

Background calculations for the article "Modified use of the Component Method to get more realistic force distribution in joints of steel structures"

Material properties:

Steel:

$$S235 \quad f_u := 360 \frac{\text{N}}{\text{mm}^2} \quad f_y := 235 \frac{\text{N}}{\text{mm}^2} \quad E := 210000 \frac{\text{N}}{\text{mm}^2}$$

$$\varepsilon := \sqrt{\frac{235 \frac{\text{N}}{\text{mm}^2}}{f_y}} = 1$$

Bolt:

$$M 20 \quad 8.8 \quad f_{yb} := 640 \frac{\text{N}}{\text{mm}^2} \quad f_{ub} := 800 \frac{\text{N}}{\text{mm}^2}$$

$$d := 20 \text{ mm} \quad d_o := 22 \text{ mm} \quad d_m := 32,4 \text{ mm} \quad A := 314 \text{ mm}^2 \quad A_s := 245 \text{ mm}^2$$

Factors:

$$\gamma_{m0} := 1 \quad \gamma_{m1} := 1 \quad \gamma_{m2} := 1,25$$

Geometrical datas:

$$\begin{aligned} \text{Beam} \quad h_b &:= 450 \text{ mm} \quad r_b := 21 \text{ mm} \\ \text{IPE 450} \quad b_b &:= 190 \text{ mm} \quad W_{pl} := 1701,79 \text{ cm}^3 \\ t_{wb} &:= 9,4 \text{ mm} \\ t_{fb} &:= 14,6 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Column} \quad h_c &:= 450 \text{ mm} \quad r_c := 21 \text{ mm} \\ \text{IPE 450} \quad b_c &:= 190 \text{ mm} \quad A_c := 98,82 \text{ cm}^2 \\ t_{wc} &:= 9,4 \text{ mm} \\ t_{fc} &:= 14,6 \text{ mm} \end{aligned}$$

$$A_{wp} := h_c \cdot t_{wc} = 4230 \text{ mm}^2$$

$$d_{wc} := h_c - 2 \cdot (t_{fc} + r_c) = 378,8 \text{ mm}$$

Thickness of the end plate:

$$t_p := 20 \text{ mm}$$

Bolt distances:

$$\begin{aligned} h_1 &:= 488 \text{ mm} \\ h_2 &:= 388 \text{ mm} \\ p_{23} &:= 70 \text{ mm} \\ h_3 &:= 318 \text{ mm} \\ h_4 &:= 218 \text{ mm} \end{aligned}$$

Elongation length of bolts:

$$L_b := \frac{12,5 \text{ mm}}{2} + 3 \text{ mm} + t_{fc} + t_p + 3 \text{ mm} + \frac{19 \text{ mm}}{2} = 56,35 \text{ mm}$$

Punching shear test:

$$B_{pRd} := \frac{0,6 \cdot \pi \cdot d_m \cdot t_p \cdot f_u}{\gamma_{m2}} = 351,778 \text{ kN} \quad - \geq - \quad F_{tzRd} := 0,9 \cdot \frac{A_s \cdot f_{ub}}{\gamma_{m2}} = 141,12 \text{ kN}$$

$$B_{pRd} := \frac{0,6 \cdot \pi \cdot d_m \cdot t_{fc} \cdot f_u}{\gamma_{m2}} = 256,7979 \text{ kN} \quad - \geq - \quad F_{tzRd} := 0,9 \cdot \frac{A_s \cdot f_{ub}}{\gamma_{m2}} = 141,12 \text{ kN}$$

Passed! Punching shear failure is not relevant

Defining the effective length values

Side of the column

$$e := 45 \text{ mm}$$

$$m_c := \frac{100 \text{ mm} - t_{wc}}{2} - 0,8 \cdot r_c = 28,5 \text{ mm}$$

1st bolt row

$$e_c := 35 \text{ mm} \quad m_{xc} := 83 \text{ mm} - 35 \text{ mm} - 0,8 \cdot 4 \text{ mm} \cdot \sqrt{2} = 43,4745 \text{ mm}$$

$$\alpha := \min \left(\left[\max \left(\left[\begin{array}{c} 4 + 1,25 \cdot \frac{e}{m_c} \\ 4 + 1,67 \cdot \frac{e}{m_c} \cdot \left(\frac{m_c}{m_{xc}} \right)^{0,67} \\ 8 \end{array} \right] \right) \right] \right] = 5,9871$$

$$L_{1eff1c} := \min \left(\left[\begin{array}{c} 2 \cdot \pi \cdot m_c \\ \pi \cdot m_c + 2 \cdot e_c \\ \alpha \cdot m_c \\ \alpha \cdot m_c - (2 \cdot m_c + 0,625 \cdot e) + e_c \end{array} \right] \right] = 120,506 \text{ mm}$$

$$L_{1eff2c} := \min \left(\left[\begin{array}{c} \alpha \cdot m_c \\ \alpha \cdot m_c - (2 \cdot m_c + 0,625 \cdot e) + e_c \end{array} \right] \right] = 120,506 \text{ mm}$$

2nd bolt row

$$m_2 := 135 \text{ mm} - 80 \text{ mm} - \frac{t_{fb}}{2} - \frac{8 \text{ mm}}{2} - 0,8 \cdot 4 \text{ mm} \cdot \sqrt{2} = 39,1745 \text{ mm}$$

$$\alpha := \min \left(\left[\left[\max \left(\left[\begin{array}{c} 4 + 1,25 \cdot \frac{e}{m_c} \\ 4 + 1,67 \cdot \frac{e}{m_c} \cdot \left(\frac{m_c}{m_2} \right)^{0,67} \\ 8 \end{array} \right] \right] \right] \right] = 6,1307$$

$$L_{2eff1c} := \min \left(\left[\begin{array}{c} 2 \cdot \pi \cdot m_c \\ \alpha \cdot m_c \\ \pi \cdot m_c + p_{23} \\ 0,5 \cdot p_{23} + \alpha \cdot m_c - (2 \cdot m_c + 0,625 \cdot e) \end{array} \right] \right] = 124,599 \text{ mm}$$

$$L_{2eff2c} := \min \left(\left[\begin{array}{c} \alpha \cdot m_c \\ 0,5 \cdot p_{23} + \alpha \cdot m_c - (2 \cdot m_c + 0,625 \cdot e) \end{array} \right] \right] = 124,599 \text{ mm}$$

$$\left[\begin{array}{c} 2 \cdot \pi \cdot m_c \\ \alpha \cdot m_c \\ \pi \cdot m_c + p_{23} \\ 0,5 \cdot p_{23} + \alpha \cdot m_c - (2 \cdot m_c + 0,625 \cdot e) \end{array} \right] = \begin{bmatrix} 179,0708 \\ 174,7243 \\ 159,5354 \\ 124,5993 \end{bmatrix} \text{ mm}$$

3rd bolt row

$$m_2 := 46 \text{ mm} - 0,8 \cdot 4 \text{ mm} \cdot \sqrt{2} = 41,4745 \text{ mm}$$

$$\alpha := \min \left(\left[\left[\max \left(\left[\begin{array}{c} 4 + 1,25 \cdot \frac{e}{m_c} \\ 4 + 1,67 \cdot \frac{e}{m_c} \cdot \left(\frac{m_c}{m_2} \right)^{0,67} \\ 8 \end{array} \right] \right] \right] \right] = 6,0508$$

$$L_{3eff1c} := \min \left(\left[\begin{array}{c} 2 \cdot \pi \cdot m_c \\ \alpha \cdot m_c \\ \pi \cdot m_c + p_{23} \\ 0,5 \cdot p_{23} + \alpha \cdot m_c - (2 \cdot m_c + 0,625 \cdot e) \end{array} \right] \right] = 122,322 \text{ mm}$$

$$L_{3eff2c} := \min \left(\left[\begin{array}{c} \alpha \cdot m_c \\ 0,5 \cdot p_{23} + \alpha \cdot m_c - (2 \cdot m_c + 0,625 \cdot e) \end{array} \right] \right] = 122,322 \text{ mm}$$

$$\left[\begin{array}{c} 2 \cdot \pi \cdot m_c \\ \alpha \cdot m_c \\ \pi \cdot m_c + p_{23} \\ 0,5 \cdot p_{23} + \alpha \cdot m_c - (2 \cdot m_c + 0,625 \cdot e) \end{array} \right] = \begin{bmatrix} 179,0708 \\ 172,4469 \\ 159,5354 \\ 122,3219 \end{bmatrix} \text{ mm}$$

Note: Bolt row 2 and 3 are checked as a group too. Non-circular group failure is the relevant. It defines the Leff value.

4th bolt row

$$m_2 := 46 \text{ mm} - 0,8 \cdot 4 \text{ mm} \cdot \sqrt{2} = 41,4745 \text{ mm}$$

$$\alpha := \min \left(\left[\max \left(\left[\begin{array}{c} 4 + 1,25 \cdot \frac{e}{m_c} \\ 4 + 1,67 \cdot \frac{e}{m_c} \cdot \left(\frac{m_c}{m_2} \right)^{0,67} \\ 8 \end{array} \right] \right) \right] \right] = 6,0508$$

$$L_{4eff1c} := \min \left(\left[\begin{array}{c} 2 \cdot \pi \cdot m_c \\ \alpha \cdot m_c \end{array} \right] \right] = 172,447 \text{ mm}$$

$$L_{4eff2c} := \alpha \cdot m_c = 172,447 \text{ mm}$$

Side of the end plate (beam)

$$e := 45 \text{ mm}$$

$$m_b := \frac{100 \text{ mm} - t_{wb}}{2} - 0,8 \cdot 4 \text{ mm} \cdot \sqrt{2} = 40,7745 \text{ mm}$$

1st bolt row (Outside the top flange)

$$e_x := 35 \text{ mm} \quad b_p := 190 \text{ mm} \quad w := 100 \text{ mm} \quad m_{xb} := 80 \text{ mm} - 35 \text{ mm} - 0,8 \cdot 7 \text{ mm} \cdot \sqrt{2} = 37,0804 \text{ mm}$$

$$L_{1eff1b} := \min \left(\left[\begin{array}{c} 2 \cdot \pi \cdot m_{xb} \\ \pi \cdot m_{xb} + w \\ \pi \cdot m_{xb} + 2 \cdot e \\ 4 \cdot m_{xb} + 1,25 \cdot e_x \\ e + 2 \cdot m_{xb} + 0,625 \cdot e_x \\ 0,5 \cdot b_p \\ 0,5 \cdot w + 2 \cdot m_{xb} + 0,625 \cdot e_x \end{array} \right] \right] = 95 \text{ mm}$$

$$L_{1eff2b} := \min \left(\left[\begin{array}{c} 4 \cdot m_{xb} + 1,25 \cdot e_x \\ e + 2 \cdot m_{xb} + 0,625 \cdot e_x \\ 0,5 \cdot b_p \\ 0,5 \cdot w + 2 \cdot m_{xb} + 0,625 \cdot e_x \end{array} \right] \right] = 95 \text{ mm}$$

2nd bolt row

$$m_2 := 135 \text{ mm} - 80 \text{ mm} - t_{fb} - 0,8 \cdot 7 \text{ mm} \cdot \sqrt{2} = 32,48 \text{ mm}$$

$$\alpha := \min \left(\left[\left[\max \left(\left[\begin{array}{c} 4 + 1,25 \cdot \frac{e}{m_b} \\ 4 + 1,67 \cdot \frac{e}{m_b} \cdot \left(\frac{m_b}{m_2} \right)^{0,67} \\ 8 \end{array} \right] \right] \right] \right] = 6,1464$$

$$L_{2eff1b} := \min \left(\left[\begin{array}{c} 2 \cdot \mathbf{n} \cdot m_b \\ \alpha \cdot m_b \\ \mathbf{n} \cdot m_b + p_{23} \\ 0,5 \cdot p_{23} + \alpha \cdot m_b - (2 \cdot m_b + 0,625 \cdot e) \end{array} \right] \right] = 175,943 \text{ mm}$$

$$L_{2eff2b} := \min \left(\left[\begin{array}{c} \alpha \cdot m_b \\ 0,5 \cdot p_{23} + \alpha \cdot m_b - (2 \cdot m_b + 0,625 \cdot e) \end{array} \right] \right] = 175,943 \text{ mm}$$

$$\left[\begin{array}{c} 2 \cdot \mathbf{n} \cdot m_b \\ \alpha \cdot m_b \\ \mathbf{n} \cdot m_b + p_{23} \\ 0,5 \cdot p_{23} + \alpha \cdot m_b - (2 \cdot m_b + 0,625 \cdot e) \end{array} \right] = \left[\begin{array}{c} 256,1938 \\ 250,6172 \\ 198,0969 \\ 175,9432 \end{array} \right] \text{ mm}$$

3rd bolt row

$$m_2 := 46 \text{ mm} - 0,8 \cdot 4 \text{ mm} \cdot \sqrt{2} = 41,475 \text{ mm}$$

$$\alpha := \min \left(\left[\left[\max \left(\left[\begin{array}{c} 4 + 1,25 \cdot \frac{e}{m_b} \\ 4 + 1,67 \cdot \frac{e}{m_b} \cdot \left(\frac{m_b}{m_2} \right)^{0,67} \\ 8 \end{array} \right] \right] \right] \right] = 5,8222$$

$$L_{3eff1b} := \min \left(\left[\begin{array}{c} 2 \cdot \mathbf{n} \cdot m_b \\ \alpha \cdot m_b \\ \mathbf{n} \cdot m_b + p_{23} \\ 0,5 \cdot p_{23} + \alpha \cdot m_b - (2 \cdot m_b + 0,625 \cdot e) \end{array} \right] \right] = 162,722 \text{ mm}$$

$$L_{3eff2b} := \min \left(\left[\begin{array}{c} \alpha \cdot m_b \\ 0,5 \cdot p_{23} + \alpha \cdot m_b - (2 \cdot m_b + 0,625 \cdot e) \end{array} \right] \right] = 162,722 \text{ mm}$$

$$\left[\begin{array}{c} 2 \cdot \mathbf{n} \cdot m_b \\ \alpha \cdot m_b \\ \mathbf{n} \cdot m_b + p_{23} \\ 0,5 \cdot p_{23} + \alpha \cdot m_b - (2 \cdot m_b + 0,625 \cdot e) \end{array} \right] = \left[\begin{array}{c} 256,1938 \\ 237,3959 \\ 198,0969 \\ 162,7218 \end{array} \right] \text{ mm}$$

Note: Bolt row 2 and 3 are checked as a group too. Non-circular group failure is the relevant. It defines the Leff value.

4th bolt row

$$m_2 := 46 \text{ mm} - 0,8 \cdot 4 \text{ mm} \cdot \sqrt{2} = 41,475 \text{ mm}$$

$$\alpha := \min \left(\left[\max \left(\left[\begin{array}{c} 4 + 1,25 \cdot \frac{e}{m_b} \\ 4 + 1,67 \cdot \frac{e}{m_b} \cdot \left(\frac{m_b}{m_2} \right)^{0,67} \\ 8 \end{array} \right] \right) \right] \right] = 5,8222$$

$$L_{4eff1b} := \min \left(\left[\begin{array}{c} 2 \cdot \pi \cdot m_b \\ \alpha \cdot m_b \end{array} \right] \right] = 237,396 \text{ mm}$$

$$L_{4eff2b} := \alpha \cdot m_b = 237,396 \text{ mm}$$

Defining the tension resistance of each bolt rows. (considering the following components: bolt in tension, end plate in bending, column flange in bending, column web in tension, beam flange/web in tension)

1st bolt row

$$\frac{8,8 \cdot m_c^3 \cdot A_s}{L_{1eff1c} \cdot t_{fc}} = 133,0805 \text{ mm} \quad - > - \quad L_b = 56,35 \text{ mm} \quad - < - \quad \frac{8,8 \cdot m_{xb}^3 \cdot A_s}{L_{1eff1b} \cdot t_p} = 144,6334 \text{ mm}$$

Prying force can be taken into account!

$$M_{P11Rd_c} := \frac{0,25 \cdot L_{1eff1c} \cdot t_{fc}^2 \cdot f_y}{Y_{m0}} = 1,509 \text{ kN m}$$

$$M_{P11Rd_b} := \frac{0,25 \cdot L_{1eff1b} \cdot t_p^2 \cdot f_y}{Y_{m0}} = 2,232 \text{ kN m}$$

$$M_{P12Rd_c} := \frac{0,25 \cdot L_{1eff2c} \cdot t_{fc}^2 \cdot f_y}{Y_{m0}} = 1,509 \text{ kN m}$$

$$M_{P12Rd_b} := \frac{0,25 \cdot L_{1eff2b} \cdot t_p^2 \cdot f_y}{Y_{m0}} = 2,232 \text{ kN m}$$

$$\omega := \frac{1}{\sqrt{1 + 1,3 \cdot \left(\frac{L_{1eff1c} \cdot t_{wc}}{A_{wp}} \right)^2}} = 0,956$$

$$F_{tRdSum} := \frac{0,9 \cdot f_{ub} \cdot A_s}{Y_{m2}} \cdot 2 = 282,24 \text{ kN}$$

$$n_c := \min \left(\left[\begin{array}{c} e \\ 1,25 \cdot m_c \end{array} \right] \right] = 35,625 \text{ mm}$$

$$n_{xb} := \min \left(\left[\begin{array}{c} e_x \\ 1,25 \cdot m_{xb} \end{array} \right] \right] = 35 \text{ mm}$$

Strength components

Column flange under bending

$$\frac{4 \cdot M_{Pl1Rd_c}}{m_c} = 211,8064 \text{ kN}$$

$$\frac{2 \cdot M_{Pl2Rd_c} + n_c \cdot F_{tRdSum}}{m_c + n_c} = 203,8681 \text{ kN}$$

End plate under bending

$$\frac{4 \cdot M_{Pl1Rd_b}}{m_{xb}} = 240,828 \text{ kN}$$

$$\frac{2 \cdot M_{Pl2Rd_b} + n_{xb} \cdot F_{tRdSum}}{m_{xb} + n_{xb}} = 198,9917 \text{ kN}$$

Bolt

$$F_{tRdSum} = 282,24 \text{ kN}$$

Column web under tension

$$\frac{\omega \cdot L_{lefflc} \cdot t_{wc} \cdot f_y}{Y_{m0}} = 254,5957 \text{ kN}$$

Beam flange under tension

$$\frac{L_{lefflb} \cdot t_{fb} \cdot f_y}{Y_{m0}} = 325,945 \text{ kN}$$

Overall strength

$$F_{TRd_1} := \min \left[\begin{array}{l} \frac{4 \cdot M_{Pl1Rd_c}}{m_c} \\ \frac{2 \cdot M_{Pl2Rd_c} + n_c \cdot F_{tRdSum}}{m_c + n_c} \\ \frac{4 \cdot M_{Pl1Rd_b}}{m_{xb}} \\ \frac{2 \cdot M_{Pl2Rd_b} + n_{xb} \cdot F_{tRdSum}}{m_{xb} + n_{xb}} \\ F_{tRdSum} \\ \frac{\omega \cdot L_{lefflc} \cdot t_{wc} \cdot f_y}{Y_{m0}} \\ \frac{L_{lefflb} \cdot t_{fb} \cdot f_y}{Y_{m0}} \end{array} \right] = 198,992 \text{ kN}$$

Stiffness components

$$k_{tfc_1} := \frac{0,9 \cdot L_{lefflc} \cdot t_{fc}^3}{m_c} = 14,581 \text{ mm}$$

$$k_{tfc_1} \cdot E = 3,0619 \cdot 10^6 \frac{\text{kN}}{\text{m}}$$

$$k_{tep_1} := \frac{0,9 \cdot L_{lefflb} \cdot t_p^3}{m_{xb}} = 13,416 \text{ mm}$$

$$k_{tep_1} \cdot E = 2,8174 \cdot 10^6 \frac{\text{kN}}{\text{m}}$$

$$k_{t_1} := 1,6 \cdot \frac{A_s}{L_b} = 6,957 \text{ mm}$$

$$k_{t_1} \cdot E = 1,4609 \cdot 10^6 \frac{\text{kN}}{\text{m}}$$

$$k_{twc_1} := \frac{0,7 \cdot L_{lefflc} \cdot t_{wc}}{d_{wc}} = 2,0933 \text{ mm}$$

$$k_{twc_1} \cdot E = 4,3959 \cdot 10^5 \frac{\text{kN}}{\text{m}}$$

Stiffness of the component is infinite.

Stiffness of the bolt row

$$k_{eff1} := \frac{1}{\frac{1}{k_{tfc_1}} + \frac{1}{k_{tep_1}} + \frac{1}{k_{t_1}} + \frac{1}{k_{twc_1}}} = 1,308 \text{ mm}$$

$$S_1 := k_{eff1} \cdot E = 2,7466 \cdot 10^5 \frac{\text{kN}}{\text{m}}$$

2nd bolt row

$$\frac{8,8 \cdot m_c^3 \cdot A_s}{L_{2eff1c} \cdot t_{fc}} = 128,7091 \text{ mm} \quad _ > _ \quad L_b = 56,35 \text{ mm} \quad _ < _ \quad \frac{8,8 \cdot m_b^3 \cdot A_s}{L_{2eff1b} \cdot t_p} = 103,8372 \text{ mm}$$

Prying force can be
taken into account!

$$M_{P11Rd_c} := \frac{0,25 \cdot L_{2eff1c} \cdot t_{fc}^2 \cdot f_y}{Y_{m0}} = 1,56 \text{ kN m}$$

$$M_{P11Rd_b} := \frac{0,25 \cdot L_{2eff1b} \cdot t_p^2 \cdot f_y}{Y_{m0}} = 4,135 \text{ kN m}$$

$$M_{P12Rd_c} := \frac{0,25 \cdot L_{2eff2c} \cdot t_{fc}^2 \cdot f_y}{Y_{m0}} = 1,56 \text{ kN m}$$

$$M_{P12Rd_b} := \frac{0,25 \cdot L_{2eff2b} \cdot t_p^2 \cdot f_y}{Y_{m0}} = 4,135 \text{ kN m}$$

$$\omega := \frac{1}{\sqrt{1 + 1,3 \cdot \left(\frac{L_{2eff1c} \cdot t_{wc}}{A_{wp}} \right)^2}} = 0,954$$

$$F_{tRdSum} := \frac{0,9 \cdot f_{ub} \cdot A_s}{Y_{m2}} \cdot 2 = 282,24 \text{ kN}$$

$$n_c := \min \left(\left[\begin{array}{c} e \\ 1,25 \cdot m_c \end{array} \right] \right) = 35,625 \text{ mm}$$

$$n_b := \min \left(\left[\begin{array}{c} e \\ 1,25 \cdot m_b \end{array} \right] \right) = 45 \text{ mm}$$

Strength components

Stiffness components

Column flange under bending

$$\frac{4 \cdot M_{P11Rd_c}}{m_c} = 219,0001 \text{ kN}$$

$$k_{tfc_2} := \frac{0,9 \cdot L_{2eff1c} \cdot t_{fc}^3}{m_c} = 15,076 \text{ mm}$$

$$\frac{2 \cdot M_{P12Rd_c} + n_c \cdot F_{tRdSum}}{m_c + n_c} = 205,4667 \text{ kN}$$

$$k_{tfc_2} \cdot E = 3,1659 \cdot 10^6 \frac{\text{kN}}{\text{m}}$$

End plate under bending

$$\frac{4 \cdot M_{P11Rd_b}}{m_b} = 405,6127 \text{ kN}$$

$$k_{tep_2} := \frac{0,9 \cdot L_{2eff1b} \cdot t_p^3}{m_b} = 18,687 \text{ mm}$$

$$\frac{2 \cdot M_{P12Rd_b} + n_b \cdot F_{tRdSum}}{m_b + n_b} = 244,4797 \text{ kN}$$

$$k_{tep_2} \cdot E = 3,9243 \cdot 10^6 \frac{\text{kN}}{\text{m}}$$

Bolt failure

$$F_{tRdSum} = 282,24 \text{ kN}$$

$$k_{t_2} := 1,6 \cdot \frac{A_s}{L_b} = 6,957 \text{ mm}$$

$$k_{t_2} \cdot E = 1,4609 \cdot 10^6 \frac{\text{kN}}{\text{m}}$$

Column web under tension

$$\frac{\omega \cdot L_{2efflc} \cdot t_{wc} \cdot f_y}{Y_{m0}} = 262,4707 \text{ kN}$$

$$k_{twc_2} := \frac{0,7 \cdot L_{2efflc} \cdot t_{wc}}{d_{wc}} = 2,1644 \text{ mm}$$

$$k_{twc_2} \cdot E = 4,5452 \cdot 10^5 \frac{\text{kN}}{\text{m}}$$

Beam flange under tension

$$\frac{L_{2efflb} \cdot t_{wb} \cdot f_y}{Y_{m0}} = 388,6585 \text{ kN}$$

Stiffness of the component is infinite.

Overall strength

Stiffness of the bolt row

$$k_{eff2} := \frac{1}{\frac{1}{k_{tfc_2}} + \frac{1}{k_{tep_2}} + \frac{1}{k_{t_2}} + \frac{1}{k_{twc_2}}} = 1,378 \text{ mm}$$

$$S_2 := k_{eff2} \cdot E = 2,8941 \cdot 10^5 \frac{\text{kN}}{\text{m}}$$

$$F_{TRd_2} := \min \left[\begin{array}{l} \frac{4 \cdot M_{P11Rd_c}}{m_c} \\ \frac{2 \cdot M_{P12Rd_c} + n_c \cdot F_{tRdSum}}{m_c + n_c} \\ \frac{4 \cdot M_{P11Rd_b}}{m_b} \\ \frac{2 \cdot M_{P12Rd_b} + n_b \cdot F_{tRdSum}}{m_b + n_b} \\ F_{tRdSum} \\ \frac{\omega \cdot L_{2efflc} \cdot t_{wc} \cdot f_y}{Y_{m0}} \\ \frac{L_{2efflb} \cdot t_{wb} \cdot f_y}{Y_{m0}} \end{array} \right] = 205,467 \text{ kN}$$

3rd bolt row

$$\frac{8,8 \cdot m_c^3 \cdot A_s}{L_{3efflc} \cdot t_{fc}} = 131,1054 \text{ mm} \quad - > - \quad L_b = 56,35 \text{ mm} \quad - < - \quad \frac{8,8 \cdot m_b^3 \cdot A_s}{L_{3efflb} \cdot t_p} = 112,274 \text{ mm}$$

Prying force can be taken into account!

$$M_{P11Rd_c} := \frac{0,25 \cdot L_{3efflc} \cdot t_{fc}^2 \cdot f_y}{Y_{m0}} = 1,532 \text{ kN m}$$

$$M_{P11Rd_b} := \frac{0,25 \cdot L_{3efflb} \cdot t_p^2 \cdot f_y}{Y_{m0}} = 3,824 \text{ kN m}$$

$$M_{P12Rd_c} := \frac{0,25 \cdot L_{3eff2c} \cdot t_{fc}^2 \cdot f_y}{Y_{m0}} = 1,532 \text{ kN m}$$

$$M_{P12Rd_b} := \frac{0,25 \cdot L_{3eff2b} \cdot t_p^2 \cdot f_y}{Y_{m0}} = 3,824 \text{ kN m}$$

$$\omega := \frac{1}{\sqrt{1 + 1,3 \cdot \left(\frac{L_{3efflc} \cdot t_{wc}}{A_{wp}} \right)^2}} = 0,955$$

$$F_{tRdSum} := \frac{0,9 \cdot f_{ub} \cdot A_s}{Y_{m2}} \cdot 2 = 282,24 \text{ kN}$$

$$n_c := \min \left(\left[\begin{array}{c} e \\ 1,25 \cdot m_c \end{array} \right] \right) = 35,625 \text{ mm}$$

$$n_b := \min \left(\left[\begin{array}{c} e \\ 1,25 \cdot m_b \end{array} \right] \right) = 45 \text{ mm}$$

Strength components

Column flange under bending

$$\frac{4 \cdot M_{Pl1Rd_c}}{m_c} = 214,9973 \text{ kN}$$

$$\frac{2 \cdot M_{Pl2Rd_c} + n_c \cdot F_{tRdSum}}{m_c + n_c} = 204,5772 \text{ kN}$$

End plate under bending

$$\frac{4 \cdot M_{Pl1Rd_b}}{m_b} = 375,1327 \text{ kN}$$

$$\frac{2 \cdot M_{Pl2Rd_b} + n_b \cdot F_{tRdSum}}{m_b + n_b} = 237,2351 \text{ kN}$$

Bolt failure

$$F_{tRdSum} = 282,24 \text{ kN}$$

Column web under tension

$$\frac{\omega \cdot L_{3eff1c} \cdot t_{wc} \cdot f_y}{Y_{m0}} = 258,0973 \text{ kN}$$

Beam web under tension

$$\frac{L_{3eff1b} \cdot t_{wb} \cdot f_y}{Y_{m0}} = 359,4525 \text{ kN}$$

Stiffness components

$$k_{tfc_3} := \frac{0,9 \cdot L_{3eff1c} \cdot t_{fc}^3}{m_c^3} = 14,8 \text{ mm}$$

$$k_{tfc_3} \cdot E = 3,1081 \cdot 10^6 \frac{\text{kN}}{\text{m}}$$

$$k_{tep_3} := \frac{0,9 \cdot L_{3eff1b} \cdot t_p^3}{m_b^3} = 17,283 \text{ mm}$$

$$k_{tep_3} \cdot E = 3,6294 \cdot 10^6 \frac{\text{kN}}{\text{m}}$$

$$k_{t_3} := 1,6 \cdot \frac{A_s}{L_b} = 6,957 \text{ mm}$$

$$k_{t_3} \cdot E = 1,4609 \cdot 10^6 \frac{\text{kN}}{\text{m}}$$

$$k_{twc_3} := \frac{0,7 \cdot L_{3eff1c} \cdot t_{wc}}{d_{wc}} = 2,1248 \text{ mm}$$

$$k_{twc_3} \cdot E = 4,4621 \cdot 10^5 \frac{\text{kN}}{\text{m}}$$

Stiffness of the component is infinite.

Overall strength

$$F_{TRd_3} := \min \left(\left[\begin{array}{c} \frac{4 \cdot M_{Pl1Rd_c}}{m_c} \\ \frac{2 \cdot M_{Pl2Rd_c} + n_c \cdot F_{tRdSum}}{m_c + n_c} \\ \frac{4 \cdot M_{Pl1Rd_b}}{m_b} \\ \frac{2 \cdot M_{Pl2Rd_b} + n_b \cdot F_{tRdSum}}{m_b + n_b} \\ F_{tRdSum} \\ \frac{\omega \cdot L_{3eff1c} \cdot t_{wc} \cdot f_y}{Y_{m0}} \\ \frac{L_{3eff1b} \cdot t_{wb} \cdot f_y}{Y_{m0}} \end{array} \right] \right) = 204,577 \text{ kN}$$

Stiffness of the bolt row

$$k_{eff3} := \frac{1}{\frac{1}{k_{tfc_3}} + \frac{1}{k_{tep_3}} + \frac{1}{k_{t_3}} + \frac{1}{k_{twc_3}}} = 1,352 \text{ mm}$$

$$S_3 := k_{eff3} \cdot E = 2,8386 \cdot 10^5 \frac{\text{kN}}{\text{m}}$$

4th bolt row

$$\frac{8,8 \cdot m_c^3 \cdot A_s}{L_{4eff1c} \cdot t_{fc}} = 92,9971 \text{ mm} \quad - > - \quad L_b = 56,35 \text{ mm} \quad - < - \quad \frac{8,8 \cdot m_b^3 \cdot A_s}{L_{4eff1b} \cdot t_p} = 76,9577 \text{ mm}$$

Prying force can be taken into account!

$$M_{Pl1Rd_c} := \frac{0,25 \cdot L_{4eff1c} \cdot t_{fc}^2 \cdot f_y}{Y_{m0}} = 2,16 \text{ kN m}$$

$$M_{Pl1Rd_b} := \frac{0,25 \cdot L_{4eff1b} \cdot t_p^2 \cdot f_y}{Y_{m0}} = 5,579 \text{ kN m}$$

$$M_{Pl2Rd_c} := \frac{0,25 \cdot L_{4eff2c} \cdot t_{fc}^2 \cdot f_y}{Y_{m0}} = 2,16 \text{ kN m}$$

$$M_{Pl2Rd_b} := \frac{0,25 \cdot L_{4eff2b} \cdot t_p^2 \cdot f_y}{Y_{m0}} = 5,579 \text{ kN m}$$

$$\omega := \frac{1}{\sqrt{1 + 1,3 \cdot \left(\frac{L_{4eff1c} \cdot t_{wc}}{A_{wp}} \right)^2}} = 0,916$$

$$F_{tRdSum} := \frac{0,9 \cdot f_{ub} \cdot A_s}{Y_{m2}} \cdot 2 = 282,24 \text{ kN}$$

$$n_c := \min \left(\left[\begin{array}{c} e \\ 1,25 \cdot m_c \end{array} \right] \right) = 35,625 \text{ mm}$$

$$n_b := \min \left(\left[\begin{array}{c} e \\ 1,25 \cdot m_b \end{array} \right] \right) = 45 \text{ mm}$$

Strength components

Column flange under bending

$$\frac{4 \cdot M_{Pl1Rd_c}}{m_c} = 303,0987 \text{ kN}$$

$$\frac{2 \cdot M_{Pl2Rd_c} + n_c \cdot F_{tRdSum}}{m_c + n_c} = 224,1553 \text{ kN}$$

End plate under bending

$$\frac{4 \cdot M_{Pl1Rd_b}}{m_b} = 547,2833 \text{ kN}$$

$$\frac{2 \cdot M_{Pl2Rd_b} + n_b \cdot F_{tRdSum}}{m_b + n_b} = 278,1526 \text{ kN}$$

Bolt

$$F_{tRdSum} = 282,24 \text{ kN}$$

Column web under tension

$$\frac{\omega \cdot L_{4eff1c} \cdot t_{wc} \cdot f_y}{Y_{m0}} = 349,0693 \text{ kN}$$

Beam flange under tension

$$\frac{L_{4eff1b} \cdot t_{wb} \cdot f_y}{Y_{m0}} = 524,4075 \text{ kN}$$

Overall strength

$$F_{TRd_4} := \min \left[\begin{array}{l} \frac{4 \cdot M_{Pl1Rd_c}}{m_c} \\ \frac{2 \cdot M_{Pl2Rd_c} + n_c \cdot F_{tRdSum}}{m_c + n_c} \\ \frac{4 \cdot M_{Pl1Rd_b}}{m_b} \\ \frac{2 \cdot M_{Pl2Rd_b} + n_b \cdot F_{tRdSum}}{m_b + n_b} \\ F_{tRdSum} \\ \frac{\omega \cdot L_{4eff1c} \cdot t_{wc} \cdot f_y}{Y_{m0}} \\ \frac{L_{4eff1b} \cdot t_{wb} \cdot f_y}{Y_{m0}} \end{array} \right] = 224,155 \text{ kN}$$

Stiffness components

$$k_{tfc_4} := \frac{0,9 \cdot L_{4eff1c} \cdot t_{fc}^3}{m_c^3} = 20,865 \text{ mm}$$

$$k_{tfc_4} \cdot E = 4,3817 \cdot 10^6 \frac{\text{kN}}{\text{m}}$$

$$k_{tep_4} := \frac{0,9 \cdot L_{4eff1b} \cdot t_p^3}{m_b^3} = 25,214 \text{ mm}$$

$$k_{tep_4} \cdot E = 5,2949 \cdot 10^6 \frac{\text{kN}}{\text{m}}$$

$$k_{t_4} := 1,6 \cdot \frac{A_s}{L_b} = 6,957 \text{ mm}$$

$$k_{t_4} \cdot E = 1,4609 \cdot 10^6 \frac{\text{kN}}{\text{m}}$$

$$k_{twc_4} := \frac{0,7 \cdot L_{4eff1c} \cdot t_{wc}}{d_{wc}} = 2,9955 \text{ mm}$$

$$k_{twc_4} \cdot E = 6,2906 \cdot 10^5 \frac{\text{kN}}{\text{m}}$$

Stiffness of the component is infinite.

Stiffness of the bolt row

$$k_{eff4} := \frac{1}{\frac{1}{k_{tfc_4}} + \frac{1}{k_{tep_4}} + \frac{1}{k_{t_4}} + \frac{1}{k_{twc_4}}} = 1,769 \text{ mm}$$

$$S_4 := k_{eff4} \cdot E = 3,7157 \cdot 10^5 \frac{\text{kN}}{\text{m}}$$

Equivalent arm of forces

$$z_{eq} := \frac{k_{eff1} \cdot h_1^2 + k_{eff2} \cdot h_2^2 + k_{eff3} \cdot h_3^2 + k_{eff4} \cdot h_4^2}{k_{eff1} \cdot h_1 + k_{eff2} \cdot h_2 + k_{eff3} \cdot h_3 + k_{eff4} \cdot h_4} = 371,99 \text{ mm}$$

Common stiffness of the tension part

$$k_{eq} := \frac{k_{eff1} \cdot h_1 + k_{eff2} \cdot h_2 + k_{eff3} \cdot h_3 + k_{eff4} \cdot h_4}{z_{eq}} = 5,346 \text{ mm}$$

$$S_{eq} := k_{eq} \cdot E = 1,1226 \cdot 10^6 \frac{\text{kN}}{\text{m}}$$

Shear resistance of the column web

$$\eta := 1,2$$

$$\frac{h_c - 2 \cdot t_{fc}}{t_{wc}} = 44,766 \quad - \leq - \quad 72 \cdot \frac{\sqrt{\frac{235 \frac{\text{N}}{\text{mm}^2}}{f_y}}}{\eta} = 60$$

$$V_{wpRd} := \frac{0,9 \cdot f_y \cdot A_{wp}}{\sqrt{3} \cdot \gamma_{m0}} = 516,524 \text{ kN}$$

Additional resistance generated by the bending failure of the flanges and the stiffeners.

$$t_{st} := 8 \text{ mm} \quad b_{st} := 180 \text{ mm}$$

$$d_s := 435 \text{ mm} \quad h_r := 388 \text{ mm}$$

$$M_{pl_fc_Rd} := \frac{b_c \cdot t_{fc}^2 \cdot f_y}{4 \cdot \gamma_{m0}} = 2,379 \text{ kN m} \quad M_{pl_st_Rd} := \frac{b_{st} \cdot t_{st}^2 \cdot f_y}{4 \cdot \gamma_{m0}} = 0,677 \text{ kN m}$$

$$V_{wpaddRd} := \frac{4 \cdot M_{pl_fc_Rd}}{h_r} + \frac{2 \cdot M_{pl_st_Rd}}{h_r} = 28,019 \text{ kN}$$

$$V_{wpRd} := \frac{0,9 \cdot f_y \cdot A_{wp}}{\sqrt{3} \cdot \gamma_{m0}} + V_{wpaddRd} = 544,5421 \text{ kN}$$

Stiffness component

$$\beta := 1 \quad z_{eq} = 371,9896 \text{ mm}$$

$$k_{wp} := \frac{0,38 \cdot A_{wp}}{\beta \cdot z_{eq}} = 4,3211 \text{ mm}$$

$$S_{shear} := k_{wp} \cdot E = 9,0743 \cdot 10^5 \frac{\text{kN}}{\text{m}}$$

Compression resistance of the column web (with stiffener)

$$h_{st} := 90 \text{ mm} \quad t_{st} := 8 \text{ mm} \quad s := r_c = 21 \text{ mm} \quad s_{p1} := 20 \text{ mm} \quad s_{p2} := 0 \text{ mm}$$

$$a_p := 7 \text{ mm} \quad s_p := s_{p1} + s_{p2} = 20 \text{ mm}$$

$$b_{eff_c_wc} := t_{fb} + 2 \cdot \sqrt{2} \cdot a_p + 5 \cdot (t_{fc} + s) + s_p = 232,399 \text{ mm}$$

$$A_{eff} := b_{eff_c_wc} \cdot t_{wc} + h_{st} \cdot t_{st} \cdot 2 = 36,246 \text{ cm}^2$$

$$\sigma_{comEd} := 0 \frac{\text{N}}{\text{mm}^2}$$

$$\omega := \frac{1}{\sqrt{1 + 1,3 \cdot \left(\frac{b_{eff_c_wc} \cdot t_{wc}}{A_{wp}} \right)^2}} = 0,862$$

$$k_{wc} := \text{if } \sigma_{comEd} > 0,7 \cdot f_y = 1$$

$$1,7 - \frac{\sigma_{comEd}}{f_y}$$

else

$$1$$

Buckling check of the stiffener:

$$I := \frac{b_{eff_c_wc} \cdot t_{wc}^3}{12} + 2 \cdot \left(\frac{t_{st} \cdot h_{st}^3}{12} + h_{st} \cdot t_{st} \cdot \left(\frac{t_{wc}}{2} + \frac{h_{st}}{2} \right)^2 \right) = 454,502 \text{ cm}^4$$

$$i := \sqrt{\frac{I}{A_{eff}}} = 35,411 \text{ mm}$$

$$\lambda := \frac{0,75 \cdot d_{wc}}{i} = 8,023 \quad \lambda_1 := \pi \cdot \sqrt{\frac{E}{f_y}} = 93,913$$

$$\lambda_{rel} := \frac{\lambda}{\lambda_1} = 0,085$$

$$\alpha := 0,49$$

$$\varphi := \frac{1 + \alpha \cdot (\lambda_{rel} - 0,2) - \lambda_{rel}^2}{2} = 0,468$$

$$\chi := \min \left(\left[\frac{1}{\varphi + \sqrt{\varphi^2 - \lambda_{rel}^2}} \right], 1 \right) = 1$$

$$F_{cwcRd} := \min \left(\left[\frac{\omega \cdot k_{wc} \cdot A_{eff} \cdot f_y}{Y_{m0}}, \frac{\omega \cdot k_{wc} \cdot \chi \cdot A_{eff} \cdot f_y}{Y_{m1}} \right] \right) = 733,977 \text{ kN}$$

Stiffness of the component is infinite because of the stiffener applied.

Compression resistance of the flange of the beam

$$M_{cRd} := \frac{W_{pl} \cdot f_y}{\gamma_{m0}} = 399,921 \text{ kN m}$$

$$F_{c_fc_Rd} := \frac{M_{cRd}}{h_b - t_{fb}} = 918,513 \text{ kN}$$

Stiffness of the component is infinite.

Resistance of relevant components:

tension part overall tension resistance

compression resistance of the web of the column

compression resistance of the flange of the beam

shear resistance of the web of the column

$$\begin{bmatrix} F_{TRd_1} + F_{TRd_2} + F_{TRd_3} + F_{TRd_4} \\ F_{cwcRd} \\ F_{c_fc_Rd} \\ V_{wpRd} \end{bmatrix} = \begin{bmatrix} 833,1908 \\ 733,9768 \\ 918,5132 \\ 544,5421 \end{bmatrix} \text{ kN}$$

Defining the actual tension forces in the bolt rows at the failure state.

$$\Delta\alpha := 0,00005 \text{ deg}$$

$$\begin{aligned} & \alpha := 0 \text{ deg} \\ & m := 0 \\ & F_{T_1_mod} := 0 \text{ kN} \\ & F_{T_2_mod} := 0 \text{ kN} \\ & F_{T_3_mod} := 0 \text{ kN} \\ & F_{T_4_mod} := 0 \text{ kN} \\ & \text{while } F_{T_1_mod} + F_{T_2_mod} + F_{T_3_mod} + F_{T_4_mod} < \min \begin{bmatrix} F_{TRd_1} + F_{TRd_2} + F_{TRd_3} + F_{TRd_4} \\ F_{cwcRd} \\ F_{c_fc_Rd} \\ V_{wpRd} \end{bmatrix} \\ & \quad \begin{aligned} & m := m + 1 \\ & \alpha := \alpha + \Delta\alpha \\ & F_{T_1_mod} := \min \begin{bmatrix} k_{eff1} \cdot E \cdot h_1 \cdot \tan(\alpha) \\ F_{TRd_1} \end{bmatrix} \\ & F_{T_2_mod} := \min \begin{bmatrix} k_{eff2} \cdot E \cdot h_2 \cdot \tan(\alpha) \\ F_{TRd_2} \end{bmatrix} \\ & F_{T_3_mod} := \min \begin{bmatrix} k_{eff3} \cdot E \cdot h_3 \cdot \tan(\alpha) \\ F_{TRd_3} \end{bmatrix} \\ & F_{T_4_mod} := \min \begin{bmatrix} k_{eff4} \cdot E \cdot h_4 \cdot \tan(\alpha) \\ F_{TRd_4} \end{bmatrix} \\ & z_{eqm} := \frac{F_{T_1_mod} \cdot h_1 + F_{T_2_mod} \cdot h_2 + F_{T_3_mod} \cdot h_3 + F_{T_4_mod} \cdot h_4}{F_{T_1_mod} + F_{T_2_mod} + F_{T_3_mod} + F_{T_4_mod}} \end{aligned} \end{aligned}$$

$$m = 1495$$

$$\alpha = 0,0748 \text{ deg}$$

$$\alpha = 0,0013 \text{ rad}$$

$$z_{eqm} = 371,9896 \text{ mm}$$

Original tension
resistances:

Modified tension
values:

$$F_{TRd_1} = 198,9917 \text{ kN} \quad F_{TRd_1_mod} := \min \left(\left[\begin{array}{c} k_{eff1} \cdot E \cdot h_1 \cdot \tan(\alpha - \Delta\alpha) \\ F_{TRd_1} \end{array} \right] \right) = 174,7461 \text{ kN}$$

$$F_{TRd_2} = 205,4667 \text{ kN} \quad F_{TRd_2_mod} := \min \left(\left[\begin{array}{c} k_{eff2} \cdot E \cdot h_2 \cdot \tan(\alpha - \Delta\alpha) \\ F_{TRd_2} \end{array} \right] \right) = 146,3991 \text{ kN}$$

$$F_{TRd_3} = 204,5772 \text{ kN} \quad F_{TRd_3_mod} := \min \left(\left[\begin{array}{c} k_{eff3} \cdot E \cdot h_3 \cdot \tan(\alpha - \Delta\alpha) \\ F_{TRd_3} \end{array} \right] \right) = 117,6863 \text{ kN}$$

$$F_{TRd_4} = 224,1553 \text{ kN} \quad F_{TRd_4_mod} := \min \left(\left[\begin{array}{c} k_{eff4} \cdot E \cdot h_4 \cdot \tan(\alpha - \Delta\alpha) \\ F_{TRd_4} \end{array} \right] \right) = 105,6075 \text{ kN}$$

$$F_{TRd_1_mod} + F_{TRd_2_mod} + F_{TRd_3_mod} + F_{TRd_4_mod} = 544,439 \text{ kN}$$

$$M_{JRd} := F_{TRd_1_mod} \cdot h_1 + F_{TRd_2_mod} \cdot h_2 + F_{TRd_3_mod} \cdot h_3 + F_{TRd_4_mod} \cdot h_4 = 202,5256 \text{ kN m}$$

Initial stiffness of the joint.

$$S_{jini} := \frac{E \cdot z_{eqm}^2}{\frac{1}{k_{wp}} + \frac{1}{k_{eq}}} = 69437,601 \frac{\text{kN m}}{\text{rad}}$$