Supplementary Materials:

Detailed Comparison of Index Decomposition Analysis Methods

Comparison Criteria	Logarithmic Mean Divisia Index (LMDI)	AMDI	Laspeyres Index	Paasche Index	Fisher Index
Perfect Decomposition	Achieves perfect decomposition, where the sum of the decomposed terms equals the total change.	Does not guarantee perfect decomposition and may generate residuals.	Does not guarantee perfect decomposition; residuals may arise due to different base year selections.	Does not guarantee perfect decomposition; base year selection issues are significant.	Combines the strengths of the Laspeyres and Paasche indices, but the calculations are complex.
Logarithmic Form	Uses a logarithmic form, making it more natural and accurate in handling proportional changes.	Uses the arithmetic mean, which may be less flexible and accurate in handling index changes.	Uses a base year analysis method, which may not be natural or accurate in handling proportional changes.	Uses a base year analysis method, which may not be natural or accurate in handling proportional changes.	Combines various methods but may not be natural or accurate in handling proportional changes.
Time Series Consistency	Excels in time series analysis, maintaining consistency over time.	May lack consistency in time series analysis.	The base year selection may affect time series consistency.	The base year selection may affect time series consistency.	May lack consistency in time series analysis.
No Residual	Avoids residual problems, reflecting the true impact of each factor more accurately.	Typically generates residuals.	Prone to generating residuals.	Prone to generating residuals.	Calculations are complex and may still generate residuals.
Flexibility	Highly flexible, applicable to various types of datasets and decomposition analyses.	Less flexible in handling complex data structures.	Less flexible when dealing with complex data structures.	Less flexible when dealing with complex data structures.	Suitable for diverse analyses, but the calculations are complex.

The LMDI method demonstrates clear advantages in terms of accuracy, flexibility, time series consistency, and the absence of residuals compared to the AMDI, the Laspeyres index, and other methods. Consequently, the LMDI is the most suitable choice for analyzing quarterly carbon emission intensity. The other methods' residual issues make them less robust for validating results obtained from a no-residual method like the LMDI. Moreover, most of the literature on carbon emission and energy driving factor decomposition predominantly uses the LMDI method without subsequent robustness checks, affirming its reliability and scientific rigor. Therefore, using the LMDI is both scientifically sound and supported by the precedent in the field.