Supplementary materials for "Demonstrating that the autoregressive distributed lag bounds test can detect a long-run levels relationship when the dependent variable is $I(0)$"

The supporting information contained in this document are extended or unreported tables referred to in the main paper. A brief description of each of these tables is given below followed by the tables.

- Table S1 to Table S4 report lower and upper bound critical values for the F_x bounds tests at the 1%, 2.5%, 5% and 10% significance levels for cases 2 and 4. This extends the results reported in Table 2 and Table 3 of the paper which only gives these critical values at the 5% level.
- Table S5 reports the eight estimated short-run Law of One Price models that are used for inference in the paper.
- Table S6 reports the misspecification test results for the eight models reported in Table S5.

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Table S1: $I(0)$ lower bound ARDL F_x-test critical values: case 2 (restricted intercept, no trend)

	Sample size (T)												Significance
K	30	40	50	60	70	80	90	100	200	500	1000	10000	Level
1	*7.992*	*7.583*	*7.433*	*7.272*	*7.212*	*7.177*	*7.114*	*7.062*	*6.925*	*6.746*	*6.733*	*6.758*	*1%*
	6.462	6.231	6.097	5.996	5.964	5.928	5.918	5.869	5.765	5.679	5.687	5.661	2.5%
	5.332	***5.187***	***5.093***	***5.049***	***5.001***	***4.973***	***4.984***	***4.963***	***4.883***	***4.824***	***4.836***	***4.810***	***5%***
	4.230	*4.126*	*4.102*	*4.061*	*4.042*	*4.034*	*4.032*	*4.019*	*3.986*	*3.944*	*3.948*	*3.933*	10%
2	*6.219*	*5.831*	*5.677*	*5.579*	*5.492*	*5.413*	*5.394*	*5.361*	*5.237*	*5.145*	*5.125*	*5.083*	*1%*
	5.061	4.812	4.698	4.640	4.598	4.556	4.522	4.519	4.426	4.343	4.347	4.323	2.5%
	4.195	***4.049***	***3.993***	***3.942***	***3.914***	***3.890***	***3.875***	***3.845***	***3.792***	***3.741***	***3.749***	***3.722***	***5%***
	3.385	3.295	3.259	3.237	3.218	3.200	3.200	3.173	3.149	3.112	3.123	3.092	10%
3	*5.299*	*4.926*	*4.795*	*4.704*	*4.600*	*4.546*	*4.526*	*4.480*	*4.360*	*4.291*	*4.287*	*4.246*	*1%*
	4.353	4.112	4.004	3.937	3.899	3.871	3.845	3.809	3.740	3.680	3.660	3.634	2.5%
	3.655	***3.488***	***3.409***	***3.376***	***3.357***	***3.332***	***3.307***	***3.288***	***3.243***	***3.185***	***3.197***	***3.161***	***5%***
	2.968	2.869	2.826	2.806	2.788	2.774	2.762	2.749	2.714	2.691	2.687	2.668	10%
4	*4.838*	*4.409*	*4.230*	*4.150*	*4.098*	*4.026*	*3.969*	*3.932*	*3.840*	*3.770*	*3.733*	*3.735*	*1%*
	3.946	3.697	3.578	3.513	3.471	3.422	3.405	3.363	3.309	3.260	3.242	3.214	2.5%
	3.317	***3.165***	***3.083***	***3.031***	***3.003***	***2.981***	***2.961***	***2.936***	***2.897***	***2.853***	***2.846***	***2.828***	***5%***
	2.710	2.615	2.568	2.543	2.519	2.501	2.495	2.484	2.451	2.425	2.426	2.410	10%
5	*4.478*	*4.085*	*3.873*	*3.785*	*3.717*	*3.664*	*3.623*	*3.576*	*3.485*	*3.402*	*3.382*	*3.384*	*1%*
	3.659	3.397	3.302	3.230	3.180	3.146	3.119	3.085	3.016	2.961	2.948	2.931	2.5%
	3.098	***2.923***	***2.844***	***2.800***	***2.773***	***2.739***	***2.720***	***2.698***	***2.653***	***2.614***	***2.603***	***2.593***	***5%***
	2.547	2.444	2.385	2.364	2.348	2.331	2.318	2.306	2.274	2.247	2.243	2.231	10%
6	*4.253*	*3.819*	*3.638*	*3.542*	*3.483*	*3.427*	*3.361*	*3.337*	*3.224*	*3.147*	*3.137*	*3.115*	*1%*
	3.477	3.223	3.096	3.026	2.987	2.947	2.910	2.882	2.807	2.756	2.741	2.723	2.5%
	2.949	***2.761***	***2.691***	***2.638***	***2.603***	***2.571***	***2.561***	***2.530***	***2.477***	***2.446***	***2.436***	***2.416***	***5%***
	2.436	2.326	2.267	2.237	2.211	2.198	2.186	2.175	2.145	2.119	2.112	2.094	10%
7	*4.072*	*3.628*	*3.472*	*3.326*	*3.277*	*3.208*	*3.164*	*3.119*	*3.026*	*2.941*	*2.926*	*2.923*	*1%*
	3.374	3.063	2.939	2.874	2.819	2.777	2.745	2.722	2.644	2.594	2.572	2.571	2.5%
	2.844	***2.648***	***2.565***	***2.504***	***2.470***	***2.437***	***2.424***	***2.400***	***2.349***	***2.313***	***2.301***	***2.287***	***5%***
	2.358	2.227	2.170	2.135	2.113	2.096	2.084	2.070	2.041	2.015	2.008	1.993	10%

Table S1 (continued): $I(0)$ lower bound ARDL F_x-test critical values: case 2 (restricted intercept, no trend)

	Sample size (T)												Significance
K	30	40	50	60	70	80	90	100	200	500	1000	10000	Level
8	*3.948*	*3.491*	*3.307*	*3.191*	*3.111*	*3.045*	*3.004*	*2.951*	*2.868*	*2.790*	*2.765*	*2.761*	*1%*
	3.283	2.954	2.822	2.741	2.697	2.650	2.623	2.588	2.518	2.466	2.447	2.447	2.5%
	2.787	***2.557***	***2.453***	***2.403***	***2.367***	***2.345***	***2.321***	***2.299***	***2.249***	***2.212***	***2.202***	***2.190***	***5%***
	2.295	2.156	2.093	2.054	2.037	2.015	2.004	1.991	1.957	1.937	1.933	1.919	10%
9	*3.880*	*3.406*	*3.191*	*3.071*	*2.993*	*2.926*	*2.884*	*2.852*	*2.738*	*2.676*	*2.654*	*2.632*	*1%*
	3.228	2.869	2.721	2.645	2.601	2.547	2.521	2.497	2.414	2.363	2.352	2.341	2.5%
	2.740	***2.488***	***2.376***	***2.323***	***2.293***	***2.259***	***2.239***	***2.213***	***2.161***	***2.128***	***2.113***	***2.103***	***5%***
	2.258	2.106	2.033	1.997	1.973	1.954	1.943	1.927	1.893	1.871	1.866	1.852	10%
10	*3.882*	*3.311*	*3.088*	*2.978*	*2.888*	*2.825*	*2.776*	*2.749*	*2.636*	*2.564*	*2.539*	*2.529*	*1%*
	3.180	2.800	2.645	2.567	2.513	2.464	2.442	2.410	2.323	2.276	2.270	2.255	2.5%
	2.699	***2.427***	***2.318***	***2.258***	***2.223***	***2.198***	***2.175***	***2.144***	***2.086***	***2.055***	***2.049***	***2.033***	***5%***
	2.232	2.060	1.985	1.949	1.927	1.905	1.891	1.874	1.837	1.816	1.814	1.801	10%
11	*3.848*	*3.246*	*3.021*	*2.877*	*2.794*	*2.740*	*2.694*	*2.660*	*2.533*	*2.475*	*2.456*	*2.440*	*1%*
	3.161	2.753	2.581	2.496	2.444	2.394	2.370	2.345	2.254	2.202	2.194	2.177	2.5%
	2.677	***2.389***	***2.266***	***2.205***	***2.169***	***2.140***	***2.119***	***2.095***	***2.030***	***1.996***	***1.986***	***1.973***	***5%***
	2.214	2.025	1.948	1.909	1.889	1.865	1.849	1.832	1.795	1.768	1.766	1.751	10%
12	*3.889*	*3.212*	*2.959*	*2.813*	*2.715*	*2.679*	*2.626*	*2.590*	*2.468*	*2.402*	*2.375*	*2.362*	*1%*
	3.176	2.717	2.538	2.448	2.389	2.342	2.317	2.283	2.197	2.148	2.132	2.112	2.5%
	2.676	***2.355***	***2.223***	***2.162***	***2.128***	***2.092***	***2.071***	***2.046***	***1.981***	***1.944***	***1.936***	***1.920***	***5%***
	2.209	1.999	1.918	1.876	1.850	1.830	1.814	1.795	1.757	1.728	1.727	1.711	10%

Number of replications, $N = 100000$. T is the number of observations in the sample (after 100 discarded initial observations). Lower bound DGPs: $x_{k,t} = \varepsilon_{k,t}$, $\varepsilon_{k,t} \sim N(0,1)$; $y_t = y_{t-1} + \varepsilon_{y,t}$, $\varepsilon_{y,t} \sim N(0,1)$. The regression is, $\Delta y_t = c_0 + \pi_y y_{t-1} + \sum_{k=1}^{K} \pi_{xk} x_{k,t-1} + u_t$. Critical values are given for an F_x test of the null hypothesis of no equilibrium, $H_0^x: c_0 = \pi_{x1} = \cdots = \pi_{xK} = 0$. The following levels of significance are indicated by the following font types: 1% (red, italic), 2.5% (green), 5% (black, bold, italic) and 10% (blue).

Table S2: $I(1)$ upper bound ARDL F_x-test critical values: case 2 (restricted intercept, no trend)

	Sample size (T)												Significance
K	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**200**	**500**	**1000**	**10000**	**Level**
1	*9.270*	*8.847*	*8.661*	*8.335*	*8.224*	*8.172*	*8.106*	*8.066*	*7.916*	*7.741*	*7.763*	*7.601*	*1%*
	7.622	7.291	7.130	7.008	6.896	6.868	6.853	6.790	6.667	6.552	6.552	6.494	2.5%
	6.329	***6.098***	***6.015***	***5.948***	***5.884***	***5.863***	***5.836***	***5.811***	***5.726***	***5.667***	***5.641***	***5.619***	***5%***
	5.067	4.941	4.908	4.862	4.836	4.830	4.816	4.787	4.754	4.706	4.708	4.668	10%
2	*7.789*	*7.283*	*7.085*	*6.883*	*6.813*	*6.733*	*6.671*	*6.591*	*6.468*	*6.282*	*6.250*	*6.188*	*1%*
	6.469	6.098	5.960	5.855	5.782	5.725	5.681	5.638	5.539	5.443	5.459	5.378	2.5%
	5.457	***5.230***	***5.130***	***5.072***	***4.998***	***4.951***	***4.946***	***4.926***	***4.836***	***4.784***	***4.775***	***4.727***	***5%***
	4.467	4.326	4.275	4.233	4.203	4.185	4.176	4.164	4.105	4.073	4.063	4.039	10%
3	*7.064*	*6.588*	*6.305*	*6.138*	*6.014*	*5.952*	*5.846*	*5.815*	*5.668*	*5.539*	*5.519*	*5.453*	*1%*
	5.884	5.555	5.381	5.234	5.174	5.125	5.082	5.053	4.946	4.844	4.838	4.791	2.5%
	5.003	***4.767***	***4.649***	***4.582***	***4.528***	***4.498***	***4.452***	***4.436***	***4.374***	***4.306***	***4.303***	***4.247***	***5%***
	4.152	4.003	3.932	3.896	3.852	3.829	3.823	3.807	3.747	3.730	3.725	3.687	10%
4	*6.581*	*6.107*	*5.818*	*5.645*	*5.550*	*5.463*	*5.380*	*5.331*	*5.186*	*5.063*	*5.071*	*4.994*	*1%*
	5.527	5.204	5.003	4.874	4.795	4.737	4.702	4.668	4.534	4.470	4.476	4.416	2.5%
	4.738	***4.490***	***4.367***	***4.276***	***4.221***	***4.195***	***4.163***	***4.156***	***4.050***	***4.008***	***4.015***	***3.959***	***5%***
	3.958	3.799	3.719	3.677	3.640	3.628	3.597	3.590	3.534	3.508	3.502	3.476	10%
5	*6.361*	*5.823*	*5.459*	*5.287*	*5.189*	*5.109*	*5.061*	*5.008*	*4.848*	*4.711*	*4.686*	*4.676*	*1%*
	5.337	4.955	4.742	4.614	4.539	4.491	4.443	4.420	4.289	4.203	4.197	4.159	2.5%
	4.583	***4.313***	***4.165***	***4.077***	***4.024***	***3.983***	***3.956***	***3.948***	***3.832***	***3.788***	***3.801***	***3.750***	***5%***
	3.830	3.666	3.582	3.530	3.488	3.470	3.456	3.444	3.375	3.342	3.352	3.317	10%
6	*6.159*	*5.569*	*5.253*	*5.058*	*4.978*	*4.873*	*4.852*	*4.795*	*4.585*	*4.481*	*4.434*	*4.423*	*1%*
	5.181	4.771	4.554	4.425	4.346	4.306	4.266	4.244	4.071	4.001	3.992	3.966	2.5%
	4.470	***4.174***	***4.025***	***3.951***	***3.879***	***3.850***	***3.814***	***3.797***	***3.683***	***3.624***	***3.625***	***3.594***	***5%***
	3.762	3.576	3.475	3.429	3.394	3.364	3.344	3.337	3.256	3.226	3.231	3.198	10%
7	*6.108*	*5.393*	*5.107*	*4.905*	*4.780*	*4.662*	*4.652*	*4.616*	*4.380*	*4.271*	*4.250*	*4.213*	*1%*
	5.115	4.657	4.442	4.287	4.211	4.146	4.118	4.096	3.920	3.848	3.834	3.810	2.5%
	4.416	***4.091***	***3.926***	***3.833***	***3.776***	***3.733***	***3.703***	***3.684***	***3.555***	***3.499***	***3.493***	***3.468***	***5%***
	3.722	3.507	3.400	3.347	3.305	3.283	3.257	3.252	3.166	3.136	3.129	3.108	10%

Table S2 (continued): $I(1)$ upper bound ARDL F_x-test critical values: case 2 (restricted intercept, no trend)

	Sample size (T)												Significance
K	30	40	50	60	70	80	90	100	200	500	1000	10000	Level
8	*6.062*	*5.306*	*4.964*	*4.765*	*4.643*	*4.570*	*4.496*	*4.419*	*4.213*	*4.100*	*4.089*	*4.061*	*1%*
	5.062	4.556	4.325	4.195	4.106	4.037	3.997	3.962	3.793	3.711	3.708	3.688	2.5%
	4.369	***4.011***	***3.842***	***3.748***	***3.685***	***3.639***	***3.602***	***3.589***	***3.459***	***3.399***	***3.384***	***3.374***	***5%***
	3.691	3.452	3.338	3.284	3.243	3.216	3.194	3.181	3.093	3.062	3.050	3.038	10%
9	*6.014*	*5.233*	*4.878*	*4.656*	*4.545*	*4.451*	*4.360*	*4.303*	*4.105*	*3.966*	*3.949*	*3.915*	*1%*
	5.076	4.502	4.240	4.100	4.013	3.947	3.892	3.862	3.690	3.604	3.601	3.568	2.5%
	4.356	***3.963***	***3.779***	***3.679***	***3.607***	***3.559***	***3.526***	***3.506***	***3.377***	***3.309***	***3.299***	***3.285***	***5%***
	3.671	3.417	3.300	3.234	3.184	3.156	3.134	3.120	3.031	2.996	2.982	2.974	10%
10	*6.051*	*5.173*	*4.797*	*4.572*	*4.456*	*4.352*	*4.253*	*4.212*	*3.984*	*3.849*	*3.839*	*3.816*	*1%*
	5.097	4.462	4.183	4.044	3.934	3.880	3.813	3.776	3.614	3.510	3.506	3.472	2.5%
	4.379	***3.925***	***3.729***	***3.612***	***3.547***	***3.501***	***3.458***	***3.441***	***3.315***	***3.237***	***3.226***	***3.209***	***5%***
	3.672	3.381	3.264	3.189	3.145	3.114	3.091	3.078	2.983	2.941	2.933	2.925	10%
11	*6.129*	*5.121*	*4.712*	*4.527*	*4.361*	*4.273*	*4.171*	*4.126*	*3.913*	*3.768*	*3.749*	*3.713*	*1%*
	5.117	4.445	4.139	3.975	3.868	3.814	3.742	3.713	3.545	3.439	3.428	3.397	2.5%
	4.397	***3.900***	***3.682***	***3.572***	***3.495***	***3.443***	***3.404***	***3.381***	***3.255***	***3.176***	***3.168***	***3.146***	***5%***
	3.688	3.366	3.227	3.153	3.105	3.075	3.048	3.033	2.940	2.894	2.886	2.875	10%
12	*6.278*	*5.124*	*4.668*	*4.464*	*4.294*	*4.190*	*4.109*	*4.064*	*3.843*	*3.683*	*3.667*	*3.615*	*1%*
	5.195	4.426	4.107	3.935	3.822	3.757	3.686	3.674	3.490	3.372	3.368	3.334	2.5%
	4.432	***3.901***	***3.670***	***3.536***	***3.457***	***3.411***	***3.371***	***3.347***	***3.202***	***3.125***	***3.113***	***3.092***	***5%***
	3.714	3.360	3.205	3.126	3.078	3.045	3.018	2.999	2.906	2.857	2.848	2.833	10%

Number of replications, $N = 100000$. T is the number of observations in the sample (after 100 discarded initial observations). Upper bound DGPs: $x_{k,t} = x_{k,t-1} + \varepsilon_{k,t}$, $\varepsilon_{k,t} \sim N(0,1)$; $y_t = y_{t-1} + \varepsilon_{y,t}$, $\varepsilon_{y,t} \sim N(0,1)$. The regression is, $\Delta y_t = c_0 + \pi_y y_{t-1} + \sum_{k=1}^{K} \pi_{xk} x_{k,t-1} + u_t$. Critical values are given for an F_x test of the null hypothesis of no equilibrium, $H_0^x: c_0 = \pi_{x1} = \cdots = \pi_{xK} = 0$. The following levels of significance are indicated by the following font types: 1% (red, italic), 2.5% (green), 5% (black, bold, italic) and 10% (blue).

Table S3: $I(0)$ lower bound ARDL F_x-test critical values: case 4 (unrestricted intercept, restricted trend)

	Sample size (T)												Significance
K	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**200**	**500**	**1000**	**10000**	**Level**
1	*9.195*	*8.710*	*8.543*	*8.410*	*8.323*	*8.238*	*8.126*	*8.128*	*7.919*	*7.803*	*7.782*	*7.806*	*1%*
	7.487	7.169	7.066	6.931	6.883	6.809	6.772	6.743	6.674	6.545	6.549	6.578	2.5%
	6.108	***5.979***	***5.904***	***5.834***	***5.768***	***5.771***	***5.721***	***5.709***	***5.646***	***5.596***	***5.570***	***5.579***	***5%***
	4.846	*4.761*	*4.733*	*4.692*	*4.650*	*4.657*	*4.622*	*4.610*	*4.585*	*4.565*	*4.543*	*4.551*	10%
2	*7.036*	*6.571*	*6.400*	*6.280*	*6.212*	*6.095*	*6.048*	*6.040*	*5.867*	*5.767*	*5.714*	*5.736*	*1%*
	5.707	5.407	5.323	5.224	5.163	5.120	5.075	5.079	4.975	4.881	4.842	4.890	2.5%
	4.732	***4.585***	***4.502***	***4.423***	***4.379***	***4.367***	***4.331***	***4.326***	***4.265***	***4.217***	***4.182***	***4.212***	***5%***
	3.784	3.707	3.656	3.614	3.578	3.596	3.555	3.551	3.529	3.494	3.484	3.490	10%
3	*5.941*	*5.506*	*5.310*	*5.180*	*5.112*	*5.043*	*4.980*	*4.961*	*4.824*	*4.720*	*4.676*	*4.703*	*1%*
	4.835	4.571	4.452	4.361	4.282	4.275	4.224	4.225	4.111	4.046	4.015	4.038	2.5%
	4.051	***3.851***	***3.787***	***3.723***	***3.681***	***3.674***	***3.644***	***3.641***	***3.567***	***3.512***	***3.504***	***3.510***	***5%***
	3.252	3.166	3.119	3.078	3.054	3.048	3.035	3.019	2.986	2.964	2.946	2.941	10%
4	*5.308*	*4.817*	*4.659*	*4.537*	*4.455*	*4.409*	*4.326*	*4.290*	*4.222*	*4.114*	*4.048*	*4.063*	*1%*
	4.332	4.033	3.908	3.825	3.772	3.746	3.700	3.704	3.591	3.532	3.518	3.527	2.5%
	3.621	***3.433***	***3.364***	***3.294***	***3.261***	***3.242***	***3.208***	***3.204***	***3.137***	***3.097***	***3.086***	***3.087***	***5%***
	2.939	2.842	2.791	2.751	2.727	2.710	2.700	2.692	2.655	2.629	2.620	2.610	10%
5	*4.859*	*4.390*	*4.203*	*4.100*	*4.015*	*3.995*	*3.910*	*3.871*	*3.779*	*3.685*	*3.636*	*3.638*	*1%*
	4.000	3.703	3.559	3.490	3.440	3.404	3.346	3.346	3.241	3.204	3.181	3.176	2.5%
	3.366	***3.157***	***3.073***	***3.026***	***2.983***	***2.947***	***2.918***	***2.914***	***2.846***	***2.810***	***2.795***	***2.795***	***5%***
	2.745	2.623	2.578	2.532	2.509	2.490	2.478	2.472	2.436	2.408	2.403	2.399	10%
6	*4.618*	*4.103*	*3.920*	*3.803*	*3.726*	*3.673*	*3.593*	*3.575*	*3.472*	*3.393*	*3.329*	*3.347*	*1%*
	3.772	3.467	3.325	3.236	3.196	3.163	3.101	3.095	3.002	2.955	2.926	2.919	2.5%
	3.177	***2.965***	***2.882***	***2.825***	***2.782***	***2.755***	***2.721***	***2.718***	***2.646***	***2.611***	***2.596***	***2.592***	***5%***
	2.602	2.475	2.426	2.386	2.357	2.334	2.324	2.316	2.279	2.247	2.248	2.238	10%
7	*4.419*	*3.906*	*3.680*	*3.549*	*3.482*	*3.420*	*3.362*	*3.331*	*3.223*	*3.146*	*3.104*	*3.117*	*1%*
	3.626	3.280	3.143	3.053	2.996	2.962	2.913	2.900	2.815	2.764	2.728	2.734	2.5%
	3.042	***2.820***	***2.732***	***2.665***	***2.628***	***2.595***	***2.565***	***2.565***	***2.487***	***2.460***	***2.441***	***2.441***	***5%***
	2.504	2.360	2.308	2.268	2.243	2.221	2.204	2.197	2.156	2.129	2.123	2.116	10%

Table S3 (continued): $I(0)$ lower bound ARDL F_x-test critical values: case 4 (unrestricted intercept, restricted trend)

	Sample size (T)												Significance
K	30	40	50	60	70	80	90	100	200	500	1000	10000	Level
8	*4.283*	*3.734*	*3.478*	*3.357*	*3.286*	*3.246*	*3.181*	*3.157*	*3.022*	*2.969*	*2.912*	*2.922*	*1%*
	3.481	3.144	2.984	2.907	2.840	2.812	2.765	2.748	2.661	2.614	2.583	2.586	2.5%
	2.949	***2.709***	***2.608***	***2.548***	***2.504***	***2.477***	***2.440***	***2.437***	***2.370***	***2.328***	***2.317***	***2.311***	***5%***
	2.428	2.277	2.217	2.177	2.147	2.128	2.112	2.100	2.057	2.031	2.028	2.018	10%
9	*4.181*	*3.588*	*3.360*	*3.229*	*3.140*	*3.073*	*3.039*	*3.004*	*2.897*	*2.830*	*2.764*	*2.778*	*1%*
	3.422	3.047	2.873	2.779	2.724	2.685	2.643	2.629	2.553	2.494	2.460	2.455	2.5%
	2.873	***2.628***	***2.519***	***2.451***	***2.406***	***2.381***	***2.345***	***2.335***	***2.271***	***2.230***	***2.216***	***2.213***	***5%***
	2.376	2.210	2.145	2.100	2.073	2.054	2.035	2.025	1.981	1.955	1.950	1.942	10%
10	*4.120*	*3.499*	*3.259*	*3.110*	*3.021*	*2.952*	*2.911*	*2.879*	*2.769*	*2.701*	*2.644*	*2.673*	*1%*
	3.360	2.967	2.777	2.691	2.625	2.595	2.546	2.521	2.457	2.390	2.364	2.361	2.5%
	2.841	***2.563***	***2.443***	***2.366***	***2.327***	***2.300***	***2.263***	***2.251***	***2.193***	***2.145***	***2.130***	***2.130***	***5%***
	2.338	2.153	2.081	2.041	2.009	1.988	1.970	1.963	1.918	1.890	1.887	1.879	10%
11	*4.147*	*3.430*	*3.173*	*3.030*	*2.924*	*2.867*	*2.813*	*2.785*	*2.667*	*2.598*	*2.544*	*2.564*	*1%*
	3.340	2.909	2.722	2.615	2.557	2.519	2.470	2.445	2.361	2.306	2.281	2.281	2.5%
	2.815	***2.515***	***2.378***	***2.303***	***2.258***	***2.232***	***2.201***	***2.186***	***2.128***	***2.080***	***2.065***	***2.062***	***5%***
	2.326	2.119	2.038	1.994	1.960	1.942	1.922	1.912	1.863	1.837	1.830	1.824	10%
12	*4.166*	*3.382*	*3.081*	*2.936*	*2.839*	*2.775*	*2.722*	*2.709*	*2.567*	*2.506*	*2.465*	*2.478*	*1%*
	3.352	2.859	2.651	2.547	2.487	2.444	2.405	2.375	2.293	2.239	2.212	2.207	2.5%
	2.811	***2.470***	***2.324***	***2.249***	***2.205***	***2.176***	***2.146***	***2.129***	***2.066***	***2.022***	***2.006***	***2.002***	***5%***
	2.315	2.087	1.997	1.949	1.920	1.902	1.881	1.869	1.819	1.794	1.785	1.781	10%

Number of replications, $N = 100000$. T is the number of observations in the sample (after 100 discarded initial observations). Lower bound DGPs: $x_{k,t} = \varepsilon_{k,t}$, $\varepsilon_{k,t} \sim N(0,1)$; $y_t = y_{t-1} + \varepsilon_{y,t}$, $\varepsilon_{y,t} \sim N(0,1)$. The regression is, $\Delta y_t = c_0 + c_1 t + \pi_y y_{t-1} + \sum_{k=1}^{K} \pi_{xk} x_{k,t-1} + u_t$. Critical values are given for an F_x test of the null hypothesis of no equilibrium, $H_0^x: c_1 = \pi_{x1} = \cdots = \pi_{xK} = 0$. The following levels of significance are indicated by the following font types: 1% (red, italic), 2.5% (green), 5% (black, bold, italic) and 10% (blue).

Table S4: $I(1)$ upper bound ARDL F_x-test critical values: case 4 (unrestricted intercept, restricted trend)

	Sample size (T)												Significance
K	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**200**	**500**	**1000**	**10000**	**Level**
1	*10.377*	*9.967*	*9.607*	*9.382*	*9.255*	*9.190*	*9.119*	*9.086*	*8.739*	*8.664*	*8.614*	*8.695*	*1%*
	8.476	8.145	8.000	7.892	7.760	7.717	7.681	7.642	7.465	7.411	7.354	7.383	2.5%
	7.030	***6.848***	***6.745***	***6.671***	***6.608***	***6.547***	***6.559***	***6.535***	***6.428***	***6.376***	***6.332***	***6.373***	***5%***
	5.624	5.541	5.487	5.432	5.415	5.390	5.378	5.348	5.317	5.299	5.277	5.289	10%
2	*8.487*	*7.991*	*7.690*	*7.547*	*7.424*	*7.307*	*7.264*	*7.212*	*6.959*	*6.871*	*6.767*	*6.866*	*1%*
	6.988	6.685	6.469	6.388	6.305	6.255	6.191	6.190	6.008	5.965	5.931	5.941	2.5%
	5.889	***5.667***	***5.561***	***5.497***	***5.442***	***5.414***	***5.368***	***5.369***	***5.276***	***5.223***	***5.187***	***5.220***	***5%***
	4.788	4.668	4.613	4.581	4.548	4.529	4.526	4.521	4.467	4.432	4.400	4.415	10%
3	*7.511*	*7.049*	*6.709*	*6.550*	*6.439*	*6.330*	*6.310*	*6.288*	*6.062*	*5.959*	*5.856*	*5.897*	*1%*
	6.236	5.947	5.707	5.616	5.556	5.483	5.431	5.419	5.292	5.194	5.147	5.193	2.5%
	5.292	***5.113***	***4.963***	***4.890***	***4.836***	***4.801***	***4.786***	***4.772***	***4.669***	***4.600***	***4.577***	***4.593***	***5%***
	4.366	4.237	4.184	4.146	4.100	4.088	4.079	4.073	4.004	3.973	3.950	3.968	10%
4	*6.971*	*6.450*	*6.140*	*5.934*	*5.839*	*5.753*	*5.718*	*5.724*	*5.516*	*5.367*	*5.302*	*5.315*	*1%*
	5.827	5.450	5.253	5.155	5.078	5.018	4.976	4.956	4.850	4.717	4.697	4.691	2.5%
	4.971	***4.717***	***4.600***	***4.532***	***4.479***	***4.425***	***4.401***	***4.381***	***4.305***	***4.220***	***4.187***	***4.216***	***5%***
	4.106	3.985	3.901	3.870	3.828	3.809	3.792	3.779	3.729	3.684	3.659	3.681	10%
5	*6.663*	*6.083*	*5.750*	*5.579*	*5.451*	*5.366*	*5.306*	*5.281*	*5.087*	*4.953*	*4.899*	*4.909*	*1%*
	5.570	5.153	4.960	4.826	4.763	4.704	4.656	4.644	4.506	4.389	4.381	4.371	2.5%
	4.738	***4.463***	***4.342***	***4.265***	***4.207***	***4.166***	***4.149***	***4.136***	***4.040***	***3.954***	***3.931***	***3.941***	***5%***
	3.939	3.801	3.720	3.674	3.640	3.617	3.613	3.592	3.534	3.484	3.465	3.483	10%
6	*6.448*	*5.804*	*5.509*	*5.303*	*5.151*	*5.083*	*4.994*	*4.986*	*4.815*	*4.666*	*4.598*	*4.600*	*1%*
	5.391	4.957	4.754	4.621	4.517	4.466	4.416	4.394	4.275	4.166	4.120	4.125	2.5%
	4.616	***4.306***	***4.169***	***4.076***	***4.024***	***3.984***	***3.954***	***3.930***	***3.840***	***3.762***	***3.750***	***3.747***	***5%***
	3.835	3.677	3.589	3.535	3.498	3.472	3.455	3.443	3.394	3.341	3.323	3.331	10%
7	*6.308*	*5.579*	*5.294*	*5.095*	*4.945*	*4.855*	*4.784*	*4.771*	*4.575*	*4.448*	*4.387*	*4.371*	*1%*
	5.283	4.797	4.597	4.440	4.331	4.276	4.242	4.218	4.079	3.985	3.953	3.947	2.5%
	4.518	***4.192***	***4.040***	***3.951***	***3.877***	***3.841***	***3.808***	***3.795***	***3.694***	***3.619***	***3.588***	***3.595***	***5%***
	3.767	3.580	3.483	3.437	3.392	3.361	3.348	3.335	3.278	3.226	3.213	3.215	10%

Table S4 (continued): $I(1)$ upper bound ARDL F_x-test critical values: case 4 (unrestricted intercept, restricted trend)

	Sample size (T)												Significance
K	30	40	50	60	70	80	90	100	200	500	1000	10000	Level
8	*6.260*	*5.471*	*5.153*	*4.933*	*4.775*	*4.710*	*4.627*	*4.580*	*4.380*	*4.248*	*4.197*	*4.178*	*1%*
	5.225	4.684	4.459	4.304	4.198	4.142	4.101	4.086	3.935	3.834	3.805	3.789	2.5%
	4.457	***4.102***	***3.939***	***3.838***	***3.761***	***3.719***	***3.697***	***3.679***	***3.568***	***3.497***	***3.476***	***3.475***	***5%***
	3.719	3.513	3.402	3.342	3.303	3.277	3.266	3.250	3.185	3.137	3.123	3.122	10%
9	*6.271*	*5.355*	*5.027*	*4.796*	*4.646*	*4.574*	*4.501*	*4.426*	*4.242*	*4.118*	*4.045*	*4.052*	*1%*
	5.196	4.623	4.368	4.206	4.100	4.033	3.993	3.966	3.809	3.722	3.690	3.670	2.5%
	4.446	***4.051***	***3.867***	***3.753***	***3.669***	***3.634***	***3.604***	***3.586***	***3.475***	***3.408***	***3.379***	***3.371***	***5%***
	3.710	3.480	3.359	3.287	3.242	3.211	3.196	3.181	3.114	3.064	3.054	3.049	10%
10	*6.257*	*5.291*	*4.925*	*4.717*	*4.545*	*4.451*	*4.386*	*4.315*	*4.107*	*3.993*	*3.934*	*3.921*	*1%*
	5.216	4.578	4.296	4.125	4.002	3.944	3.900	3.871	3.711	3.615	3.582	3.576	2.5%
	4.437	***4.003***	***3.809***	***3.684***	***3.610***	***3.551***	***3.528***	***3.507***	***3.392***	***3.324***	***3.299***	***3.290***	***5%***
	3.715	3.439	3.313	3.236	3.186	3.154	3.135	3.117	3.056	3.007	2.988	2.986	10%
11	*6.388*	*5.270*	*4.859*	*4.617*	*4.453*	*4.346*	*4.290*	*4.232*	*4.007*	*3.891*	*3.826*	*3.821*	*1%*
	5.256	4.522	4.236	4.057	3.939	3.878	3.827	3.778	3.622	3.533	3.495	3.495	2.5%
	4.486	***3.971***	***3.761***	***3.629***	***3.552***	***3.493***	***3.459***	***3.434***	***3.324***	***3.255***	***3.228***	***3.226***	***5%***
	3.720	3.415	3.274	3.192	3.142	3.105	3.087	3.071	3.000	2.959	2.934	2.933	10%
12	*6.572*	*5.233*	*4.803*	*4.554*	*4.401*	*4.275*	*4.198*	*4.135*	*3.919*	*3.809*	*3.741*	*3.729*	*1%*
	5.358	4.504	4.190	4.020	3.891	3.808	3.754	3.706	3.550	3.471	3.428	3.419	2.5%
	4.547	***3.942***	***3.727***	***3.594***	***3.502***	***3.445***	***3.405***	***3.378***	***3.264***	***3.203***	***3.170***	***3.166***	***5%***
	3.760	3.394	3.244	3.152	3.103	3.069	3.042	3.032	2.956	2.914	2.890	2.889	10%

Number of replications, $N = 100000$. T is the number of observations in the sample (after 100 discarded initial observations). Upper bound DGPs: $x_{k,t} = x_{k,t-1} + \varepsilon_{k,t}$, $\varepsilon_{k,t} \sim N(0,1)$; $y_t = y_{t-1} + \varepsilon_{y,t}$, $\varepsilon_{y,t} \sim N(0,1)$. The regression is, $\Delta y_t = c_0 + c_1 t + \pi_y y_{t-1} + \sum_{k=1}^{K} \pi_{xk} x_{k,t-1} + u_t$. Critical values are given for an F_x test of the null hypothesis of no equilibrium, $H_0^x: c_1 = \pi_{x1} = \cdots = \pi_{xK} = 0$. The following levels of significance are indicated by the following font types: 1% (red, italic), 2.5% (green), 5% (black, bold, italic) and 10% (blue).

Table S5. Estimated short-run ARDL models

Countries	Bulgaria-Denmark				Denmark-Euro			
Regressor	LRP_PH_BD	LRP_PH_BD	LRP_PH_BD	LRP_PH_BD	LRP_HT_DE	LRP_HT_DE	LRP_RR_DE	LRP_RR_DE
Model	ARDL(1,1)	ARDL(1,1)	ARDL(1,1) ARCH(5)	ARDL(1,1) ARCH(5)	ARDL(1,3)	ARDL(1,3)	ARDL(1,1)	ARDL(1,1)
Intercept	-0.255	-0.258	-0.229	-0.248	0.220	0.220	0.178	0.178
	(-5.636)*	(-5.714)*	(-6.008)*	(-6.657)*	(4.077)*	(4.069)*	(3.662)*	(3.646)*
Trend		4.11×10^{-7}		1.17×10^{-6}		6.21×10^{-8}		-2.47×10^{-7}
		(0.544)		(2.282)*		(0.141)		(-0.082)
lnS_{t-1}	-0.191	-0.193	-0.171	-0.185	-0.109	-0.110	-0.089	-0.089
	(-5.635)*	(-5.712)*	(-6.008)*	(-6.653)*	(-4.077)*	(-4.069)*	(-3.662)*	(-3.642)*
$lnRP_{t-1}$	0.001	0.001	0.002	0.001	0.002	0.002	2.25×10^{-4}	5.19×10^{-4}
	(2.370)*	(1.959)	(2.915)*	(2.621)*	(1.439)	(1.395)	(0.430)	(0.1433)
$\Delta lnRP_t$	0.004	0.004	0.007	0.006	0.004	0.004	0.011	0.011
	(1.248)	(1.239)	(3.234)*	(3.275)*	(2.184)*	(2.179)*	(1.958)	(1.877)
$\Delta lnRP_{t-1}$					0.005	0.005		
					(2.212)*	(2.212)*		
$\Delta lnRP_{t-2}$					0.003	0.003		
					(1.577)	(1.581)		
ARCH(intercept)			1.44×10^{-7}	1.41×10^{-7}				
			(3.711)*	(3.648)*				
ARCH(1)			0.048	0.032				
			(1.342)	(1.086)				
ARCH(2)			0.075	0.070				
			(1.593)	(1.469)				
ARCH(3)			0.245	0.231				
			(1.966)*	(1.872)				
ARCH(4)			0.186	0.195				
			(1.655)	(1.681)				
ARCH(5)			0.317	0.346				
			(2.455)*	(2.507)*				
$\bar{R}^2$	0.105	0.102	0.057	0.051	0.071	0.068	0.049	0.045
s	8.78×10^{-4}	8.79×10^{-4}	9.01×10^{-4}	9.04×10^{-4}	6.14×10^{-4}	6.15×10^{-4}	6.19×10^{-4}	6.20×10^{-4}
AIC	-11.223	-11.217	-11.514	-11.522	-11.934	-11.927	-11.922	-11.916
Observations	282	282	282	282	291	291	293	293

Coefficients with t-ratios in parentheses are reported for the short-run ARDL model, (2). For the Bulgaria-Denmark equations White's heteroscedasticity consistent covariance is used for test statistics in the model without an ARCH process and the Bollerslev-Wooldridge covariance in the model with ARCH effects. The standard OLS covariance matrix is used to calculate test statistics for Denmark-euro equations. The adjusted coefficient of determination is denoted $\bar{R}^2$, s is the standard error of regression while the Akaike information criterion is given by AIC. The countries involved in the exchange rate are specified in the row labelled "Country". An asterisk (*) indicates that the associated test's null hypothesis is rejected at the 5% significance level. The samples size is given in the row labelled "Observations" and the values of p and q_1 of the estimated ARDL(p,q_1) specification are indicated in the row labelled "Model".

Table S6. Misspecification tests

	Bulgaria-Denmark				Denmark-euro			
Regressor	LRP_PH_BD	LRP_PH_BD	LRP_PH_BD	LRP_PH_BD	LRP_HT_DE	LRP_HT_DE	LRP_RR_DE	LRP_RR_DE
Model	ARDL(1,1)	ARDL(1,1)	ARDL(1,1) ARCH(5)	ARDL(1,1) ARCH(5)	ARDL(1,3)	ARDL(1,3)	ARDL(1,1)	ARDL(1,1)
Case	II and III	IV	II and III	IV	II and III	IV	II and III	IV
LMA 1	0.894	0.871	0.837	0.856	0.775	0.779	0.967	0.965
LMA 2	0.894	0.875	0.667	0.675	0.914	0.916	0.941	0.940
LMA 3	0.970	0.960	0.626	0.600	0.519	0.515	0.770	0.770
LMA 4	0.820	0.825	0.768	0.743	0.538	0.532	0.830	0.831
LMA 5	0.488	0.506	0.787	0.750	0.531	0.530	0.805	0.805
LMA 6	0.444	0.467	0.821	0.799	0.562	0.557	0.769	0.771
LMA 7	0.200	0.207	0.618	0.626	0.365	0.350	0.477	0.475
LMA 8	0.249	0.258	0.713	0.717	0.455	0.442	0.583	0.582
LMA 9	0.313	0.322	0.277	0.308	0.291	0.268	0.405	0.403
LMA 10	0.260	0.266	0.345	0.389	0.364	0.342	0.478	0.476
LMA 11	0.332	0.339	0.429	0.477	0.317	0.284	0.410	0.408
LMA 12	0.387	0.396	0.512	0.562	0.394	0.358	0.476	0.473
LMH 1	0.001*	0.001*	0.255	0.388	0.540	0.650	0.695	0.751
LMH 2	0.351	0.411	0.894	0.076	0.957	0.984	0.119	0.118
LMN	0.000*	0.000*	0.031*	0.018*	0.001*	0.001*	0.000*	0.000*
LMARCH 1	0.806	0.811	0.635	0.624	0.054	0.052	0.106	0.107
LMARCH 2	0.963	0.966	0.823	0.827	0.149	0.147	0.256	0.257
LMARCH 3	0.983	0.982	0.883	0.877	0.281	0.277	0.433	0.434
LMARCH 4	0.000*	0.000*	0.759	0.753	0.162	0.158	0.429	0.432
LMARCH 5	0.000*	0.000*	0.784	0.777	0.232	0.225	0.514	0.519
LMARCH 6	0.000*	0.000*	0.815	0.826	0.214	0.206	0.601	0.607
LMARCH 7	0.000*	0.000*	0.883	0.889	0.294	0.287	0.663	0.670
LMARCH 8	0.000*	0.000*	0.818	0.736	0.387	0.377	0.747	0.754
LMARCH 9	0.000*	0.000*	0.820	0.796	0.436	0.428	0.788	0.794
LMARCH 10	0.000*	0.000*	0.833	0.765	0.488	0.480	0.821	0.826
LMARCH 11	0.000*	0.000*	0.875	0.833	0.564	0.555	0.880	0.884
LMARCH 12	0.000*	0.000*	0.847	0.828	0.425	0.418	0.812	0.816
LRFF	0.095	0.152			0.986	0.978	0.414	0.406
FC2007m8	0.125	0.203			0.951	0.976	0.808	0.730
FC2016m6	0.221	0.332			0.860	0.473	0.906	0.874
FC2020m3	0.325	0.428			0.513	0.279	0.056	0.086
UDMax	11.875	12.132			14.763	18.569	9.886	19.273*
	{16.370}	{18.420}			{20.300}	{22.040}	{16.370}	{18.420}
WDMax	11.875	12.651			14.922	24.802*	16.447	29.241*
	{17.830}	{19.960}			{21.860}	{23.810}	{17.830}	{19.960}

Probability values are reported for the following misspecification tests. The Breusch-Godfrey LM test for autocorrelation of order s is denoted 'LMA s', except for the model with an ARCH process where 'LMA s' denotes the Ljung-Box test for autocorrelation of order s. White's LM test for heteroscedasticity including only squared regressors is denoted LMH1 while an asymptotic equivalent to White's test that parsimoniously captures the regressors, squared regressors and cross terms by using the squared fitted value is given by 'LMH2'. That is, the squared residuals are regressed on an intercept and the squared fitted value where the number of observations multiplied by this auxiliary regression's R-squared gives LMH2. 'LMN' represents the Jarque-Bera LM test for non-normally distributed residuals. LM tests for r^{th} order ARCH effects are denoted 'LMARCH r', Ramsey's RESET LR test for non-linear functional form is given by 'LRFF', Chow's first F-test for structural change with 'date' as the date of the first period of the second sub-sample is 'FCdate'. The Bai and Perron global UDMax and WDMax test statistics for up to 5 multiple breaks at estimated break dates against zero breaks are denoted 'UDMax' and 'WDMax', respectively. The corresponding 5% critical values are given in braces next to the associated statistics. An asterisk (*) indicates significance at the 5% level.