [1]. Present-day gig workers recognize that the structural forces of economic recessions restrict their autonomy in flexible scheduling as service demand abates [4]. In addition to experiencing decreased autonomy, it is reasonable that low-skill laborers increase their preference for employee roles which come with additional financial stability and labor protections. As low-skill gig labor generally supports operations regarding consumer goods and services, decreased demand for such services may implicate decreased demand for low-skill gig labor to fulfill service operations. Therefore, it is sensible that low-skill firms decrease their demand for gig labor during a bear market.

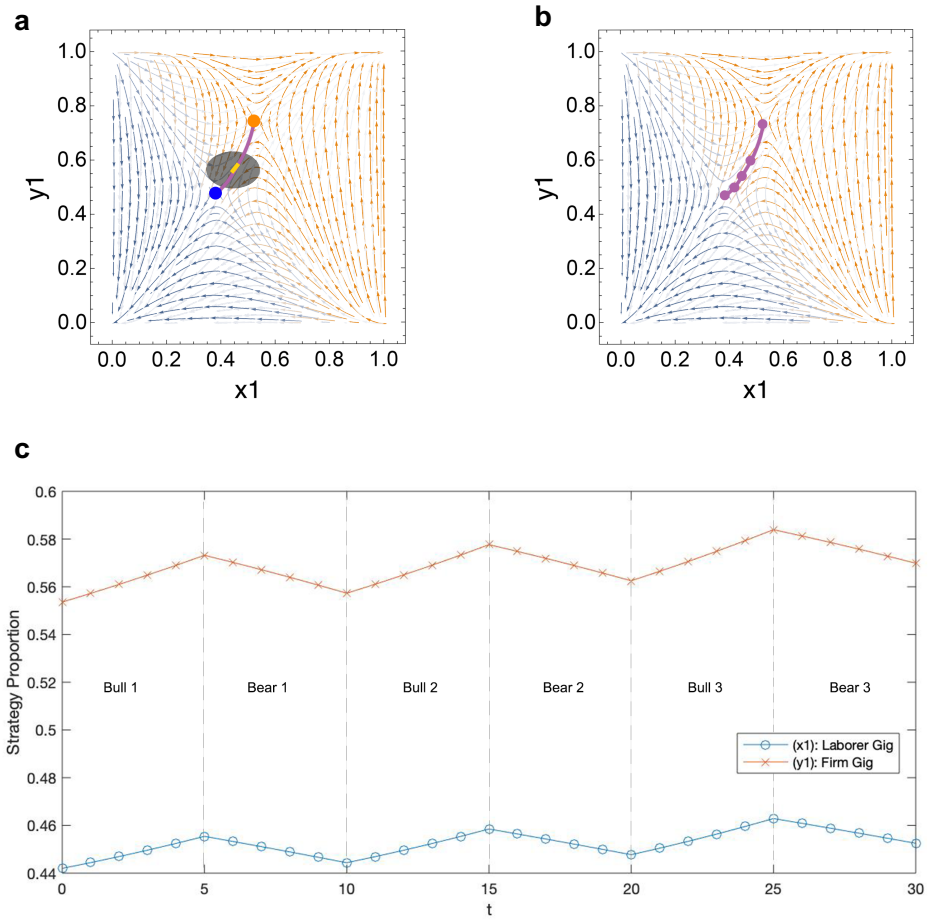


Figure S1. Evolution of Strategy Densities for Small Low-Skill Firm with Initial Conditions $(0.4417, 0.5554)$, the attractor position at $n = 0.5$, an approximation for a point in the trapping zone; $\omega = 0.00000001$; and Payoff Matrices Small Low Bear and Small Low Bull, see Table S1 and S2. (a) Trapping Zone Orbit (b) Attractor Arc (c) Labor Strategy Oscillation Over Three Market Cycles. In (a), the trapping zone orbit is plotted in yellow, and attractor positions at $n = 0$ and $n = 1$ are represented in orange and blue respectively. In (b), we plot a reference attractor arc in purple with attractor positions when $n = 0$, $n = 0.25$, $n = 0.5$, $n = 0.75$ and $n = 1$. (c) visualizes the fluctuation in firm and laborer preferences for gig strategies over three market cycles.

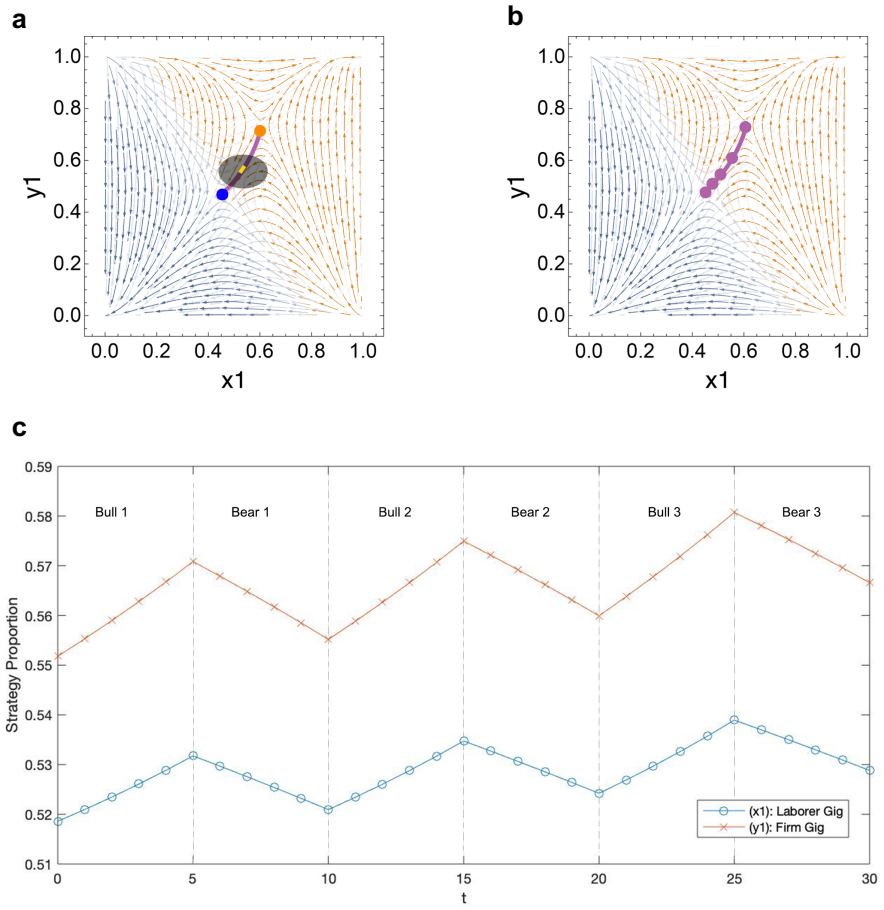


Figure S2. Evolution of Strategy Densities for Large Low-Skill Firm with Initial Conditions $(0.5186, 0.5535)$, the attractor position at $n = 0.5$, an approximation for a point in the trapping zone; $\omega = 0.0000000002$; and Payoff Matrices Large Low Bear and Large Low Bull, see Table S3 and S4 (a) Trapping Zone Orbit (b) Attractor Arc (c) Labor Strategy Oscillation Over Three Market Cycles. In (a), the trapping zone orbit is plotted in yellow, and attractor positions at $n = 0$ and $n = 1$ are represented in orange and blue respectively. In (b), we plot a reference attractor arc in purple with attractor positions when $n = 0$, $n = 0.25$, $n = 0.5$, $n = 0.75$ and $n = 1$. (c) visualizes the fluctuation in firm and laborer preferences for gig strategies over three market cycles.

When the market environment changes to a bull market, low-skill firms and laborers increase their preference for the gig strategy, see Figures S1 and S2. As market optimism rises, demand for consumer goods and services grows. For laborers pursuing gig roles, this implies additional financial stability as demand for services stabilizes [4,5]. Uber drivers, for instance, can complete more rides each day as a result of increased rider demand. Since the salary of a low-skill gig worker is directly impacted by the number of tasks completed, more rides implies higher compensation. Accordingly, it is logical that low-skill laborers increase their preference for gig strategies during a bull market to capture this increased demand for consumer goods and services. Correspondingly, low-skill firms must adjust their labor demands to accommodate this interval of increased consumerism. It is therefore reasonable that low-skill firms increase their demand for gig labor as they accelerate service operations during bull market conditions.

Market Influence on High-skill Firms and Laborers

In bear market conditions, high-skill firms increase preference for employees while high-skill laborers increase preference for gig work, see Figures S3 and S4. While recessionary economic conditions are unsympathetic to low-skill laborers, laborers with specific

skill sets are more impervious to the impacts of an economic downturn [2,3]. Therefore, high-skill laborers have increased leverage during bear market conditions. Coupled with increased flexibility and the opportunity to take on low-skill contracts [6], it is reasonable that high-skill laborers increase their preference to work gig contracts during bear markets. Conversely, high-skill firms are particularly sensitive to talent retention costs and worker reliability. It is plausible, for example, that firms can expect lower employee churn during a bear market as workers who choose to participate as employees want to retain their positions. Additionally, the cost of error can be particularly detrimental to a high-skill firm during a bear market; therefore, it is sensible that the reliability of an employee over that of a gig worker is more valuable to a high-skill enterprise during a bear market.

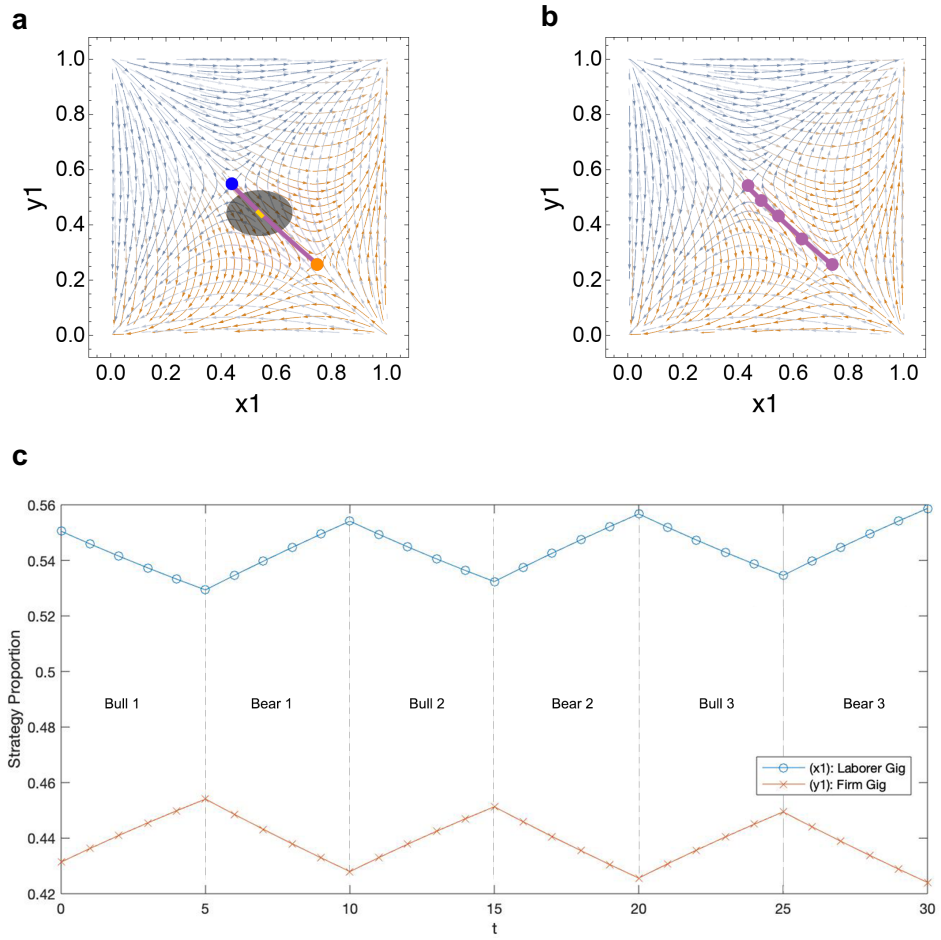


Figure S3. Evolution of Strategy Densities for Small High-Skill Firm with Initial Conditions $(0.5498, 0.4298)$, the attractor position at $n = 0.5$, an approximation for a point in the trapping zone; $\omega = 0.00000001$; and Payoff Matrices Small High Bear and Small High Bull, see Table S5 and S6 (a) Trapping Zone Orbit (b) Attractor Arc (c) Labor Strategy Oscillation Over Three Market Cycles. In (a), the trapping zone orbit is plotted in yellow, and attractor positions at $n = 0$ and $n = 1$ are represented in orange and blue respectively. In (b), we plot a reference attractor arc in purple with attractor positions when $n = 0$, $n = 0.25$, $n = 0.5$, $n = 0.75$ and $n = 1$. (c) visualizes the fluctuation in firm and laborer preferences for gig strategies over three market cycles.

When the market shifts to bull market conditions, high-skill firms increase their preference for gig work while laborers increase their preference for employee roles, see Figures S3 and S4. High-skill employees have the opportunity to accrue additional compensation with a carried interest bonus, a share of profits that depend on the company's performance. As optimistic market conditions can serve as an appropriate proxy for increasing company revenues, the value of this carried interest bonus is highest during a bull market. Therefore,

high-skill laborers have an increased incentive to participate in the labor market as an employee during a bull market to capture this bonus. An encouraging market outlook can also champion high-skill firms to pursue a broader range of new programs and products in different industry domains. For firms, gig labor provides a flexible on-demand pool of diverse skills that can accommodate these new risk-seeking programs. Simultaneously, employee talent acquisition and retention costs increase as laborers gain access to additional alternative work options during bull market conditions, further incentivizing firms to hire gig workers over employees during a bull market.

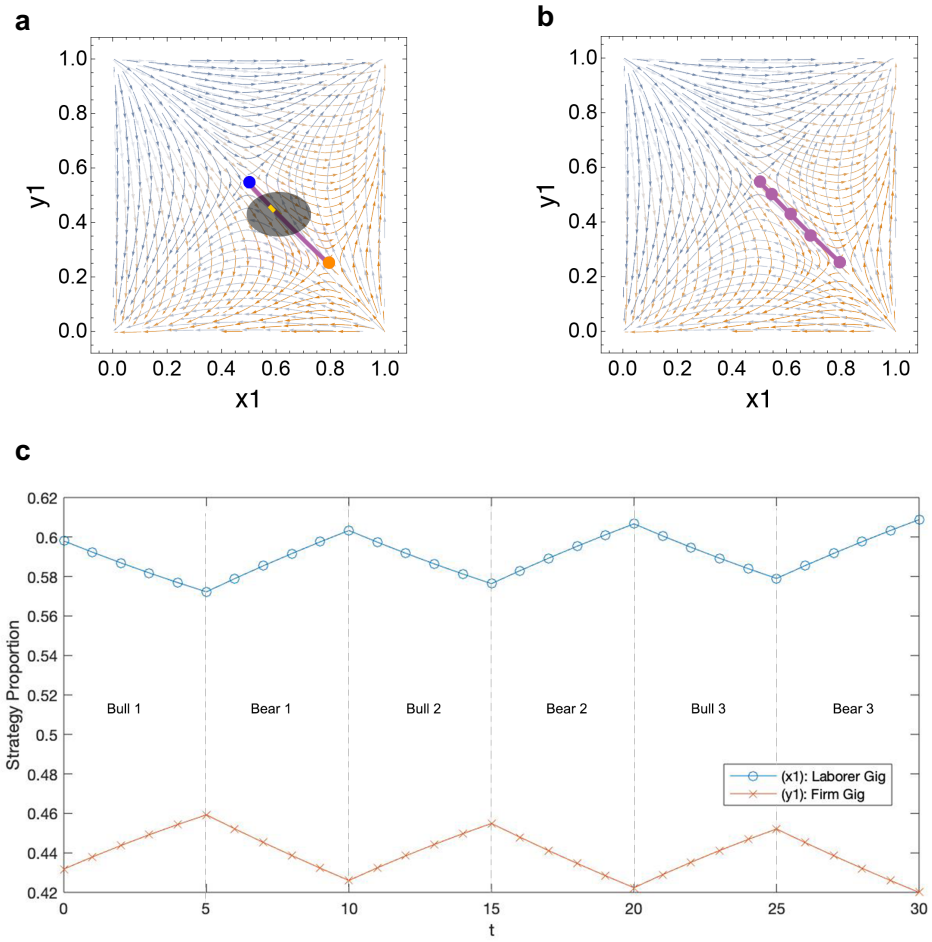


Figure S4. Evolution of Strategy Densities for Large High-Skill Firm with Initial Conditions $(0.5973, 0.4302)$, the attractor position at $n = 0.5$, an approximation for a point in the trapping zone; $\omega = 0.0000000002$; and Payoff Matrices Large High Bear and Large High Bull, see Table S7 and S8 (a) Trapping Zone Orbit (b) Attractor Arc (c) Labor Strategy Oscillation Over Three Market Cycles. In (a), the trapping zone orbit is plotted in yellow, and attractor positions at $n = 0$ and $n = 1$ are represented in orange and blue respectively. In (b), we plot a reference attractor arc in purple with attractor positions when $n = 0$, $n = 0.25$, $n = 0.5$, $n = 0.75$ and $n = 1$. (c) visualizes the fluctuation in firm and laborer preferences for gig strategies over three market cycles.

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