Supplementary Materials

Survival Analysis of Type-II Lehmann Fréchet Parameters via Progressive Type-II Censoring with Applications

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File S1: Fisher Elements of MLEs

From log-LF in (8), the Fisher's elements of $\mathbf{I}^{-1}(\hat{\underline{\theta}})$ in (29) are obtained as

$$\mathcal{L}_{\delta\delta} = -\frac{m}{\delta^2},$$

$$\mathcal{L}_{\alpha\alpha} = -\frac{m}{\alpha^2} - \sum_{i=1}^{m} \psi''_{\alpha}(x_i) + \sum_{i=1}^{m} \left(\delta\left(R_i+1\right)-1\right)\exp\left(-\psi\left(x_i\right)\right)\left(1-\exp\left(-\psi\left(x_i\right)\right)\right)^{-1} \times \left\{\psi''_{\alpha}(x_i) - \left(\psi'_{\alpha}(x_i)\right)^2 - \left(\psi'_{\alpha}(x_i)\right)^2 \exp\left(-\psi\left(x_i\right)\right)\left(1-\exp\left(-\psi\left(x_i\right)\right)\right)^{-1}\right\},$$

$$\mathcal{L}_{\mu\mu} = -\frac{m\alpha}{\mu^2} - \sum_{i=1}^{m} \psi''_{\mu}(x_i) + \sum_{i=1}^{m} \left(\delta\left(R_i+1\right)-1\right)\exp\left(-\psi\left(x_i\right)\right)\left(1-\exp\left(-\psi\left(x_i\right)\right)\right)^{-1} \times \left\{\psi''_{\mu}(x_i) - \left(\psi'_{\mu}(x_i)\right)^2 - \left(\psi'_{\mu}(x_i)\right)^2 \exp\left(-\psi\left(x_i\right)\right)\left(1-\exp\left(-\psi\left(x_i\right)\right)\right)^{-1}\right\},$$

$$\mathcal{L}_{\delta\alpha} = \sum_{i=1}^{m} \left(R_i+1\right)\psi'_{\alpha}(x_i)\exp\left(-\psi\left(x_i\right)\right)\left(1-\exp\left(-\psi\left(x_i\right)\right)\right)^{-1},$$

$$\mathcal{L}_{\delta\mu} = \sum_{i=1}^{m} \left(R_i+1\right)\psi'_{\mu}(x_i)\exp\left(-\psi\left(x_i\right)\right)\left(1-\exp\left(-\psi\left(x_i\right)\right)\right)^{-1},$$

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and

$$\mathcal{L}_{\alpha\mu} = \frac{m}{\mu} - \sum\nolimits_{i=1}^{m} \psi''_{\alpha\mu}(x_i) + \sum\nolimits_{i=1}^{m} (\delta(R_i+1)-1)\exp(-\psi(x_i))(1-\exp(-\psi(x_i)))^{-1}$$
$$\times \left\{\psi''_{\alpha\mu}(x_i) - \psi'_{\alpha}(x_i)\psi'_{\mu}(x_i) - \psi'_{\alpha}(x_i)\psi'_{\mu}(x_i)\exp(-\psi(x_i))(1-\exp(-\psi(x_i)))^{-1}\right\},$$

where $\psi''_{\theta}(\cdot)$ is the second-partial derivative with respect to the parameter θ such as
$\psi''_{\alpha}(x_i) = \psi(x_i;\alpha,\mu)(\log(\mu/x_i))^2,\ i=1,2,...,m,,$
$\psi''_{\mu}(x_i) = \psi(x_i;\alpha,\mu)(\alpha(\alpha-1)/\mu^2),\ i=1,2,...,m,$
and
$\psi''_{\alpha\mu}(x_i) = \psi(x_i;\alpha,\mu)[\mu^{-1}+(\alpha/\mu)\log(\mu/x_i)],\ i=1,2,...,m.$

File S2: Fisher Elements of MPSEs

From log-PS in (11), the Fisher's elements of $\mathbf{I}^{-1}(\widehat{\underline{\theta}})$ in (30) are obtained as

$$\zeta''_{\delta}(\underline{\theta}) = 0,$$

$$\zeta''_{\alpha}(\underline{\theta}) = \sum\nolimits_{i=1}^{m} \delta R_i \exp(-\psi(x_i;\alpha,\mu))(1-\exp(-\psi(x_i;\alpha,\mu)))^{-2}\{(1-\exp(-\psi(x_i;\alpha,\mu)))$$
$$\times\left[\psi''_{\alpha}(x_i;\alpha,\mu) - (\psi'_{\alpha}(x_i;\alpha,\mu))^2\right] - (\psi'_{\alpha}(x_i;\alpha,\mu))^2\exp(-\psi(x_i;\alpha,\mu))\Big\},$$

$$\zeta''_{\mu}(\underline{\theta}) = \sum\nolimits_{i=1}^{m} \delta R_i \exp(-\psi(x_i;\alpha,\mu))(1-\exp(-\psi(x_i;\alpha,\mu)))^{-2}\{(1-\exp(-\psi(x_i;\alpha,\mu)))$$
$$\times\left[\psi''_{\mu}(x_i;\alpha,\mu) - \left(\psi'_{\mu}(x_i;\alpha,\mu)\right)^2\right] - \left(\psi'_{\mu}(x_i;\alpha,\mu)\right)^2\exp(-\psi(x_i;\alpha,\mu))\Big\},$$

$$\xi''_{\delta}(\underline{\theta}) = \sum\nolimits_{i=1}^{m+1}\left\{\left[\bar{F}(x_{i-1})-\bar{F}(x_i)\right]\left[\bar{F}''_{\delta}(x_{i-1})-\bar{F}''_{\delta}(x_i)\right] - \left[\bar{F}'_{\delta}(x_{i-1})-\bar{F}'_{\delta}(x_i)\right]^2\right\}$$
$$\times\left[\bar{F}(x_{i-1})-\bar{F}(x_i)\right]^{-2},$$

$$\xi''_{\alpha}(\underline{\theta}) = \sum\nolimits_{i=1}^{m+1}\left\{\left[\bar{F}(x_{i-1})-\bar{F}(x_i)\right]\left[\bar{F}''_{\alpha}(x_{i-1})-\bar{F}''_{\alpha}(x_i)\right] - \left[\bar{F}'_{\alpha}(x_{i-1})-\bar{F}'_{\alpha}(x_i)\right]^2\right\}$$
$$\times\left[\bar{F}(x_{i-1})-\bar{F}(x_i)\right]^{-2},$$

$$\xi''_{\mu}(\underline{\theta}) = \sum\nolimits_{i=1}^{m+1}\left\{\left[\bar{F}(x_{i-1})-\bar{F}(x_i)\right]\left[\bar{F}''_{\mu}(x_{i-1})-\bar{F}''_{\mu}(x_i)\right] - \left[\bar{F}'_{\mu}(x_{i-1})-\bar{F}'_{\mu}(x_i)\right]^2\right\}$$
$$\times\left[\bar{F}(x_{i-1})-\bar{F}(x_i)\right]^{-2},$$

$$\xi''_{\delta\alpha}(\underline{\theta}) = \sum\nolimits_{i=1}^{m+1}\left[\bar{F}(x_{i-1})-\bar{F}(x_i)\right]^{-2}\left\{\left[\bar{F}(x_{i-1})-\bar{F}(x_i)\right]\left[\bar{F}''_{\delta\alpha}(x_{i-1})-\bar{F}''_{\delta\alpha}(x_i)\right]\right.$$
$$\left.-\left[\bar{F}'_{\delta}(x_{i-1})-\bar{F}'_{\delta}(x_i)\right]\left[\bar{F}'_{\alpha}(x_{i-1})-\bar{F}'_{\alpha}(x_i)\right]\right\},$$

$$\xi''_{\delta\mu}(\underline{\theta}) = \sum_{i=1}^{m+1} \left[\bar{F}(x_{i-1}) - \bar{F}(x_i)\right]^{-2} \left\{ \left[\bar{F}(x_{i-1}) - \bar{F}(x_i)\right] \left[\bar{F}''_{\delta\mu}(x_{i-1}) - \bar{F}''_{\delta\mu}(x_i)\right] \right.$$
$$\left. - \left[\bar{F}'_{\delta}(x_{i-1}) - \bar{F}'_{\delta}(x_i)\right] \left[\bar{F}'_{\mu}(x_{i-1}) - \bar{F}'_{\mu}(x_i)\right] \right\},$$

$$\xi''_{\alpha\mu}(\underline{\theta}) = \sum_{i=1}^{m+1} \left[\bar{F}(x_{i-1}) - \bar{F}(x_i)\right]^{-2} \left\{ \left[\bar{F}(x_{i-1}) - \bar{F}(x_i)\right] \left[\bar{F}''_{\alpha\mu}(x_{i-1}) - \bar{F}''_{\alpha\mu}(x_i)\right] \right.$$
$$\left. - \left[\bar{F}'_{\alpha}(x_{i-1}) - \bar{F}'_{\alpha}(x_i)\right] \left[\bar{F}'_{\mu}(x_{i-1}) - \bar{F}'_{\mu}(x_i)\right] \right\},$$

$$\zeta''_{\delta\alpha}(\underline{\theta}) = \sum_{i=1}^{m} R_i \psi'_{\alpha}(x_i;\alpha,\mu) \exp(-\psi(x_i;\alpha,\mu)) \left(1 - \exp(-\psi(x_i;\alpha,\mu))\right)^{-1},$$

$$\zeta''_{\delta\mu}(\underline{\theta}) = \sum_{i=1}^{m} R_i \psi'_{\mu}(x_i;\alpha,\mu) \exp(-\psi(x_i;\alpha,\mu)) \left(1 - \exp(-\psi(x_i;\alpha,\mu))\right)^{-1},$$

and

$$\zeta''_{\alpha\mu}(\underline{\theta}) = \sum_{i=1}^{m} \delta R_i \exp(-\psi(x_i;\alpha,\mu)) \left(1 - \exp(-\psi(x_i;\alpha,\mu))\right)^{-2}$$
$$\times \left\{ \left(1 - \exp(-\psi(x_i;\alpha,\mu))\right) \left[\psi''_{\alpha\mu}(x_i;\alpha,\mu) - \psi'_{\alpha}(x_i;\alpha,\mu)\, \psi'_{\mu}(x_i;\alpha,\mu)\right] \right.$$
$$\left. -\psi'_{\alpha}(x_i;\alpha,\mu)\, \psi'_{\mu}(x_i;\alpha,\mu) \exp(-\psi(x_i;\alpha,\mu)) \right\},$$

where

$$\bar{F}''_{\delta}(x_{i-1}) = \left(1 - \exp(-\psi(x_{i-1};\alpha,\mu))\right)^{\delta} \left(\log\left(1 - \exp(-\psi(x_{i-1};\alpha,\mu))\right)\right)^2,$$

$$\bar{F}''_{\delta}(x_i) = \left(1 - \exp(-\psi(x_i;\alpha,\mu))\right)^{\delta} \left(\log\left(1 - \exp(-\psi(x_i;\alpha,\mu))\right)\right)^2,$$

$$\bar{F}''_{\alpha}(x_{i-1}) = \delta \exp(-\psi(x_{i-1};\alpha,\mu)) \left(1 - \exp(-\psi(x_{i-1};\alpha,\mu))\right)^{\delta-1}$$
$$\times \left\{ (\delta - 1) \left(\psi'_{\alpha}(x_{i-1};\alpha,\mu)\right)^2 \exp(-\psi(x_{i-1};\alpha,\mu)) \left(1 - \exp(-\psi(x_{i-1};\alpha,\mu))\right)^{-1} \right.$$
$$\left. + \left[\psi''_{\alpha}(x_{i-1};\alpha,\mu) - \left(\psi'_{\alpha}(x_{i-1};\alpha,\mu)\right)^2\right] \right\},$$

$$\bar{F}''_{\alpha}(x_i) = \delta \exp(-\psi(x_i;\alpha,\mu)) \left(1 - \exp(-\psi(x_i;\alpha,\mu))\right)^{\delta-1}$$
$$\times \left\{ (\delta - 1) \left(\psi'_{\alpha}(x_i;\alpha,\mu)\right)^2 \exp(-\psi(x_i;\alpha,\mu)) \left(1 - \exp(-\psi(x_i;\alpha,\mu))\right)^{-1} \right.$$
$$\left. + \left[\psi''_{\alpha}(x_i;\alpha,\mu) - \left(\psi'_{\alpha}(x_i;\alpha,\mu)\right)^2\right] \right\},$$

$$\bar{F}''_{\mu}(x_{i-1}) = \delta \exp(-\psi(x_{i-1};\alpha,\mu)) \left(1 - \exp(-\psi(x_{i-1};\alpha,\mu))\right)^{\delta-1}$$
$$\times \left\{ (\delta - 1) \left(\psi'_{\mu}(x_{i-1};\alpha,\mu)\right)^2 \exp(-\psi(x_{i-1};\alpha,\mu)) \left(1 - \exp(-\psi(x_{i-1};\alpha,\mu))\right)^{-1} \right.$$
$$\left. + \left[\psi''_{\mu}(x_{i-1};\alpha,\mu) - \left(\psi'_{\mu}(x_{i-1};\alpha,\mu)\right)^2\right] \right\},$$

$$\bar{F}''_{\mu}(x_i) = \delta \exp(-\psi(x_i;\alpha,\mu))(1-\exp(-\psi(x_i;\alpha,\mu)))^{\delta-1}$$
$$\times \left\{(\delta-1)\left(\psi'_{\mu}(x_i;\alpha,\mu)\right)^2 \exp(-\psi(x_i;\alpha,\mu))(1-\exp(-\psi(x_i;\alpha,\mu)))^{-1}\right.$$
$$\left. + \left[\psi''_{\mu}(x_i;\alpha,\mu) - \left(\psi'_{\mu}(x_i;\alpha,\mu)\right)^2\right]\right\}.$$

$$\bar{F}''_{\delta\alpha}(x_{i-1}) = \psi'_{\alpha}(x_{i-1};\alpha,\mu)\exp(-\psi(x_{i-1};\alpha,\mu))(1-\exp(-\psi(x_{i-1};\alpha,\mu)))^{\delta-1}$$
$$\times \{1+\delta\log(1-\exp(-\psi(x_{i-1};\alpha,\mu)))\},$$

$$\bar{F}''_{\delta\alpha}(x_i) = \psi'_{\alpha}(x_i;\alpha,\mu)\exp(-\psi(x_i;\alpha,\mu))(1-\exp(-\psi(x_i;\alpha,\mu)))^{\delta-1}$$
$$\times \{1+\delta\log(1-\exp(-\psi(x_i;\alpha,\mu)))\},$$

$$\bar{F}''_{\delta\mu}(x_{i-1}) = \psi'_{\mu}(x_{i-1};\alpha,\mu)\exp(-\psi(x_{i-1};\alpha,\mu))(1-\exp(-\psi(x_{i-1};\alpha,\mu)))^{\delta-1}$$
$$\times \{1+\delta\log(1-\exp(-\psi(x_{i-1};\alpha,\mu)))\},$$

$$\bar{F}''_{\delta\mu}(x_i) = \psi'_{\mu}(x_i;\alpha,\mu)\exp(-\psi(x_i;\alpha,\mu))(1-\exp(-\psi(x_i;\alpha,\mu)))^{\delta-1}$$
$$\times \{1+\delta\log(1-\exp(-\psi(x_i;\alpha,\mu)))\},$$

$$\bar{F}''_{\alpha\mu}(x_{i-1}) = \delta\exp(-\psi(x_{i-1};\alpha,\mu))(1-\exp(-\psi(x_{i-1};\alpha,\mu)))^{\delta-1}$$
$$\times \left\{(\delta-1)\psi'_{\alpha}(x_{i-1};\alpha,\mu)\psi'_{\mu}(x_{i-1};\alpha,\mu)\exp(-\psi(x_{i-1};\alpha,\mu))(1-\exp(-\psi(x_{i-1};\alpha,\mu)))^{-1}\right.$$
$$\left. + \left[\psi''_{\alpha\mu}(x_{i-1};\alpha,\mu) - \psi'_{\alpha}(x_{i-1};\alpha,\mu)\psi'_{\mu}(x_{i-1};\alpha,\mu)\right]\right\},$$

and

$$\bar{F}''_{\alpha\mu}(x_i) = \delta\exp(-\psi(x_i;\alpha,\mu))(1-\exp(-\psi(x_i;\alpha,\mu)))^{\delta-1}$$
$$\times \left\{(\delta-1)\psi'_{\alpha}(x_i;\alpha,\mu)\psi'_{\mu}(x_i;\alpha,\mu)\exp(-\psi(x_i;\alpha,\mu))(1-\exp(-\psi(x_i;\alpha,\mu)))^{-1}\right.$$
$$\left. + \left[\psi''_{\alpha\mu}(x_i;\alpha,\mu) - \psi'_{\alpha}(x_i;\alpha,\mu)\psi'_{\mu}(x_i;\alpha,\mu)\right]\right\}.$$

Table S1: The AEs (1st column), RMSEs (2nd column) and MRABs (3rd column) of δ.

(n,m)	Scheme	MLE	MPSE	SEL				GEL($\tau=-5$)				GEL($\tau=+5$)			
				LF		PS		LF		PS		LF		PS	
Prior $\rightarrow$				1	2	1	2	1	2	1	2	1	2	1	2
(40,20)	1	1.6982	0.5509	0.5251	0.5193	0.3852	0.5931	0.5830	0.5675	0.3993	0.5943	0.4578	0.4609	0.3518	0.5914
		2.7965	0.2549	0.1212	0.1109	0.1280	0.0950	0.0830	0.0675	0.1007	0.0943	0.0422	0.0391	0.1482	0.0914
		2.0571	0.1875	0.1827	0.1698	0.2296	0.1862	0.1661	0.1350	0.2014	0.1887	0.0845	0.0782	0.2964	0.1827
	2	2.9227	0.5199	0.5251	0.5201	0.5696	0.5731	0.5876	0.5694	0.5760	0.5753	0.4564	0.4595	0.5700	0.5585
		2.3960	0.1285	0.1248	0.1124	0.0852	0.0772	0.0876	0.0694	0.0819	0.0753	0.0436	0.0405	0.0713	0.0585
		1.4353	0.1235	0.1864	0.1721	0.1550	0.1463	0.1753	0.1388	0.1639	0.1506	0.0873	0.0809	0.1426	0.1171
	3	2.5629	0.5283	0.5243	0.5210	0.5919	0.5764	0.5836	0.5710	0.5955	0.5791	0.4573	0.4597	0.5864	0.5718
		2.4070	0.0828	0.1222	0.1132	0.0976	0.0817	0.0836	0.0710	0.0955	0.0792	0.0427	0.0403	0.0865	0.0718
		1.8060	0.1220	0.1837	0.1729	0.1839	0.1527	0.1672	0.1419	0.1910	0.1584	0.0854	0.0806	0.1729	0.1436
(40,36)	1	0.7748	0.5317	0.5133	0.5132	0.5759	0.4498	0.5434	0.5406	0.5836	0.4575	0.4743	0.4773	0.5645	0.4384
		2.0834	0.0890	0.0873	0.0836	0.0894	0.0653	0.0434	0.0406	0.0836	0.0425	0.0257	0.0227	0.0645	0.0616
		1.0582	0.1275	0.1351	0.1299	0.1539	0.1137	0.0868	0.0812	0.1671	0.0850	0.0513	0.0455	0.1290	0.1233
	2	0.8261	0.5301	0.5138	0.5122	0.5775	0.4990	0.5444	0.5392	0.5819	0.5011	0.4739	0.4768	0.5713	0.4959
		2.0875	0.0836	0.0880	0.0826	0.0822	0.0253	0.0444	0.0392	0.0760	0.0156	0.0261	0.0232	0.0700	0.0451
		1.1446	0.1192	0.1358	0.1280	0.1460	0.0385	0.0889	0.0785	0.1521	0.0314	0.0522	0.0463	0.1401	0.0823
	3	0.8700	0.5281	0.5135	0.5124	0.5343	0.5117	0.5438	0.5407	0.5421	0.5153	0.4753	0.4756	0.5218	0.5058
		2.1968	0.0823	0.0877	0.0848	0.0578	0.0331	0.0438	0.0407	0.0421	0.0153	0.0247	0.0244	0.0218	0.0058
		1.2490	0.1211	0.1354	0.1301	0.0967	0.0534	0.0877	0.0813	0.0843	0.0307	0.0495	0.0487	0.0436	0.0115
(80,40)	1	0.6897	0.5217	0.5130	0.5120	0.5811	0.4593	0.5405	0.5367	0.5827	0.4631	0.4774	0.4794	0.5786	0.4532
		1.3980	0.0746	0.0834	0.0791	0.0840	0.0506	0.0405	0.0367	0.0828	0.0369	0.0226	0.0206	0.0786	0.0468
		0.7929	0.1110	0.1293	0.1229	0.1622	0.0838	0.0809	0.0734	0.1655	0.0738	0.0452	0.0412	0.1573	0.0936
	2	0.7220	0.5120	0.5144	0.5114	0.5426	0.5392	0.5422	0.5366	0.5462	0.5416	0.4774	0.4786	0.5372	0.5359
		1.6265	0.0538	0.0840	0.0801	0.0530	0.0465	0.0422	0.0366	0.0463	0.0415	0.0225	0.0214	0.0466	0.0358
		0.8601	0.0791	0.1298	0.1252	0.0880	0.0797	0.0844	0.0732	0.0926	0.0832	0.0452	0.0428	0.0932	0.0717
	3	1.8992	0.5139	0.5131	0.5117	0.4449	0.4613	0.5411	0.5366	0.4458	0.4635	0.4772	0.4781	0.4436	0.4582
		2.2132	0.0543	0.0841	0.0796	0.0569	0.0446	0.0411	0.0366	0.0542	0.0365	0.0228	0.0219	0.0564	0.0417
		1.2990	0.0833	0.1302	0.1242	0.1101	0.0793	0.0822	0.0732	0.1083	0.0730	0.0457	0.0438	0.1128	0.0835
(80,72)	1	0.5469	0.5176	0.5068	0.5069	0.4734	0.4880	0.5214	0.5210	0.4783	0.4918	0.4867	0.4874	0.4665	0.4825
		0.4151	0.0575	0.0605	0.0596	0.0431	0.0326	0.0214	0.0210	0.0217	0.0082	0.0133	0.0126	0.0335	0.0175
		0.4769	0.0882	0.0949	0.0935	0.0736	0.0533	0.0427	0.0419	0.0435	0.0163	0.0267	0.0253	0.0669	0.0350
	2	0.5699	0.5151	0.5072	0.5063	0.4609	0.5139	0.5218	0.5201	0.4648	0.5157	0.4868	0.4872	0.4534	0.5113
		0.4252	0.0522	0.0607	0.0588	0.0501	0.0230	0.0218	0.0201	0.0352	0.0105	0.0132	0.0128	0.0372	0.0113
		0.5071	0.0807	0.0953	0.0922	0.0805	0.0411	0.0436	0.0403	0.0703	0.0234	0.0264	0.0256	0.0745	0.0228
	3	0.5644	0.5166	0.5072	0.5062	0.4939	0.5006	0.5223	0.5202	0.4987	0.5012	0.4867	0.4868	0.4866	0.4996
		0.4449	0.0518	0.0615	0.0593	0.0352	0.0129	0.0223	0.0202	0.0013	0.0012	0.0133	0.0132	0.0134	0.0004
		0.5132	0.0808	0.0964	0.0932	0.0613	0.0218	0.0446	0.0404	0.0026	0.0025	0.0266	0.0263	0.0269	0.0008

Table S2: The AEs (1st column), RMSEs (2nd column) and MRABs (3rd column) of α.

(n,m)	Scheme	MLE	MPSE	SEL				GEL($\tau=-5$)				GEL($\tau=+5$)			
				LF		PS		LF		PS		LF		PS	
Prior $\rightarrow$				1	2	1	2	1	2	1	2	1	2	1	2
(40,20)	1	4.6939	2.9592	2.5034	2.5015	2.3605	2.5830	2.5040	2.5018	2.3648	2.5845	2.5026	2.5012	2.3548	2.5808
		4.4872	0.9268	0.0622	0.0444	0.1559	0.0939	0.0610	0.0372	0.1352	0.0845	0.0607	0.0383	0.1452	0.0808
		1.1638	0.2778	0.0244	0.0156	0.0577	0.0333	0.0244	0.0149	0.0541	0.0338	0.0243	0.0153	0.0581	0.0323
	2	4.1887	3.0444	2.5117	2.5200	2.4433	2.5587	2.5200	2.5122	2.5592	2.4447	2.5199	2.5110	2.4413	2.5578
		3.8786	0.9289	0.0882	0.0790	0.0906	0.0799	0.0771	0.0597	0.0744	0.0679	0.0804	0.0584	0.0760	0.0602
		0.9571	0.2774	0.0314	0.0260	0.0236	0.0300	0.0308	0.0239	0.0298	0.0272	0.0322	0.0234	0.0304	0.0241
	3	7.1977	2.7961	2.4278	2.4472	2.5189	2.5146	2.4288	2.4473	2.5208	2.5157	2.4264	2.4470	2.5161	2.5130
		5.2320	0.5314	0.1199	0.0991	0.0544	0.0439	0.0930	0.0650	0.0370	0.0305	0.0946	0.0620	0.0380	0.0314
		2.1893	0.2106	0.0392	0.0368	0.0180	0.0161	0.0372	0.0260	0.0147	0.0122	0.0378	0.0248	0.0152	0.0126
(40,36)	1	3.6956	2.7374	2.4477	2.4624	2.3905	2.5061	2.4484	2.4628	2.3909	2.5066	2.4465	2.4617	2.3897	2.5055
		2.9274	0.5738	0.0607	0.0432	0.1123	0.0819	0.0516	0.0314	0.1090	0.0763	0.0535	0.0331	0.1103	0.0743
		0.7282	0.1771	0.0210	0.0132	0.0438	0.0302	0.0206	0.0126	0.0436	0.0305	0.0214	0.0132	0.0411	0.0297
	2	3.6361	2.7385	2.4216	2.5592	2.4486	2.4678	2.4229	2.5597	2.4496	2.4690	2.4196	2.5584	2.4496	2.4662
		2.9116	0.5534	0.0787	0.0646	0.0698	0.0642	0.0673	0.0416	0.0594	0.0555	0.0680	0.0501	0.0587	0.0577
		0.7163	0.1716	0.0270	0.0238	0.0268	0.0234	0.0269	0.0166	0.0238	0.0222	0.0272	0.0200	0.0235	0.0231
	3	4.2237	2.6900	2.4197	2.5919	2.4640	2.4880	2.5930	2.5064	2.4654	2.4894	2.5903	2.5052	2.4620	2.4858
		4.9384	0.4946	0.1077	0.0749	0.0525	0.0398	0.0918	0.0584	0.0346	0.0157	0.0903	0.0611	0.0364	0.0141
		0.9748	0.1643	0.0374	0.0256	0.0167	0.0130	0.0367	0.0234	0.0139	0.0062	0.0361	0.0245	0.0145	0.0056
(80,40)	1	3.2370	2.7401	2.5248	2.4811	2.4050	2.5755	2.5250	2.4814	2.4055	2.5763	2.5246	2.4805	2.4042	2.5743
		2.1588	0.5331	0.0282	0.0288	0.0984	0.0445	0.0250	0.0186	0.0944	0.0327	0.0246	0.0195	0.0958	0.0290
		0.5328	0.1676	0.0099	0.0107	0.0380	0.0146	0.0100	0.0074	0.0378	0.0131	0.0098	0.0078	0.0383	0.0116
	2	3.1470	2.7629	2.4324	2.5393	2.5648	2.4249	2.4327	2.5395	2.4256	2.5697	2.4320	2.5390	2.4240	2.5602
		1.9791	0.5267	0.0698	0.0427	0.0635	0.0575	0.0655	0.0395	0.0592	0.0553	0.0612	0.0390	0.0583	0.0550
		0.4781	0.1625	0.0245	0.0159	0.0236	0.0221	0.0262	0.0158	0.0237	0.0221	0.0245	0.0156	0.0233	0.0220
	3	4.0306	2.6387	2.4070	2.5638	2.5367	2.4691	2.4082	2.5650	2.5396	2.4695	2.4054	2.5620	2.5364	2.4685
		4.5757	0.4188	0.0999	0.0699	0.0403	0.0377	0.0875	0.0527	0.0208	0.0106	0.0787	0.0530	0.0161	0.0129
		0.8962	0.1271	0.0372	0.0245	0.0147	0.0131	0.0350	0.0211	0.0083	0.0042	0.0315	0.0212	0.0064	0.0052
(80,72)	1	3.0793	2.6243	2.5609	2.4679	2.5313	2.5319	2.5610	2.4686	2.5329	2.5327	2.5607	2.4669	2.5308	2.5291
		1.6179	0.3704	0.0265	0.0165	0.0539	0.0236	0.0040	0.0018	0.0329	0.0065	0.0026	0.0012	0.0308	0.0055
		0.4108	0.1161	0.0089	0.0057	0.0160	0.0068	0.0016	0.0007	0.0131	0.0026	0.0010	0.0005	0.0123	0.0022
	2	2.9713	2.6196	2.4551	2.5638	2.5590	2.5553	2.5655	2.4584	2.5594	2.5555	2.5612	2.4499	2.5584	2.5550
		1.5207	0.3531	0.0271	0.0218	0.0630	0.0490	0.0200	0.0122	0.0504	0.0310	0.0199	0.0110	0.0531	0.0338
		0.3890	0.1108	0.0086	0.0080	0.0211	0.0149	0.0080	0.0049	0.0201	0.0124	0.0080	0.0044	0.0212	0.0135
	3	3.1536	2.5783	2.5841	2.4405	2.4876	2.5063	2.5875	2.4417	2.4879	2.5067	2.5787	2.4389	2.4870	2.5057
		2.0778	0.3421	0.0798	0.0542	0.0243	0.0237	0.0712	0.0064	0.0121	0.0067	0.0736	0.0052	0.0129	0.0056
		0.4453	0.1068	0.0289	0.0211	0.0081	0.0082	0.0285	0.0026	0.0048	0.0027	0.0295	0.0021	0.0052	0.0023

Table S3: The AEs (1st column), RMSEs (2nd column) and MRABs (3rd column) of μ.

(n,m)	Scheme	MLE	MPSE	SEL				GEL($\tau=-5$)				GEL($\tau=+5$)			
				LF		PS		LF		PS		LF		PS	
Prior $\rightarrow$				1	2	1	2	1	2	1	2	1	2	1	2
(40,20)	1	1.1665	0.6523	0.5031	0.4801	0.4409	0.5135	0.4820	0.5124	0.4427	0.5171	0.4771	0.4933	0.4382	0.5081
		2.0474	0.6340	0.0571	0.0468	0.0761	0.0414	0.0455	0.0405	0.0740	0.0277	0.0551	0.0333	0.0687	0.0167
		1.5760	0.6784	0.0989	0.0795	0.1438	0.0664	0.0910	0.0811	0.1482	0.0556	0.1101	0.0667	0.1374	0.0335
	2	1.0476	0.6589	0.5388	0.5112	0.5308	0.4868	0.5399	0.5137	0.5347	0.4906	0.5372	0.5076	0.5244	0.4795
		1.1663	0.7229	0.0423	0.0448	0.0449	0.0396	0.0399	0.0281	0.0349	0.0336	0.0463	0.0144	0.0305	0.0266
		1.2950	0.6878	0.0778	0.0618	0.0759	0.0686	0.0797	0.0561	0.0698	0.0674	0.0925	0.0289	0.0610	0.0534
	3	1.6092	0.6670	0.5328	0.5229	0.4736	0.5091	0.5332	0.5247	0.4745	0.5110	0.5321	0.5204	0.4702	0.5064
		2.3928	0.4448	0.0344	0.0352	0.0476	0.0391	0.0551	0.0261	0.0455	0.0323	0.0503	0.0204	0.0429	0.0362
		1.4513	0.6032	0.0655	0.0606	0.0889	0.0684	0.1101	0.0521	0.0910	0.0645	0.1006	0.0408	0.0858	0.0725
(40,36)	1	0.6377	0.6664	0.4509	0.5377	0.4903	0.5058	0.4545	0.5405	0.5073	0.4960	0.4449	0.5333	0.4803	0.5032
		1.4429	0.3639	0.0460	0.0293	0.0398	0.0212	0.0350	0.0237	0.0573	0.0171	0.0362	0.0257	0.0618	0.0081
		0.4418	0.5106	0.0710	0.0482	0.0634	0.0332	0.0701	0.0474	0.1147	0.0342	0.0724	0.0515	0.1235	0.0162
	2	0.6438	0.6813	0.5502	0.5217	0.5167	0.5074	0.5528	0.5281	0.5183	0.5089	0.5463	0.5144	0.5140	0.5053
		1.1009	0.3227	0.0570	0.0349	0.0266	0.0209	0.0528	0.0137	0.0346	0.0094	0.0372	0.0097	0.0244	0.0205
		0.4504	0.4811	0.1013	0.0583	0.0485	0.0315	0.1057	0.0273	0.0694	0.0188	0.0745	0.0193	0.0487	0.0411
	3	0.6586	0.6886	0.5355	0.5083	0.4972	0.5148	0.4676	0.5102	0.5162	0.5003	0.4623	0.5057	0.5127	0.4925
		1.0609	0.3287	0.0625	0.0344	0.0472	0.0239	0.0332	0.0247	0.0432	0.0110	0.0377	0.0202	0.0384	0.0075
		0.4836	0.5054	0.1012	0.0555	0.0834	0.0385	0.0663	0.0494	0.0866	0.0220	0.0753	0.0403	0.0769	0.0149
(80,40)	1	0.6110	0.6721	0.4812	0.4773	0.5719	0.5234	0.4783	0.4835	0.5741	0.5278	0.4759	0.4777	0.5687	0.5167
		1.6104	0.3326	0.0370	0.0281	0.0623	0.0333	0.0217	0.0165	0.0253	0.0040	0.0241	0.0223	0.0231	0.0031
		0.3847	0.4864	0.0619	0.0491	0.1181	0.0566	0.0434	0.0329	0.0507	0.0079	0.0482	0.0446	0.0461	0.0064
	2	0.5632	6700	0.5148	0.4866	0.5310	0.5332	0.5178	0.4874	0.5349	0.5337	0.4853	0.5097	0.5305	0.5267
		0.3848	0.3071	0.0323	0.0275	0.0411	0.0340	0.0316	0.0126	0.0183	0.0089	0.0289	0.0076	0.0139	0.0053
		0.2713	0.4550	0.0567	0.0438	0.0691	0.0454	0.0632	0.0252	0.0367	0.0179	0.0578	0.0153	0.0279	0.0105
	3	0.7865	0.7014	0.4994	0.5270	0.5414	0.4661	0.5289	0.5054	0.5433	0.4677	0.5239	0.4911	0.5385	0.4638
		1.9486	0.3248	0.0579	0.0314	0.0346	0.0237	0.0324	0.0102	0.0242	0.0022	0.0321	0.0089	0.0297	0.0063
		0.7495	0.4965	0.1070	0.0500	0.0556	0.0347	0.0648	0.0203	0.0486	0.0044	0.0643	0.0178	0.0594	0.0127
(80,72)	1	0.5229	0.6915	0.4645	0.4755	0.5245	0.5011	0.4650	0.4763	0.5253	0.5027	0.4638	0.4743	0.5230	0.4985
		0.1662	0.2770	0.0303	0.0274	0.0289	0.0205	0.0180	0.0124	0.0074	0.0027	0.0229	0.0067	0.0197	0.0015
		0.1856	0.4415	0.0500	0.0460	0.0513	0.0337	0.0361	0.0248	0.0147	0.0055	0.0458	0.0134	0.0394	0.0030
	2	0.5308	0.6983	0.5039	0.5305	0.5092	0.5030	0.5316	0.5083	0.5112	0.5040	0.5289	0.4960	0.5062	0.5015
		0.1575	0.2749	0.0345	0.0195	0.0244	0.0161	0.0178	0.0083	0.0111	0.0040	0.0147	0.0040	0.0061	0.0015
		0.1897	0.4405	0.0607	0.0322	0.0419	0.0265	0.0357	0.0166	0.0224	0.0080	0.0293	0.0079	0.0124	0.0031
	3	0.5271	0.7157	0.5533	0.5237	0.5445	0.4966	0.5551	0.5261	0.5455	0.4978	0.5503	0.5202	0.5429	0.4950
		0.1580	0.3060	0.0383	0.0231	0.0279	0.0173	0.0289	0.0054	0.0161	0.0003	0.0239	0.0057	0.0127	0.0050
		0.1868	0.4828	0.0619	0.0347	0.0467	0.0264	0.0578	0.0107	0.0323	0.0005	0.0479	0.0114	0.0255	0.0101

Table S4: The AEs (1st column), RMSEs (2nd column) and MRABs (3rd column) of $R(t)$.

(n,m)	Scheme	MLE	MPSE	SEL				GEL($\tau=-5$)				GEL($\tau=+5$)			
				LF		PS		LF		PS		LF		PS	
Prior →				1	2	1	2	1	2	1	2	1	2	1	2
(40,20)	1	0.9968	0.9704	0.9973	0.9964	0.9910	0.9985	0.9973	0.9964	0.9910	0.9985	0.9963	0.9973	0.9909	0.9985
		0.0055	0.1288	0.0045	0.0030	0.0083	0.0016	0.0071	0.0023	0.0072	0.0003	0.0073	0.0023	0.0072	0.0002
		0.0033	0.0307	0.0025	0.0023	0.0072	0.0012	0.0071	0.0023	0.0072	0.0003	0.0073	0.0023	0.0073	0.0002
	2	0.9972	0.9715	0.9984	0.9994	0.9984	0.9966	0.9984	0.9994	0.9966	0.9983	0.9994	0.9984	0.9965	0.9983
		0.0045	0.1379	0.0025	0.0013	0.0017	0.0046	0.0017	0.0012	0.0017	0.0004	0.0017	0.0009	0.0017	0.0006
		0.0029	0.0297	0.0017	0.0012	0.0013	0.0022	0.0017	0.0012	0.0017	0.0004	0.0018	0.0009	0.0017	0.0007
	3	0.9973	0.9669	0.9985	0.9991	0.9955	0.9982	0.9991	0.9985	0.9955	0.9982	0.9991	0.9985	0.9954	0.9982
		0.0047	0.1526	0.0025	0.0012	0.0038	0.0013	0.0033	0.0006	0.0029	0.0013	0.0033	0.0006	0.0030	0.0012
		0.0030	0.0344	0.0018	0.0009	0.0029	0.0012	0.0033	0.0006	0.0029	0.0013	0.0033	0.0006	0.0029	0.0012
(40,36)	1	0.9972	0.9808	0.9911	0.9991	0.9941	0.9985	0.9912	0.9991	0.9941	0.9985	0.9910	0.9991	0.9940	0.9984
		0.0045	0.1127	0.0025	0.0026	0.0073	0.0013	0.0022	0.0019	0.0041	0.0002	0.0022	0.0016	0.0043	0.0002
		0.0028	0.0205	0.0017	0.0021	0.0044	0.0011	0.0022	0.0019	0.0041	0.0002	0.0022	0.0016	0.0043	0.0002
	2	0.9971	0.9776	0.9992	0.9989	0.9977	0.9986	0.9992	0.9989	0.9977	0.9986	0.9992	0.9988	0.9977	0.9985
		0.0046	0.1181	0.0012	0.0012	0.0013	0.0012	0.0014	0.0009	0.0011	0.0003	0.0012	0.0006	0.0011	0.0003
		0.0029	0.0238	0.0011	0.0012	0.0010	0.0011	0.0014	0.0009	0.0011	0.0003	0.0012	0.0006	0.0011	0.0003
	3	0.9970	0.9749	0.9979	0.9989	0.9973	0.9984	0.9950	0.9989	0.9973	0.9984	0.9949	0.9989	0.9972	0.9984
		0.0044	0.1293	0.0025	0.0009	0.0036	0.0010	0.0014	0.0002	0.0010	0.0002	0.0014	0.0003	0.0027	0.0003
		0.0028	0.0265	0.0017	0.0008	0.0028	0.0008	0.0014	0.0002	0.0010	0.0002	0.0014	0.0003	0.0027	0.0003
(80,40)	1	0.9973	0.9844	0.9967	0.9962	0.9998	0.9982	0.9962	0.9967	0.9998	0.9982	0.9962	0.9966	0.9998	0.9982
		0.0038	0.0954	0.0027	0.0026	0.0016	0.0010	0.0020	0.0016	0.0015	0.0001	0.0020	0.0010	0.0015	0.0001
		0.0025	0.0169	0.0022	0.0021	0.0015	0.0008	0.0020	0.0016	0.0016	0.0001	0.0020	0.0010	0.0016	0.0001
	2	0.9974	0.9857	0.9965	0.9986	0.9986	0.9994	0.9965	0.9986	0.9994	0.9986	0.9965	0.9986	0.9993	0.9986
		0.0033	0.0816	0.0022	0.0012	0.0011	0.0009	0.0011	0.0006	0.0008	0.0004	0.0011	0.0004	0.0006	0.0004
		0.0023	0.0156	0.0018	0.0011	0.0009	0.0008	0.0011	0.0006	0.0008	0.0003	0.0011	0.0004	0.0006	0.0004
	3	0.9977	0.9838	0.9976	0.9985	0.9953	0.9995	0.9976	0.9985	0.9953	0.9995	0.9976	0.9985	0.9953	0.9995
		0.0031	0.1015	0.0015	0.0010	0.0024	0.0010	0.0008	0.0002	0.0027	0.0003	0.0008	0.0004	0.0010	0.0003
		0.0021	0.0177	0.0015	0.0008	0.0017	0.0007	0.0008	0.0002	0.0028	0.0003	0.0008	0.0005	0.0010	0.0003
(80,72)	1	0.9977	0.9964	0.9960	0.9960	0.9992	0.9983	0.9960	0.9960	0.9991	0.9983	0.9960	0.9960	0.9991	0.9983
		0.0032	0.0395	0.0026	0.0012	0.0011	0.0010	0.0009	0.0008	0.0009	0.0001	0.0019	0.0008	0.0009	0.0001
		0.0021	0.0051	0.0020	0.0010	0.0010	0.0008	0.0009	0.0008	0.0009	0.0001	0.0019	0.0008	0.0009	0.0001
	2	0.9976	0.9967	0.9973	0.9994	0.9987	0.9987	0.9994	0.9973	0.9988	0.9987	0.9994	0.9973	0.9987	0.9987
		0.0032	0.0325	0.0012	0.0010	0.0009	0.0007	0.0009	0.0004	0.0005	0.0001	0.0009	0.0001	0.0004	0.0001
		0.0021	0.0049	0.0010	0.0009	0.0008	0.0006	0.0010	0.0004	0.0005	0.0001	0.0009	0.0001	0.0005	0.0001
	3	0.9978	0.9935	0.9997	0.9986	0.9995	0.9979	0.9997	0.9986	0.9994	0.9979	0.9997	0.9986	0.9994	0.9979
		0.0031	0.0672	0.0010	0.0009	0.0012	0.0009	0.0007	0.0003	0.0009	0.0001	0.0007	0.0002	0.0009	0.0001
		0.0020	0.0081	0.0008	0.0007	0.0012	0.0007	0.0007	0.0003	0.0010	0.0001	0.0007	0.0002	0.0009	0.0001

Table S5: The AEs (1st column), RMSEs (2nd column) and MRABs (3rd column) of $h(t)$.

(n,m)	Scheme	MLE	MPSE	SEL				GEL($\tau=-5$)				GEL($\tau=+5$)			
				LF		PS		LF		PS		LF		PS	
Prior →				1	2	1	2	1	2	1	2	1	2	1	2
(40,20)	1	0.1052	0.3882	0.1257	0.1341	0.3151	0.0859	0.2595	0.2468	0.3799	0.1490	0.0018	0.0931	0.2455	0.0182
		0.1448	1.3086	0.1209	0.1034	0.1246	0.0664	0.1204	0.1176	0.3520	0.0603	0.0973	0.0679	0.1464	0.0808
		1.0785	1.4943	0.8699	0.7393	0.9777	0.5811	0.8175	0.5588	1.1014	0.6082	0.9814	0.6847	1.0762	0.8158
	2	0.0953	0.3545	0.0393	0.0900	0.0874	0.1655	0.1477	0.0616	0.1774	0.1414	0.0215	0.0214	0.0224	0.0264
		0.1165	1.3302	0.1027	0.0644	0.0762	0.0659	0.1040	0.0469	0.0782	0.0422	0.0968	0.0831	0.0891	0.0728
		0.9528	1.3215	0.7257	0.5699	0.6120	0.5706	0.8431	0.4726	0.7890	0.4258	0.9166	0.8381	0.8984	0.7339
	3	0.0904	0.3537	0.0840	0.0573	0.2203	0.0991	0.1304	0.0705	0.3848	0.3017	0.0427	0.9985	0.1431	0.0601
		0.1200	1.3773	0.0474	0.0461	0.1561	0.0697	0.1032	0.0408	0.2856	0.2025	0.0940	0.0752	0.0841	0.0437
		0.9629	1.7427	0.4312	0.3862	1.2254	0.6692	0.3687	0.1112	0.8808	0.7425	0.8484	0.6579	0.8487	0.4411
(40,36)	1	0.0993	0.2237	0.3441	0.0559	0.2420	0.0847	0.5331	0.1126	0.4511	0.1321	0.0174	0.1670	0.0428	0.0857
		0.1197	1.0009	0.1075	0.1005	0.0805	0.0659	0.1034	0.0834	0.2807	0.0498	0.0818	0.0572	0.0958	0.0783
		0.9475	1.2726	0.7026	0.5338	0.8587	0.5497	0.7625	0.4352	0.9544	0.5028	0.8248	0.5765	0.9664	0.7898
	2	0.1008	0.2651	0.0476	0.0683	0.1207	0.0798	0.1094	0.0961	0.1519	0.1179	0.0023	0.0160	0.0420	0.0554
		0.1150	1.1247	0.0887	0.0626	0.0621	0.0523	0.1032	0.0376	0.0527	0.0270	0.0814	0.0757	0.0767	0.0616
		0.9503	1.2430	0.7339	0.5863	0.5821	0.4577	0.7891	0.3788	0.5320	0.2723	0.8211	0.7739	0.7737	0.6218
	3	0.1006	0.2820	0.0982	0.0700	0.1357	0.0869	0.3143	0.0967	0.2230	0.1106	0.0201	0.1063	0.0496	0.0856
		0.1168	1.1930	0.0999	0.0452	0.1322	0.0492	0.0852	0.0287	0.1496	0.0447	0.0884	0.0651	0.0816	0.0391
		0.9562	1.5952	0.7709	0.3777	1.1101	0.3619	0.3431	0.2893	0.5084	0.4513	0.8914	0.6566	0.8226	0.3939
(80,40)	1	0.1057	0.1970	0.1639	0.1829	0.0140	0.0888	0.2389	0.2259	0.0223	0.1594	0.0777	0.1203	0.0033	0.0208
		0.1076	0.9079	0.0952	0.0595	0.0756	0.0483	0.1007	0.0767	0.0769	0.0329	0.0576	0.0212	0.0787	0.0548
		0.8517	0.9340	0.6883	0.4718	0.5697	0.3967	0.7442	0.4780	0.7756	0.3320	0.6162	0.2137	0.7935	0.5525
	2	0.1017	0.2073	0.1685	0.0783	0.0424	0.0763	0.2032	0.1460	0.0661	0.1262	0.0265	0.1018	0.0101	0.0375
		0.0990	0.9104	0.0636	0.0579	0.0542	0.0432	0.0485	0.0300	0.0330	0.0188	0.0777	0.0521	0.0637	0.0473
		0.8174	1.1167	0.6093	0.4856	0.4304	0.3859	0.4896	0.3025	0.3333	0.1899	0.7831	0.5250	0.6425	0.4769
	3	0.0905	0.1810	0.1216	0.0826	0.2075	0.0328	0.2323	0.1399	0.2487	0.0544	0.0108	0.0339	0.0150	0.1298
		0.1042	0.9323	0.0914	0.0540	0.0937	0.0437	0.0666	0.0147	0.1238	0.0391	0.0790	0.0072	0.0495	0.0306
		0.8038	1.2401	0.7541	0.4423	0.6962	0.3282	0.2700	0.0252	0.3943	0.2490	0.7969	0.0721	0.4999	0.3086
(80,72)	1	0.0969	0.0652	0.1970	0.1911	0.0509	0.0948	0.2225	0.2206	0.1371	0.0837	0.1568	0.1563	0.0205	0.0444
		0.0963	0.4054	0.0810	0.0463	0.0552	0.0473	0.0834	0.0215	0.0380	0.0154	0.0214	0.0061	0.0563	0.0134
		0.7852	0.8207	0.671	0.4270	0.5196	0.3698	0.6088	0.4251	0.5828	0.0155	0.5809	0.0610	0.5681	0.1360
	2	0.0997	0.0607	0.1305	0.0424	0.0785	0.0747	0.2323	0.0692	0.1091	0.0916	0.0177	0.0471	0.0354	0.0519
		0.0923	0.4434	0.0626	0.0548	0.0471	0.0354	0.0102	0.0031	0.0099	0.0076	0.0727	0.0027	0.0572	0.0438
		0.7592	1.0763	0.5864	0.4473	0.4183	0.3157	0.1033	0.0310	0.0999	0.0766	0.6329	0.0269	0.5765	0.4416
	3	0.0927	0.0654	0.0228	0.0789	0.0362	0.1122	0.0626	0.1138	0.0485	0.1383	0.0051	0.0340	0.0176	0.0554
		0.0921	0.0650	0.0795	0.0475	0.0530	0.0406	0.0313	0.0025	0.0506	0.0138	0.0564	0.0042	0.0439	0.0221
		0.7526	1.1388	0.7013	0.4143	0.6353	0.3521	0.2152	0.1478	0.1530	0.1104	0.5692	0.0452	0.4435	0.1934

Table S6: The ACLs (1st column) and CPs (2nd column) of 95% ACI/HPD interval estimates of δ.

(n,m)	Scheme	ACI				HPD							
		MLE		MPSE		LF				PS			
Prior →						1		2		1		2	
(40,20)	1	4.0769	0.872	4.6310	0.867	0.4471	0.943	0.4111	0.946	0.1939	0.976	0.1669	0.982
	2	4.4910	0.864	2.6198	0.899	0.4563	0.940	0.4118	0.946	0.1629	0.981	0.0971	0.990
	3	5.2716	0.822	0.8957	0.917	0.4476	0.942	0.4183	0.944	0.1610	0.980	0.1125	0.987
(40,36)	1	3.4290	0.903	4.5408	0.874	0.3271	0.952	0.3146	0.955	0.1713	0.980	0.1196	0.986
	2	4.1489	0.876	2.2412	0.908	0.3336	0.947	0.3122	0.954	0.1244	0.987	0.0849	0.993
	3	4.3877	0.872	0.8943	0.920	0.3308	0.948	0.3199	0.953	0.1165	0.986	0.0999	0.989
(80,40)	1	2.4598	0.919	4.3154	0.888	0.3138	0.955	0.2978	0.959	0.1174	0.985	0.0826	0.991
	2	2.7093	0.914	1.6502	0.921	0.3123	0.956	0.3030	0.958	0.1217	0.984	0.0847	0.990
	3	2.9400	0.910	0.6384	0.932	0.3129	0.954	0.2992	0.958	0.1158	0.987	0.0506	0.993
(80,72)	1	1.2000	0.932	4.1398	0.896	0.2300	0.970	0.2287	0.970	0.0999	0.988	0.0723	0.989
	2	1.2709	0.927	1.4444	0.923	0.2335	0.968	0.2253	0.971	0.1120	0.986	0.0706	0.991
	3	1.3926	0.921	0.6137	0.935	0.2331	0.967	0.2259	0.970	0.0807	0.992	0.0478	0.995

Table S7: The ACLs (1st column) and CPs (2nd column) of 95% ACI/HPD interval estimates of α.

(n,m)	Scheme	ACI				HPD							
		MLE		MPSE		LF				PS			
Prior →						1		2		1		2	
(40,20)	1	2.1201	0.896	2.4200	0.893	0.1245	0.918	0.0909	0.924	0.1455	0.913	0.1229	0.920
	2	2.6717	0.889	2.9012	0.885	0.1792	0.914	0.1271	0.919	0.2022	0.911	0.1460	0.915
	3	2.8445	0.885	2.9855	0.881	0.2795	0.908	0.1387	0.915	0.2733	0.909	0.1529	0.912
(40,36)	1	1.9756	0.905	2.0349	0.902	0.0964	0.923	0.0559	0.934	0.1320	0.918	0.0983	0.925
	2	2.0275	0.902	2.2323	0.895	0.1286	0.917	0.1107	0.921	0.1331	0.918	0.1224	0.919
	3	2.1841	0.893	2.4126	0.888	0.2085	0.915	0.1229	0.919	0.1513	0.921	0.1301	0.917
(80,40)	1	1.1317	0.917	1.7990	0.911	0.0889	0.929	0.0451	0.941	0.0819	0.928	0.0696	0.937
	2	1.3595	0.912	1.8863	0.906	0.0927	0.924	0.0600	0.929	0.1057	0.923	0.0928	0.926
	3	1.7443	0.905	2.1734	0.894	0.1378	0.918	0.1199	0.922	0.1247	0.922	0.1008	0.924
(80,72)	1	0.2792	0.932	0.5425	0.924	0.0654	0.932	0.0344	0.947	0.0796	0.931	0.0552	0.942
	2	0.6871	0.924	0.7524	0.919	0.0789	0.929	0.0454	0.943	0.0861	0.929	0.0566	0.940
	3	0.7574	0.918	0.8622	0.902	0.1164	0.926	0.0530	0.938	0.0947	0.928	0.0894	0.932

Table S8: The ACLs (1st column) and CPs (2nd column) of 95% ACI/HPD interval estimates of μ.

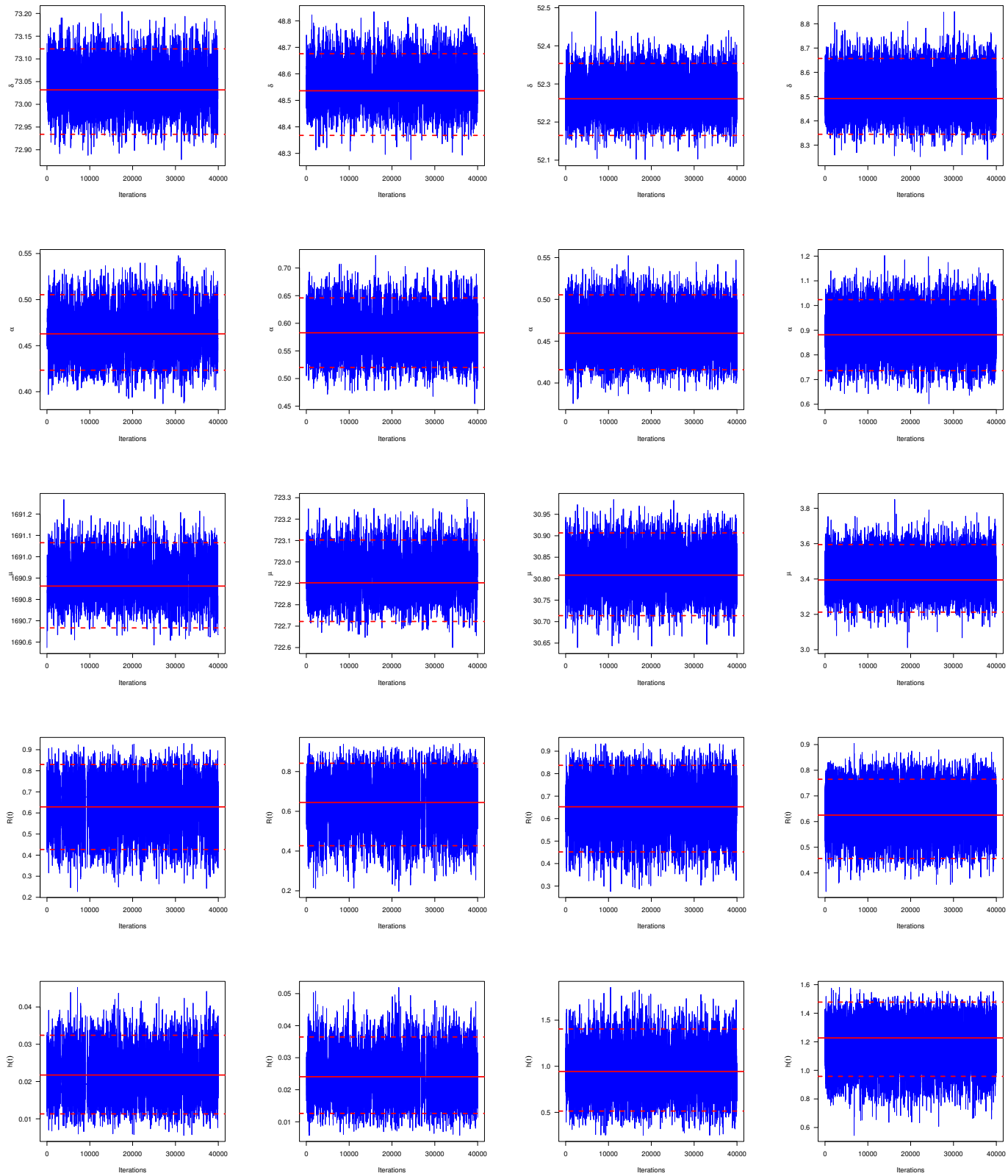
(n,m)	Scheme	ACI				HPD							
		MLE		MPSE		LF				PS			
Prior →						1		2		1		2	
(40,20)	1	1.5202	0.909	1.2762	0.918	0.1928	0.929	0.0987	0.965	0.1308	0.937	0.0890	0.969
	2	1.4377	0.912	0.5254	0.925	0.1433	0.937	0.0996	0.963	0.1225	0.935	0.1146	0.965
	3	1.8080	0.904	0.5832	0.921	0.1896	0.933	0.0848	0.968	0.1022	0.940	0.0800	0.971
(40,36)	1	0.9102	0.920	0.8803	0.923	0.1118	0.948	0.0716	0.974	0.1221	0.946	0.0835	0.972
	2	1.2860	0.917	0.4860	0.928	0.1087	0.952	0.0630	0.979	0.0952	0.954	0.0861	0.975
	3	1.4066	0.913	0.2486	0.933	0.1338	0.943	0.0847	0.968	0.0946	0.956	0.0725	0.977
(80,40)	1	0.6488	0.927	0.5256	0.928	0.0773	0.957	0.0615	0.979	0.0984	0.955	0.0688	0.976
	2	0.8321	0.925	0.3854	0.934	0.0987	0.948	0.0590	0.982	0.0793	0.959	0.0739	0.979
	3	0.5154	0.931	0.1355	0.937	0.0907	0.950	0.0810	0.974	0.0762	0.960	0.0666	0.978
(80,72)	1	0.5105	0.931	0.4389	0.935	0.0468	0.963	0.0413	0.983	0.0746	0.961	0.0584	0.983
	2	0.4864	0.935	0.2457	0.943	0.0954	0.951	0.0561	0.978	0.0726	0.961	0.0582	0.982
	3	0.4719	0.937	0.0988	0.954	0.0848	0.954	0.0398	0.981	0.0711	0.963	0.0654	0.980

Table S9: The ACLs (1st column) and CPs (2nd column) of 95% ACI/HPD interval estimates of $R(t)$.

(n,m)	Scheme	ACI				HPD							
		MLE		MPSE		LF				PS			
Prior →						1		2		1		2	
(40,20)	1	0.0217	0.936	0.0186	0.938	0.0124	0.946	0.0075	0.954	0.0161	0.944	0.0048	0.957
	2	0.0132	0.946	0.0146	0.944	0.0074	0.953	0.0041	0.957	0.0103	0.950	0.0059	0.955
	3	0.0147	0.944	0.0169	0.940	0.0073	0.953	0.0039	0.958	0.0081	0.951	0.0039	0.958
(40,36)	1	0.0101	0.948	0.0145	0.943	0.0076	0.950	0.0059	0.958	0.0112	0.947	0.0037	0.960
	2	0.0109	0.947	0.0098	0.946	0.0052	0.957	0.0026	0.964	0.0042	0.956	0.0028	0.963
	3	0.0119	0.945	0.0091	0.947	0.0063	0.955	0.0021	0.965	0.0072	0.954	0.0029	0.962
(80,40)	1	0.0076	0.952	0.0067	0.954	0.0069	0.954	0.0047	0.963	0.0051	0.958	0.0015	0.965
	2	0.0076	0.952	0.0048	0.959	0.0038	0.960	0.0016	0.966	0.0032	0.961	0.0019	0.965
	3	0.0103	0.948	0.0068	0.953	0.0034	0.961	0.0012	0.965	0.0047	0.958	0.0012	0.966
(80,72)	1	0.0059	0.957	0.0045	0.959	0.0048	0.958	0.0025	0.968	0.0037	0.961	0.0004	0.974
	2	0.0063	0.956	0.0037	0.960	0.0024	0.962	0.0012	0.965	0.0025	0.963	0.0015	0.966
	3	0.0062	0.955	0.0034	0.961	0.0029	0.962	0.0011	0.966	0.0025	0.962	0.0010	0.966

Table S10: The ACLs (1st column) and CPs (2nd column) of 95% ACI/HPD interval estimates of $h(t)$.

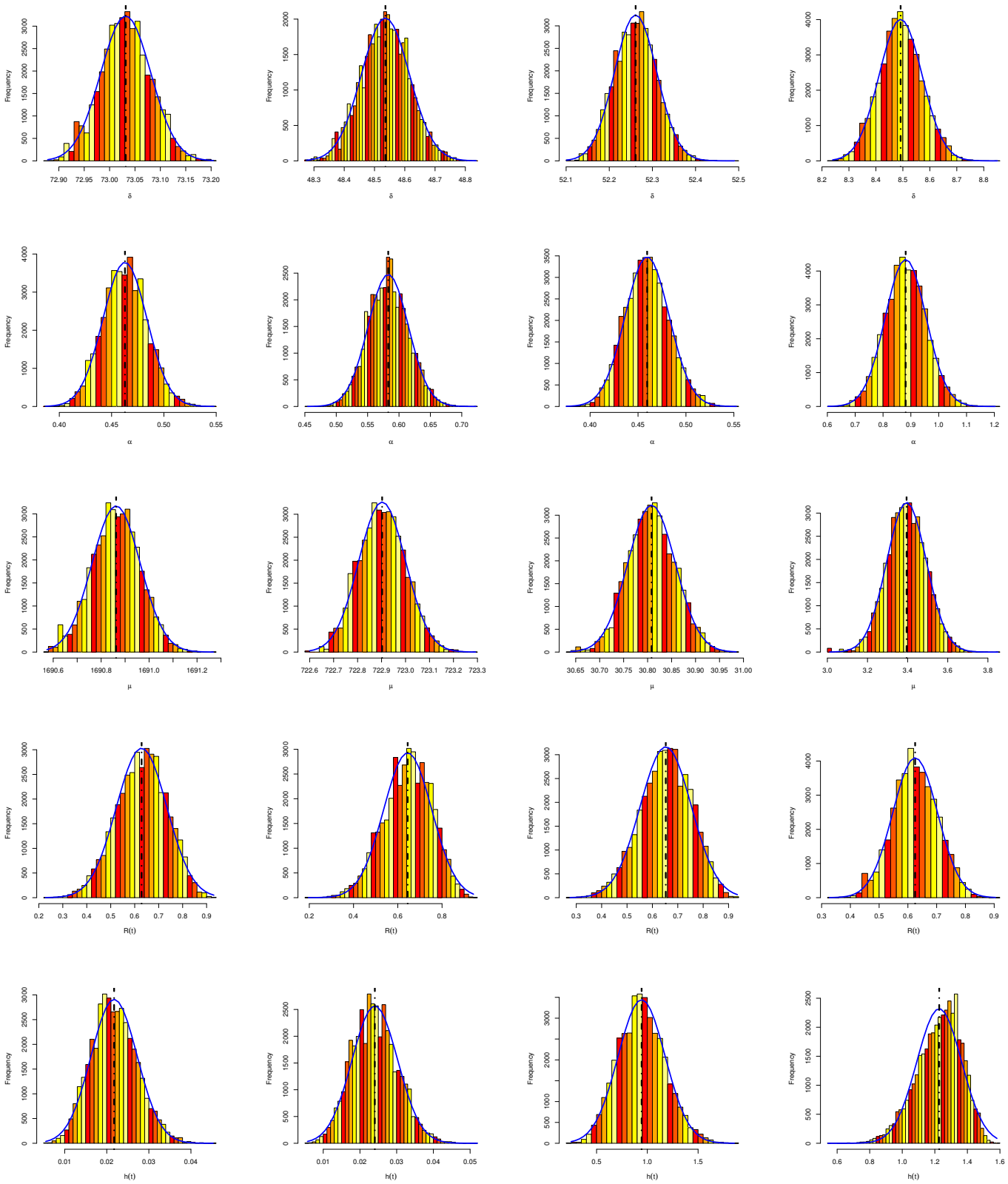
(n,m)	Scheme	ACI				HPD							
		MLE		MPSE		LF				PS			
Prior →						1		2		1		2	
(40,20)	1	0.2677	0.921	0.4104	0.913	0.2286	0.928	0.2110	0.931	0.2301	0.926	0.1958	0.931
	2	0.3033	0.918	0.2697	0.921	0.2159	0.930	0.1962	0.932	0.2481	0.924	0.1894	0.934
	3	0.3250	0.916	0.3638	0.915	0.3080	0.922	0.1747	0.934	0.2891	0.920	0.1893	0.932
(40,36)	1	0.2550	0.925	0.2738	0.922	0.2012	0.936	0.1915	0.939	0.2069	0.936	0.1712	0.943
	2	0.2695	0.924	0.2445	0.928	0.1882	0.939	0.1167	0.943	0.1987	0.938	0.1381	0.945
	3	0.2821	0.922	0.2320	0.929	0.2456	0.931	0.1209	0.943	0.2126	0.934	0.1387	0.944
(80,40)	1	0.2053	0.931	0.1777	0.933	0.1899	0.941	0.1278	0.947	0.1538	0.944	0.0825	0.950
	2	0.2173	0.930	0.1548	0.934	0.1450	0.948	0.0867	0.952	0.1118	0.954	0.0956	0.956
	3	0.2720	0.925	0.1448	0.936	0.1629	0.945	0.0694	0.954	0.1192	0.953	0.0684	0.955
(80,72)	1	0.1911	0.936	0.1651	0.936	0.1803	0.943	0.1183	0.952	0.1158	0.946	0.0302	0.961
	2	0.1988	0.932	0.1279	0.939	0.1308	0.950	0.0724	0.956	0.0869	0.955	0.0570	0.959
	3	0.1958	0.934	0.1296	0.938	0.1406	0.948	0.0613	0.959	0.0650	0.957	0.0599	0.959



(a-1) LF Approach (a-2) PS Approach (b-1) LF Approach (b-2) PS Approach

(a) Data-I (b) Data-II

Figure S1: Trace plots of δ, α, μ, $R(t)$ and $h(t)$ from data sets I and II.



(a-1) LF Approach (a-2) PS Approach (b-1) LF Approach (b-2) PS Approach

(a) Data-I (b) Data-II

Figure S2: Histograms of δ, α, μ, $R(t)$ and $h(t)$ from data sets I and II.