Supplementary Information S1: Method of biomass potential productivity (BPP)

The calculation of the biomass potential productivity (BPP) is searching $\tilde{S}$ that maximizes the objective function from a given interval of stand density index ($S_{\min}, S_{\min}$), corresponding to the maximum annual growth of biomass.

The method of searching $\tilde{S}$ we utilized the golden section method [24]. The detailed calculation of BPP is as follows:

(1) Given $[S_{\min}, S_{\max}]$, Calculation of initial values of stand density index (S)at 4 points:

$$\begin{aligned} S_1^{(Age_0)} &= S_{\min} \\ S_2^{(Age_0)} &= S_{\min} + 0.382(S_{\max} - S_{\min}) \\ S_3^{(Age_0)} &= S_{\min} + 0.618(S_{\max} - S_{\min}) \\ S_4^{(Age_0)} &= S_{\max} \end{aligned} \quad \text{(S1)}$$

(2) The BI values of the four points of segmentation ($BI_1^{(Age_0)}, BI_2^{(Age_0)}, BI_3^{(Age_0)}, BI_4^{(Age_0)}$) corresponding to $S_1^{(Age_0)}, S_2^{(Age_0)}, S_3^{(Age_0)}, S_4^{(Age_0)}$, at the t[th] iteration is obtained using **subprogram A** (see below).

$$\begin{aligned} BI_1^{(Age_0)} &= f(L, Age_0, S_1^{(Age_0)}, \hat{\Phi}_G, \hat{\Phi}_B) \\ BI_2^{(Age_0)} &= f(L, Age_0, S_2^{(Age_0)}, \hat{\Phi}_G, \hat{\Phi}_B) \\ BI_3^{(Age_0)} &= f(L, Age_0, S_3^{(Age_0)}, \hat{\Phi}_G, \hat{\Phi}_B) \\ BI_4^{(Age_0)} &= f(L, Age_0, S_4^{(Age_0)}, \hat{\Phi}_G, \hat{\Phi}_B) \end{aligned} \quad \text{(S2)}$$

Where L is the site group, BI is the biomass annual growth.

(3) If $BI_2^{(Age_0)} > BI_3^{(Age_0)}$ and $BI_2^{(Age_0)} - BI_3^{(Age_0)} | > \mathrm{e}$, transmute

$$\begin{aligned} S_1^{(Age_1)} &= S_1^{(Age_0)} \\ S_4^{(Age_1)} &= S_3^{(Age_0)} \\ S_2^{(Age_1)} &= S_1^{(Age_1)} + 0.382(S_4^{(Age_1)} - S_1^{(Age_1)}) \\ S_3^{(Age_1)} &= S_1^{(Age_1)} + 0.618(S_4^{(Age_1)} - S_1^{(Age_1)}) \end{aligned} \quad \text{(S3)}$$

where $Age_1 = Age_0 + 1$.

Or if $BI_2^{(Age_0)} < BI_3^{(Age_0)}$ and $| BI_2^{(Age_0)} - BI_3^{(Age_0)} | > \mathrm{e}$, transmute

$$\begin{aligned} S_1^{(Age_1)} &= S_2^{(Age_0)} \\ S_4^{(Age_1)} &= S_4^{((Age_0)} \\ S_2^{(Age_1)} &= S_1^{(Age_1)} + 0.382(S_4^{(Age_1)} - S_1^{(Age_1)}) \\ S_3^{(Age_1)} &= S_1^{(Age_1)} + 0.618(S_4^{(Age_1)} - S_1^{(Age_1)}) \end{aligned} \quad \text{(S4)}$$

where $Age_1 = Age_0 + 1$.

and one must go back to (2) for the next iteration, otherwise the calculation stops.

$$BPP = (BI_2^{Age_0} + BI_3^{Age_0})/2,\ S = (S_2^{Age_0} + S_3^{Age_0})/2 \quad \text{(S5)}$$

The computational process of **Subprogram A:**

(1) computing N_0 and B_0 at Age_0 :

a) Given Age_0 , S_0 and $\hat{\Phi}_G$, compute G_0 using the following equation:

$$G_0 = f_G(Age_0, S_0, \hat{\Phi}_G) \quad \text{(S6)}$$

where G_0 is the basal area increment of different stand type in Age_0 ; Age_0 is the Initial age; S_0 is the stand density that maximum objective function. $\hat{\Phi}_G$ is the model parameter.

b) Given G_0 , S_0 compute N_0 and D_0 :

$$\begin{cases} G_0 = \pi D_0^2 N_0 / 40000 \\ S_0 = N_0 (D_0 / 20)^{\beta} = N_0 (D_0 / 20)^{1.605} \end{cases} \quad \text{(S7)}$$

The estimates of D_0 and N_0 are:

$$D_0 = \left(\frac{40000 G_0}{\pi S_0 (20)^{1.605}} \right)^{1/(1.605-1)}, N_0 = G_0 / (\pi D_0^2 / 40000) \quad \text{(S8)}$$

where D_0 is the mean of the diameter at breast height; N_0 is the initial number of trees in the stand.

c) Given Age_0 , and S_0 , compute B_0 using the following equation:

$$B_0 = f_B(Age_0, S_0, \hat{\Phi}_B) \quad \text{(S9)}$$

where B_0 is the biomass increment of one stand type at Age_0 ; Age_0 is the Initial age; S_0 is the stand density index that maximum objective function. $\hat{\Phi}_B$ is the model parameter.

(2) computing B_1 at Age_1 ($Age_0 + 1$):

In our study, it was assumed that the number of trees is the same in each year ($N_1 = N_0$). We computed B_1 according to the $Age_1 = Age_0 + 1$.

a) Given Age_1 , N_1 , and $\hat{\Phi}_G$, using dichotomy iteration algorithm to compute S_1 using the following equation [24,25]:

$$f_G(Age_1, S_1, \hat{\Phi}_G) - \pi / 100 (S_1 / N_1)^{2/1.605} N_1 = 0 \quad \text{(S10)}$$

b) Given Age_1 , S_1 ,and $\hat{\Phi}_B$, compute B_1 as follows:

$$B_1 = f_B(Age_1, S_1, \hat{\Phi}_B) \quad \text{(S11)}$$

(3)Compute BI using the following equation:

$$BI = B_1 - B_0 = f_B(Age_1, S_1, \hat{\Phi}_B) - f_B(Age_0, S_0, \hat{\Phi}_B) \quad \text{(S12)}$$

Supplementary Information S2: Method of biomass realistic productivity (BRP)

The computational procedures of BRP are similar to BPP . S from field surveys, should be provided for the BRP calculation. The stand density index (S_0) for biomass realistic productivity in Age_0 is the measured data from the sample plots. We compute biomass realistic productivity through **Subprogram A.**