Supplementary Materials

Table S1a. Sensitivity to response dispersion and correlation (baseline means and budgets, common seeds)

SD multiplier	Shared coefficient	Joint area (MW·Mvar)	Joint delivery %	Fixed 0.8 delivery %	Marginal area (MW·Mvar)	Marginal delivery %
0.5	0.0	4.801	1.95	0.05	4.911	13.20
0.5	0.3	4.797	1.40	0.05	4.908	12.85
0.5	0.55	4.811	1.75	0.00	4.917	14.00
0.5	0.8	4.834	2.80	0.00	4.916	10.15
0.5	0.95	4.864	4.30	0.00	4.920	8.50
1.0	0.0	4.488	2.65	0.15	4.689	13.30
1.0	0.3	4.478	1.90	0.10	4.682	12.95
1.0	0.55	4.493	2.20	0.05	4.699	14.00
1.0	0.8	4.557	3.60	0.05	4.701	10.65
1.0	0.95	4.614	4.85	0.10	4.705	8.60
1.5	0.0	4.175	2.55	1.45	4.471	13.35
1.5	0.3	4.164	1.90	1.65	4.463	12.95
1.5	0.55	4.177	2.20	1.50	4.485	14.00
1.5	0.8	4.270	3.40	1.55	4.488	10.60
1.5	0.95	4.360	4.80	1.90	4.494	8.55
2.0	0.0	3.867	2.45	5.15	4.259	13.40
2.0	0.3	3.861	1.45	4.90	4.247	13.00
2.0	0.55	3.879	2.25	5.20	4.277	14.10
2.0	0.8	4.000	3.45	5.40	4.280	10.65
2.0	0.95	4.114	4.80	5.50	4.288	8.55
3.0	0.0	3.295	1.75	14.55	3.849	9.90
3.0	0.3	3.286	1.45	14.55	3.832	9.60
3.0	0.55	3.312	1.30	14.05	3.874	14.15
3.0	0.8	3.496	2.90	13.95	3.879	9.15
3.0	0.95	3.648	4.15	14.05	3.890	7.80

Fixed 0.8 area is 3.737 MW·Mvar in every cell.

Table S1b. Sensitivity to activation time (baseline means and budgets, common seeds)

Median-time multiplier	Log-time SD	Joint area (MW·Mvar)	Joint delivery %	Fixed 0.8 delivery %
1.0	0.16	4.511	2.80	0.00
1.0	0.32	4.511	2.85	0.05
1.0	0.48	4.450	3.85	0.90
1.8	0.16	4.509	2.75	0.05
1.8	0.32	4.206	3.90	3.55
1.8	0.48	2.888	3.35	12.60
2.6	0.16	4.110	3.30	10.25
2.6	0.32	2.878	3.10	28.55
2.6	0.48	1.564	3.00	38.95

Table S2. One-factor-at-a-time decomposition of the compound scenario (common random numbers)

Modification	Area (MW·Mvar)	Change vs. baseline	Delivery %
Baseline	4.529	–	3.60
Response ratio only (mean −0.04; SD ×1.5)	3.848	−15.0%	2.35
Delay only (median ×1.8)	4.287	−5.3%	4.85
BESS budget only (×0.35)	4.504	−0.6%	2.65
Ratio + delay	3.684	−18.7%	3.00
Compound (all three)	3.657	−19.3%	3.25

Note: common seeds give a baseline area of 4.529 here versus 4.523 in Table 5, hence 19.3% versus 19.15%.

Table S3. Alternative joint-control rules and fixed-derating sweep

Scenario	Method	Area (MW·Mvar)	Delivery %	Network %	Joint %
Baseline	Joint calibrated	4.523	4.25	1.10	4.70
Baseline	Shared P/Q counterpart	4.523	2.75	1.10	3.45
Baseline	Bonferroni conformal	4.480	3.25	1.00	3.65
Baseline	Unscaled maximum score	4.463	3.55	0.15	3.65
Baseline	Fixed 0.70	2.860	0.00	0.00	0.00
Baseline	Fixed 0.75	3.284	0.00	0.00	0.00
Baseline	Fixed 0.80	3.737	0.05	0.00	0.05
Baseline	Fixed 0.85	4.220	4.40	0.00	4.40
Baseline	Fixed 0.90	4.732	38.30	4.30	39.45
Compound	Joint calibrated	3.657	3.25	0.90	3.60
Compound	Shared P/Q counterpart	3.657	3.25	0.90	3.60
Compound	Bonferroni conformal	3.536	2.50	0.50	2.70
Compound	Unscaled maximum score	3.610	4.35	0.45	4.75
Compound	Fixed 0.70	2.859	1.75	0.00	1.75
Compound	Fixed 0.75	3.283	4.10	0.15	4.25
Compound	Fixed 0.80	3.737	11.85	0.55	12.30
Compound	Fixed 0.85	4.219	40.30	6.15	42.65
Compound	Fixed 0.90	4.731	82.00	49.20	85.35

Shared P/Q area gain over the box counterpart: baseline 0.007%, delay 0.007%, low SOC 0.007%, compound 0.013%. Note: the baseline shared-P/Q delivery frequency differs from the box result although the regions almost coincide, because the solver returns different optimal commands for the same boundary points; boundary-wide failure depends on which optimal allocation is stored.

Table S4. Magnitude of active-power shortfall (worst boundary command per episode)

Case	Delivery failure %	Mean shortfall in failing episodes (kW)	95th percentile (kW)	Maximum (kW)	Maximum relative to declared P (%)	Expected shortfall per episode (kW)
Baseline	4.25	1.0	3.9	16	4.8	0.04
Delay	4.25	4.4	20.1	106	10.5	0.19
Low SOC	1.95	1.5	3.8	34	2.5	0.03
Compound	3.25	1.4	3.2	60	4.1	0.05
Mismatch (baseline bounds, compound responses)	71.55	18.4	90.7	282	13.6	13.14

Table S5. Effect of resource placement (joint calibrated)

Placement	Scenario	Area (MW·Mvar)	Delivery %	Joint %
Submitted (wind 6, VPP 25, aggregated PV 33)	Baseline	4.523	4.25	4.70
Submitted	Compound	3.657	3.25	3.60
Wind 25, VPP 6	Baseline	4.370	2.50	3.10
Wind 25, VPP 6	Compound	3.530	1.65	2.00
Wind 33, aggregated PV 6	Baseline	4.148	3.10	3.60
Wind 33, aggregated PV 6	Compound	3.345	2.75	3.00

Table S6. Full-boundary AC diagnostics, joint calibrated (2000 episodes × 72 commands per scenario)

Scenario	Unsafe episodes	95% upper bound (%)	Min voltage (p.u.)	Max voltage (p.u.)	Max loading (%)	Max voltage error (p.u.)
Baseline	0/2000	0.15	0.9541	1.0392	100.0	0.00116
Delay	0/2000	0.15	0.9536	1.0392	100.0	0.00116
Low SOC	0/2000	0.15	0.9541	1.0348	100.0	0.00116
Compound	0/2000	0.15	0.9539	1.0329	100.0	0.00113