

# Supplementary Material for “Bulk Operator Reconstruction in Topological Tensor Network and Generalized Free Fields”

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## I. SOLUTION FOR 2+1D FIBONACCI MODEL

The general solution for the 2+1d Fibonacci model is given by the linear combination of vectors  $v^{(p)}$  in Equation (59) plus the specific solution part  $A^{(\mu)}$  for each  $\sigma_\mu$ .  $A^{(\mu)}$  is an  $8 \times 8$  matrix whose last 3 columns are zero, i.e.

$$A^{(\mu)} = (\tilde{A}^{(\mu)} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0}), \quad (1)$$

and the  $8 \times 5$  matrices  $\tilde{A}^{(\mu)}$  are given by the following, where  $\phi = (1 + \sqrt{5})/2$  and  $\omega = e^{i\pi/5}$ :

$$\begin{aligned} \tilde{A}^{(0)} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 1 & -1 \\ \phi & \sqrt{\phi} & \sqrt{\phi} & -\frac{1}{\sqrt{\phi}} & \frac{\phi+1}{\sqrt{\phi}} \end{pmatrix}, & \tilde{A}^{(1)} &= \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{\phi} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{\phi}} & -\frac{1}{\sqrt{\phi}} \\ \phi^{3/2} & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \phi^{3/2} & 0 & 0 & \frac{1}{\sqrt{\phi}} & -\frac{1}{\sqrt{\phi}} \\ \phi^{3/2} & 0 & 1 & 0 & 0 \\ -\phi & \sqrt{\phi} & \sqrt{\phi} & 1 & 0 \end{pmatrix}, \\ \tilde{A}^{(2)} &= \begin{pmatrix} 0 & \frac{1}{\phi} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{\phi}} & -\frac{1}{\sqrt{\phi}} \\ \phi & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \sqrt{\phi} \\ 0 & 0 & 1 & 0 & 0 \\ \phi & 0 & 0 & 0 & \sqrt{\phi} \\ 0 & 0 & 0 & \frac{1}{\sqrt{\phi}} & \frac{\phi-1}{\sqrt{\phi}} \\ \phi^{3/2} & 1 & \sqrt{\phi} & 1 & -2 \end{pmatrix}, & \tilde{A}^{(3)} &= \begin{pmatrix} 0 & 0 & \frac{1}{\phi} & 0 & 0 \\ \phi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & \sqrt{\phi} & 0 & \frac{1}{\sqrt{\phi}} & -\frac{1}{\sqrt{\phi}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{\phi}} & -\frac{1}{\sqrt{\phi}} \\ 0 & \sqrt{\phi} & 0 & 0 & 1 \\ \phi & \sqrt{\phi} & 0 & 0 & 0 \\ \phi^{3/2} & -1 & 1 & 1 & \sqrt{\phi}-1 \end{pmatrix}, \\ \tilde{A}^{(4)} &= \begin{pmatrix} 0 & 0 & 0 & \frac{1}{\phi^{3/2}} & -\frac{1}{\phi^{3/2}} \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ \phi & 0 & \sqrt{\phi} & 0 & 0 \\ \phi & 0 & 0 & 0 & 0 \\ 0 & 1 & \sqrt{\phi} & 0 & 0 \\ 0 & 0 & \sqrt{\phi} & 0 & 1 \\ \phi^{3/2} & \sqrt{\phi} & -1 & \frac{1}{\sqrt{\phi}} & \frac{\phi-1}{\sqrt{\phi}} \end{pmatrix}, & \tilde{A}^{(5)} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \omega^4 & 0 & 0 & 0 \\ 0 & 0 & -\omega & 0 & 0 \\ 0 & 0 & 0 & -\omega^3 & \omega^3 + \omega^2 \\ 0 & 0 & 0 & 0 & \omega^2 \\ 0 & 0 & -\omega & -\omega^3 & \omega^3 \\ 0 & \omega^4 & 0 & -\omega^3 & \omega^3 \\ \phi & \omega^4 \sqrt{\phi} & -\omega \sqrt{\phi} & \frac{\omega^3}{\sqrt{\phi}} & \frac{\omega^2 \phi - \omega^3}{\sqrt{\phi}} \end{pmatrix}, \\ \tilde{A}^{(6)} &= \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\omega^2}{\phi} \\ 0 & 0 & -\omega & 0 & 0 \\ 0 & 0 & 0 & -\frac{\omega^3}{\sqrt{\phi}} & \frac{\omega^3}{\sqrt{\phi}} \\ \phi^{3/2} & \omega^4 & 0 & 0 & 0 \\ 0 & \omega^4 & 0 & 0 & 0 \\ \phi^{3/2} & 0 & 0 & -\frac{\omega^3}{\sqrt{\phi}} & \frac{\omega^3}{\sqrt{\phi}} \\ \phi^{3/2} & 0 & -\omega & 0 & 0 \\ -\phi & \omega^4 \sqrt{\phi} & -\omega \sqrt{\phi} & -\omega^3 & \omega^3 + \omega^2 \end{pmatrix}, & \tilde{A}^{(7)} &= \begin{pmatrix} 0 & \frac{\omega^4}{\phi} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\omega^3}{\sqrt{\phi}} & \frac{\omega^3}{\sqrt{\phi}} \\ \phi & 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega & 0 & \omega^2 \sqrt{\phi} \\ 0 & 0 & -\omega & 0 & 0 \\ \phi & 0 & 0 & 0 & \omega^2 \sqrt{\phi} \\ 0 & 0 & 0 & -\frac{\omega^3}{\sqrt{\phi}} & \frac{\omega^3 + \omega^2 \phi}{\sqrt{\phi}} \\ \phi^{3/2} & \omega^4 & -\omega \sqrt{\phi} & -\omega^3 & \omega^3 - \omega^2 \end{pmatrix}, \end{aligned}$$



$$\begin{aligned}
\tilde{A}^{(20)} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -\omega & 0 & 0 & 0 \\ 0 & 0 & \omega^4 & 0 & 0 \\ 0 & 0 & 0 & \omega^2 & -\omega^3 - \omega^2 \\ 0 & 0 & 0 & 0 & -\omega^3 \\ 0 & 0 & \omega^4 & \omega^2 & -\omega^2 \\ 0 & -\omega & 0 & \omega^2 & -\omega^2 \\ \phi & -\omega\sqrt{\phi} & \omega^4\sqrt{\phi} & -\frac{\omega^2}{\sqrt{\phi}} & \frac{\omega^2 - \omega^3\phi}{\sqrt{\phi}} \end{pmatrix}, & \tilde{A}^{(21)} &= \begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{\omega^3}{\phi} \\ 0 & 0 & \omega^4 & 0 & 0 \\ 0 & 0 & 0 & \frac{\omega^2}{\sqrt{\phi}} & -\frac{\omega^2}{\sqrt{\phi}} \\ \phi^{3/2} & -\omega & 0 & 0 & 0 \\ 0 & -\omega & 0 & 0 & 0 \\ \phi^{3/2} & 0 & 0 & \frac{\omega^2}{\sqrt{\phi}} & -\frac{\omega^2}{\sqrt{\phi}} \\ \phi^{3/2} & 0 & \omega^4 & 0 & 0 \\ -\phi & -\omega\sqrt{\phi} & \omega^4\sqrt{\phi} & \omega^2 & -\omega^3 - \omega^2 \end{pmatrix}, \\
\tilde{A}^{(22)} &= \begin{pmatrix} 0 & -\frac{\omega}{\phi} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\omega^2}{\sqrt{\phi}} & -\frac{\omega^2}{\sqrt{\phi}} \\ \phi & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega^4 & 0 & -\omega^3\sqrt{\phi} \\ 0 & 0 & \omega^4 & 0 & 0 \\ \phi & 0 & 0 & 0 & -\omega^3\sqrt{\phi} \\ 0 & 0 & 0 & \frac{\omega^2}{\sqrt{\phi}} & \frac{-\omega^3\phi - \omega^2}{\sqrt{\phi}} \\ \phi^{3/2} & -\omega & \omega^4\sqrt{\phi} & \omega^2 & \omega^3 - \omega^2 \end{pmatrix}, & \tilde{A}^{(23)} &= \begin{pmatrix} 0 & 0 & \frac{\omega^4}{\phi} & 0 & 0 \\ \phi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\omega^3 \\ 0 & -\omega\sqrt{\phi} & 0 & \frac{\omega^2}{\sqrt{\phi}} & -\frac{\omega^2}{\sqrt{\phi}} \\ 0 & 0 & 0 & \frac{\omega^2}{\sqrt{\phi}} & -\frac{\omega^2}{\sqrt{\phi}} \\ 0 & -\omega\sqrt{\phi} & 0 & 0 & -\omega^3 \\ \phi & -\omega\sqrt{\phi} & 0 & 0 & 0 \\ \phi^{3/2} & \omega & \omega^4 & \omega^2 & -\omega^3\sqrt{\phi} - \omega^2 \end{pmatrix}, \\
\tilde{A}^{(24)} &= \begin{pmatrix} 0 & 0 & 0 & \frac{\omega^2}{\phi^{3/2}} & -\frac{\omega^2}{\phi^{3/2}} \\ 0 & 0 & 0 & 0 & -\omega^3 \\ 0 & -\omega & 0 & 0 & 0 \\ \phi & 0 & \omega^4\sqrt{\phi} & 0 & 0 \\ \phi & 0 & 0 & 0 & 0 \\ 0 & -\omega & \omega^4\sqrt{\phi} & 0 & 0 \\ 0 & 0 & \omega^4\sqrt{\phi} & 0 & -\omega^3 \\ \phi^{3/2} & -\omega\sqrt{\phi} & -\omega^4 & \frac{\omega^2}{\sqrt{\phi}} & \frac{-\omega^3\phi - \omega^2}{\sqrt{\phi}} \end{pmatrix}.
\end{aligned} \tag{2}$$