CEC2021 Function Definition

1.The definition of basic function

（1）Bent Cigar Function

$$f_1(\mathbf{x}) = x_1^2 + 10^6 \sum_{i=2}^{D} x_i^2 \tag{1}$$

（2）Rastrigin's Function

$$f_2(x) = \sum_{i=1}^{D} \left(x_i^2 - 10\cos\left(2\pi x_i\right) + 10 \right) \tag{2}$$

（3）High Conditioned Elliptic Function

$$f_3(x) = \sum_{i=1}^{D} \left(10^6\right)^{\frac{i-1}{D-1}} x_i^2 \tag{3}$$

（4）HGBat Function

$$f_4(\mathbf{x}) = \left| \left(\sum_{i=1}^{D} x_i^2 \right)^2 - \left(\sum_{i=1}^{D} x_i \right)^2 \right|^{1/2} + \left(0.5 \sum_{i=1}^{D} x_i^2 + \sum_{i=1}^{D} x_i \right) / D + 0.5 \tag{4}$$

（5）Rosenbrock's Function

$$f_5(\mathbf{x}) = \sum_{i=1}^{D-1} \left(100\left(x_i^2 - x_{i+1}\right)^2 + \left(x_i - 1\right)^2 \right) \tag{5}$$

（6）Griewank's Function

$$f_6(\mathbf{x}) = \sum_{i=1}^{D} \frac{x_i^2}{4000} - \prod_{i=1}^{D} \cos\left(\frac{x_i}{\sqrt{i}} \right) + 1 \tag{6}$$

（7）Ackley's Function

$$f_7(\mathbf{x}) = -20\exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2} \right) - \exp\left(\frac{1}{D}\sum_{i=1}^{D} \cos\left(2\pi x_i\right) \right) + 20 + \mathrm{e} \tag{7}$$

（8）Happycat Function

$$f_8(\mathbf{x}) = \left| \sum_{i=1}^{D} x_i^2 - D \right|^{1/4} + \left(0.5 \sum_{i=1}^{D} x_i^2 + \sum_{i=1}^{D} x_i \right) / D + 0.5 \tag{8}$$

（9）Discus Function

$$f_9(\mathbf{x}) = 10^6 x_1^2 + \sum_{i=2}^{D} x_i^2 \tag{9}$$

（10）Lunacek bi-Rastrigin Function

$$f_{10}(\mathbf{x}) = \min\left(\sum_{i=1}^{D}(\hat{x}_i - \mu_0)^2, dD + s\sum_{i=1}^{D}(\hat{x}_i - \mu_1)^2\right) + 10\left(D - \sum_{i=1}^{D}\cos(2\pi\hat{z}_i)\right) \quad (10)$$

where:

$$\mu_0 = 2.5, \mu_1 = -\sqrt{\frac{\mu_0^2 - d}{s}}, s = 1 - \frac{1}{2\sqrt{D+20} - 8.2}, d = 1$$

$$y = \frac{10(x-o)}{100}, \frac{x_i}{x_i} = 2sign\left(x_i^*\right)y_i + \mu_0, \quad \text{for } i = 1, 2, \ldots, D$$

$$z = \Lambda^{100}(\hat{x} - \mu_0)$$

（11）Modified Schwefel's Function

$$f_{11}(x) = 418.9829 \times D - \sum_{i=1}^{D} g(z_i) \quad (11)$$

where:

$$z_i = x_i + 4.209687462275036\text{e}+002$$

$$g(z_i) = \begin{cases} z_i \sin\left(|z_i|^{1/2}\right) & \text{if } z_i \le 500 \\ \left(500 - \text{mod}(z_i, 500)\right)\sin\left(\sqrt{|500 - \text{mod}(z_i, 500)|}\right) - \dfrac{(z_i - 500)^2}{10000D} & \text{if } z_i > 500 \\ \left(\text{mod}(|z_i|, 500) - 500\right)\sin\left(\sqrt{|\text{mod}(|z_i|, 500) - 500|}\right) - \dfrac{(z_i + 500)^2}{10000D} & \text{if } z_i < -500 \end{cases}$$

（12）Expanded Schaffer's Function

$$f_{12}(\mathbf{x}) = g(x_1, x_2) + g(x_2, x_3) + \ldots + g(x_{D-1}, x_D) + g(x_D, x_1) \quad (12)$$

where:

$$g(x, y) = 0.5 + \frac{\left(\sin^2\left(\sqrt{x^2 + y^2}\right) - 0.5\right)}{\left(1 + 0.001\left(x^2 + y^2\right)\right)^2}$$

（13）Expanded Rosenbrock's plus Griewangk's Function

$$f_{13}(\mathbf{x}) = f_6\left(f_5(x_1, x_2)\right) + f_6\left(f_5(x_2, x_3)\right) + \ldots + f_6\left(f_5(x_{D-1}, x_D)\right) + f_6\left(f_5(x_D, x_1)\right) \quad (13)$$

（14）Weierstrass Function

$$f_{14}(\mathbf{x})=\sum_{i=1}^{D}\left(\sum_{k=0}^{k\max}\left[a^{k}\cos\left(2\pi b^{k}\left(x_{i}+0.5\right)\right)\right]\right)-D\sum_{k=0}^{k\max}\left[a^{k}\cos\left(2\pi b^{k}\cdot 0.5\right)\right] \quad (14)$$

where: $a=0.5$, $b=3$, $k_{\max}=20$ 。

2.CEC2021 Function Definition

1.Unimodal function

Shifted and Rotated Bent Cigar Function

$$F_{1}(\mathbf{x})=f_{1}\left(\mathrm{M}\left(\mathbf{x}-o_{1}\right)\right)+F_{1}^{*} \quad (15)$$

2.Basic function

（1）Shifted and Rotated Schwefel's Function

$$F_{2}(\mathbf{x})=f_{11}\left(\mathbf{M}\left(\frac{1000\left(\mathbf{x}-o_{2}\right)}{100}\right)\right)+F_{2}^{*} \quad (16)$$

（2）Shifted and Rotated Lunacek bi-Rastrigin Function

$$F_{3}(x)=f_{10}\left(\mathbf{M}\left(\frac{600\left(x-o_{3}\right)}{100}\right)\right)+F_{3}^{*} \quad (17)$$

（3）Expanded Rosenbrock's plus Griewangk's Function

$$F_{4}(\mathbf{x})=f_{6}\left(f_{5}\left(x_{1},x_{2}\right)\right)+f_{6}\left(f_{5}\left(x_{2},x_{3}\right)\right)+\ldots+f_{6}\left(f_{5}\left(x_{D-1},x_{D}\right)\right)+f_{6}\left(f_{5}\left(x_{D},x_{1}\right)\right)+F_{4}^{*} \quad (18)$$

3.Blending function

In the blending function,the variables are randomly divided into several sub-components, and then different basis functions are used for different basic function.

$$F(\mathbf{x})=g_{1}\left(\mathbf{M}_{1}z_{1}\right)+g_{2}\left(\mathbf{M}_{2}z_{2}\right)+\ldots+g_{N}\left(\mathbf{M}_{N}z_{N}\right)+F^{*}(\mathbf{x})\ F(\mathbf{x}) \quad (19)$$

where:

$$z=\left[z_{1},z_{2},\ldots,z_{N}\right]$$

$$z_{1}=\left[y_{S_{1}},y_{S_{2}},\ldots,y_{S_{n-1}}\right],z_{2}=\left[y_{S_{n+1}},y_{S_{n+2}},\ldots,y_{S_{n+1}}\right],\ldots,z_{N}=\left[y_{S_{N=1}},y_{S_{N=1}},\ldots,y_{S_{D}}\right]$$

$$y = x - o_i, S = randperm(1:D)$$

where: $F(\mathbf{x})$ is a composite function;

$g_i(\mathbf{x})$ is the i basic function used to construct the mixing function.;

N is the number of basic functions;

p_i Used to control the percentage of $g_i(\mathbf{x})$;

n_i For the size of each basic function, $\sum_{i=1}^{N} n_i = D$,

$n_1 = [p_1 D], n_2 = [p_2 D], \ldots, n_{N-1} = [p_{N-1} D], n_N = D - \sum_{i=1}^{N-1} n_i$ 。

（5）Hybrid Function 1

$N = 3$

$p = [0.3, 0.3, 0.4]$

g_1： Modified Schwefel's Function f_{11}

g_2： Rastrigin's Function f_2

g_3： High Conditioned Elliptic Function f_3

（6）Hybrid Function 2

$N = 4$

$p = [0.2, 0.2, 0.3, 0.3]$

g_1： Expanded Schaffer Function f_{12}

g_2： HGBat Function f_4

g_3： Rosenbrock's Function f_5

g_4： Modified Schwefel's Function f_{11}

（7）Hybrid Function 3

$N = 5$

$p = [0.1, 0.2, 0.2, 0.2, 0.3]$

g_1： Expanded Schaffer Function f_{12}

g_2： HGBat Function f_4

g_3： Rosenbrock's Function f_5

g_4： Modified Schwefel's Function f_{11}

g_5： High Conditioned Elliptic Function f_3

4.Combination function

$$F(\mathbf{x}) = \sum_{i=1}^{N} \left\{ \omega_i^* \left[\lambda_i g_i(\mathbf{x}) + bias_i \right] \right\} + F^* \qquad (20)$$

where:

$$\omega_i = w_i / \sum_{i=1}^{n} w_i$$

$$w_i = \frac{1}{\sqrt{\sum_{j=1}^{D} \left(x_j - o_{ij} \right)^2}} \exp\left(-\frac{\sum_{j=1}^{D} \left(x_j - o_{ij} \right)^2}{2D\sigma_i^2} \right)$$

where: $F(\mathbf{x})$ is a combination function;

$g_i(\mathbf{x})$ is used to construct the i basic function of the combination function;

N is the number of basic functions;

o_i is used to define the locations of global and local optimal solutions for each $g_i(\mathbf{x})$;

$bias_i$ is used to define which is the global optimal.;

σ_i is used to control the coverage of each $g_i(\mathbf{x})$;

λ_i is used to control the height of each $g_i(\mathbf{x})$;

ω_i is the weight value of each $g_i(\mathbf{x})$。

（8）Composition Function 1 (the same as F22 in CEC2017)

$N = 3$

$\sigma = [10, 20, 30]$

$\lambda = [1, 10, 1]$

$bias = [0, 100, 200]$

g_1： Rastrigin's Function f_2

g_2： Griewank's Function f_6

g_3： Modified Schwefel's Function f_{11}

（9）Composition Function 2 (the same as F24 in CEC2017)

$N = 4$

$\sigma = [10, 20, 30, 40]$

$\lambda = [10, 1e-6, 10, 1]$

$bias = [0, 100, 200, 300]$

g_1： Ackley's Function f_7

g_2： High Conditioned Elliptic Function f_3

g_3： Griewank's Function f_6

g_4： Rastrigin's Function f_2

（10）Composition Function 3 (the same as F25 in CEC2017)

$N = 5$

$\sigma = [10, 20, 30, 40, 50]$

$\lambda = [10, 1, 10, 1e-6, 1]$

$bias = [0, 100, 200, 300, 400]$

g_1： Rastrigin's Function f_2

g_2： Happycat Function f_8

g_3： Ackley's Function f_7

g_4： Discus Function f_9

g_5： Rosenbrock's Function f_5