Supplementary Materials:

Multiscale Modelling and Mechanical Anisotropy of Periodic Cellular Solids with Rigid-Jointed Truss-Like Microscopic Architecture

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S.1.: Elastic Properties of Rigid Jointed 2D Lattice Materials

The elastic mechanical properties are presented herein for all 13 lattice topologies considered in the study. The following equations were utilized to create all charts presented in the paper. For all equations, H and L are, respectively, the in-plane thickness and the length of a reference cell element. $\bar{\rho}_L$ is the relative density of the lattice material, $\bar{K}_L$ is the homogenized stiffness matrix that considers both stretching and bending terms of the rigid-jointed lattice network. $(\bar{E}_L)_{xx}$ and $(\bar{E}_L)_{yy}$ are the homogenized Young modulus computed for the x and y directions, $\bar{G}_L$ is the homogenized shear modulus, $(\nu_L)_{xy}$ and $(\nu_L)_{yx}$ are the Poisson coefficients for xy and yx directions of the lattice material.

1) Square Lattice Material

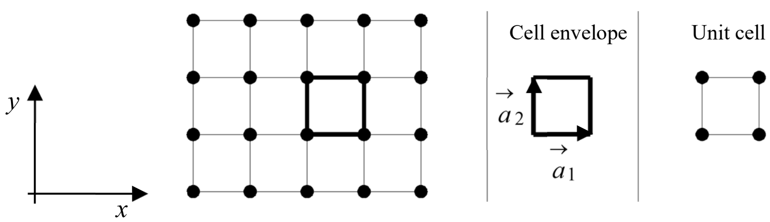

Figure (S1) Microstructure of the 2D square lattice material

$$\bar{K}_L = \frac{K_L}{E} = \bar{\rho}_L \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \left(\bar{\rho}_L\right)^3 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.0625 \end{bmatrix}, \bar{\rho}_L = 2\left(\frac{H}{L}\right)$$

$$(\bar{E}_L)_{xx} = (\bar{E}_L)_{yy} = \frac{1}{2}\bar{\rho}_L, \bar{G}_L = \frac{1}{16}\bar{\rho}_L^3, (\nu_L)_{xy} = (\nu_L)_{yx} = 0$$

2) Triangular Lattice Material

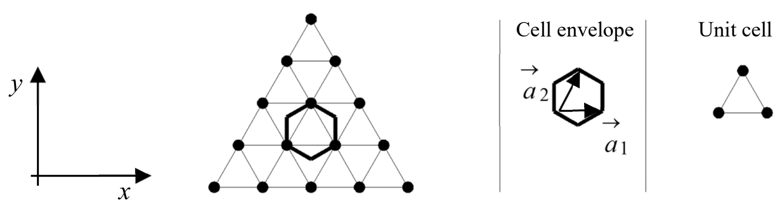

Figure (S2) Microstructure of the 2D triangular lattice material

$$\bar{K}_L = \frac{K_L}{E} = \bar{\rho}_L \begin{bmatrix} 0.375 & 0.125 & 0 \\ 0.125 & 0.375 & 0 \\ 0 & 0 & 0.125 \end{bmatrix} + \left(\bar{\rho}_L\right)^3 \begin{bmatrix} 0.0052 & -0.0052 & 0 \\ -0.0052 & 0.0052 & 0 \\ 0 & 0 & 0.0006 \end{bmatrix}, \bar{\rho}_L = 3.4641\left(\frac{H}{L}\right)$$

$$(\bar{E}_L)_{xx} = (\bar{E}_L)_{yy} = \frac{\bar{\rho}_L(625 + 26\bar{\rho}_L^2)}{1875 + 26\bar{\rho}_L^2}, \bar{G}_L = \frac{\bar{\rho}_L(625 + 3\bar{\rho}_L^2)}{5000}, (\nu_L)_{xy} = (\nu_L)_{yx} = \frac{625 - 26\bar{\rho}_L^2}{1875 + 26\bar{\rho}_L^2}$$

3) Kagome Lattice Material

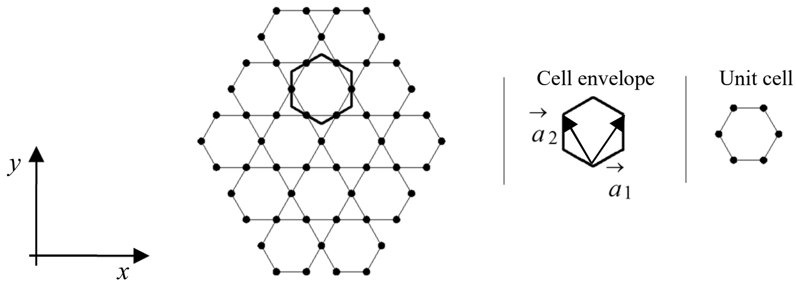

Figure (S3) Microstructure of the 2D Kagome' lattice material

$$\bar{K}_L = \frac{K_L}{E} = \bar{\rho}_L \begin{bmatrix} 0.375 & 0.125 & 0 \\ 0.125 & 0.375 & 0 \\ 0 & 0 & 0.125 \end{bmatrix} + \left(\bar{\rho}_L\right)^3 \begin{bmatrix} 0.0208 & -0.0208 & 0 \\ -0.0208 & 0.0208 & 0 \\ 0 & 0 & 0.0208 \end{bmatrix}, \bar{\rho}_L = 1.7321\left(\frac{H}{L}\right)$$

$$(\bar{E}_L)_{xx} = (\bar{E}_L)_{yy} = \frac{\bar{\rho}_L(625 + 104\bar{\rho}_L^2)}{1875 + 104\bar{\rho}_L^2}, \bar{G}_L = \frac{\bar{\rho}_L(625 + 104\bar{\rho}_L^2)}{5000}, (\nu_L)_{xy} = (\nu_L)_{yx} = \frac{625 - 104\bar{\rho}_L^2}{1875 + 104\bar{\rho}_L^2}$$

4) Lattice Material with Schlafli Symbol of $3^3.4^2$

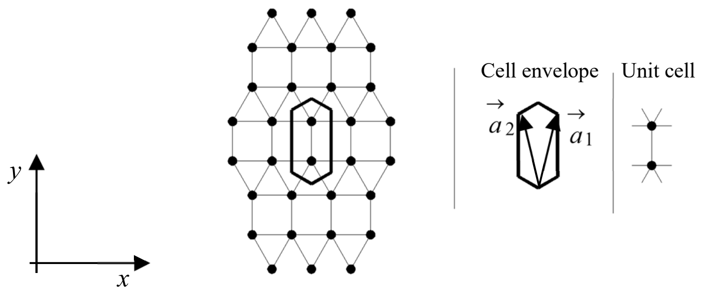

Figure (S4) Microstructure of the 2D lattice material with Schlafli symbol of $3^3.4^2$

$$\bar{K}_L = \frac{K_L}{E} = \bar{\rho}_L \begin{bmatrix} 0.41 & 0.0646 & 0 \\ 0.0646 & 0.4178 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \left(\bar{\rho}_L\right)^3 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.0279 \end{bmatrix}, \bar{\rho}_L = 2.6795\left(\frac{H}{L}\right)$$

$$(\bar{E}_L)_{xx} = (\bar{E}_L)_{yy} = 0.4\bar{\rho}_L, \bar{G}_L = 0.0279\bar{\rho}_L^2, (\nu_L)_{xy} = (\nu_L)_{yx} = 0.156$$

5) Lattice Material with Schlafli Symbol of $3^4.6$

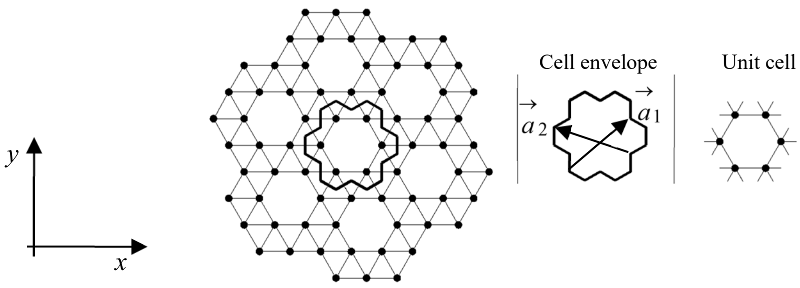

Figure (S5) Microstructure of the 2D lattice material with Schlafli symbol of $3^4.6$

$$\bar{K}_L = \frac{K_L}{E} = \bar{\rho}_L \begin{bmatrix} 0.4445 & 0.2415 & 0 \\ 0.2415 & 0.4445 & 0 \\ 0 & 0 & 0.1015 \end{bmatrix} + \left(\bar{\rho}_L\right)^3 \begin{bmatrix} 0.0095 & -0.0095 & -0.0019 \\ -0.0095 & 0.0095 & 0.0019 \\ -0.0019 & 0.0019 & 0.0124 \end{bmatrix}, \bar{\rho}_L = 2.4744\left(\frac{H}{L}\right)$$

$$(\bar{E}_L)_{xx} = (\bar{E}_L)_{yy} = 1.372\bar{\rho}_L\left(\frac{90.2 + 19.5\bar{\rho}_L^2 + \bar{\rho}_L^4}{395 + 56.7\bar{\rho}_L^2 + \bar{\rho}_L^4}\right), \bar{G}_L = \bar{\rho}_L\left(\frac{20.6 + 4.4\bar{\rho}_L^2 + 0.228\bar{\rho}_L^4}{203 + 19\bar{\rho}_L^2}\right),$$

$$(\nu_L)_{xy} = (\nu_L)_{yx} = \left(\frac{214.6 + 17.8\bar{\rho}_L^2 - \bar{\rho}_L^4}{395 + 56.7\bar{\rho}_L^2 + \bar{\rho}_L^4}\right)$$

6) Double Hexagonal Triangulation (DHT) Lattice Material

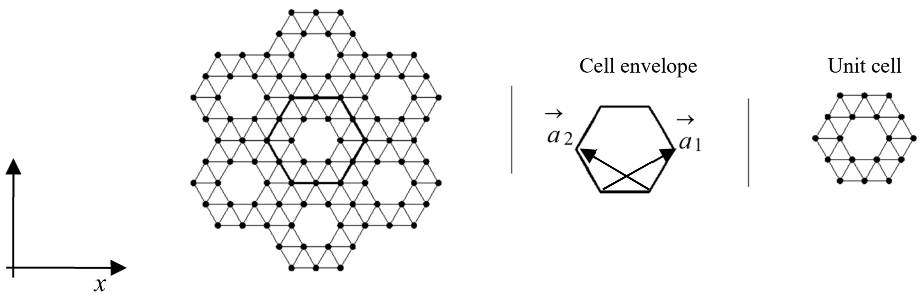

Figure (S6) Microstructure of the DHT lattice material

$$\bar{K}_L = \frac{K_L}{E} = \bar{\rho}_L \begin{bmatrix} 0.3431 & 0.1391 & 0 \\ 0.1391 & 0.3431 & 0 \\ 0 & 0 & 0.093 \end{bmatrix} + \left(\bar{\rho}_L\right)^3 \begin{bmatrix} 0.0041 & -0.0041 & 0 \\ -0.0041 & 0.0041 & 0 \\ 0 & 0 & 0.004 \end{bmatrix}, \bar{\rho}_L = 2.7905\left(\frac{H}{L}\right)$$

$$(\bar{E}_L)_{xx} = (\bar{E}_L)_{yy} = 0.964\bar{\rho}_L\left(\frac{41\bar{\rho}_L^2 + 1020}{41\bar{\rho}_L^2 + 3431}\right), \bar{G}_L = \frac{\bar{\rho}_L}{1000}\left(93 + 4\bar{\rho}_L^2\right), (\nu_L)_{xy} = (\nu_L)_{yx} = \left(\frac{1391 - 41\bar{\rho}_L^2}{3431 + 41\bar{\rho}_L^2}\right)$$

7) Semi-Uni- Braced Square (SUBS) Lattice Material

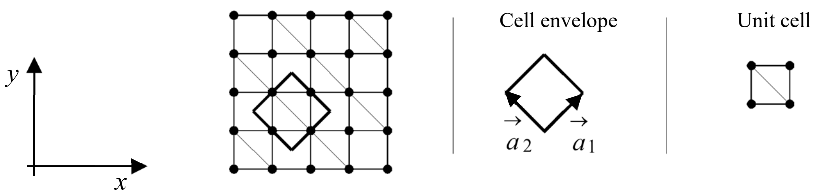

Figure (S7) Microstructure of the SUBS lattice material

$$\bar{K}_L = \frac{K_L}{E} = \bar{\rho}_L \begin{bmatrix} 0.4347 & 0.0653 & -0.0653 \\ 0.0653 & 0.4347 & -0.0653 \\ -0.0653 & -0.0653 & 0.0653 \end{bmatrix} + \left(\bar{\rho}_L\right)^3 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.0126 \end{bmatrix}, \bar{\rho}_L = 2.7071\left(\frac{H}{L}\right)$$

$$(\bar{E}_L)_{xx} = (\bar{E}_L)_{yy} = 0.3694\bar{\rho}_L\left(\frac{1.15\bar{\rho}_L^2 + 4.4}{\bar{\rho}_L^2 + 4.4}\right), \bar{G}_L = \bar{\rho}_L\left(0.0482 + 0.0126\bar{\rho}_L^2\right), (\nu_L)_{xy} = (\nu_L)_{yx} = \left(\frac{\bar{\rho}_L^2}{29.3 + 6.65\bar{\rho}_L^2}\right)$$

8) Triangular- Triangular (TT) Lattice Material

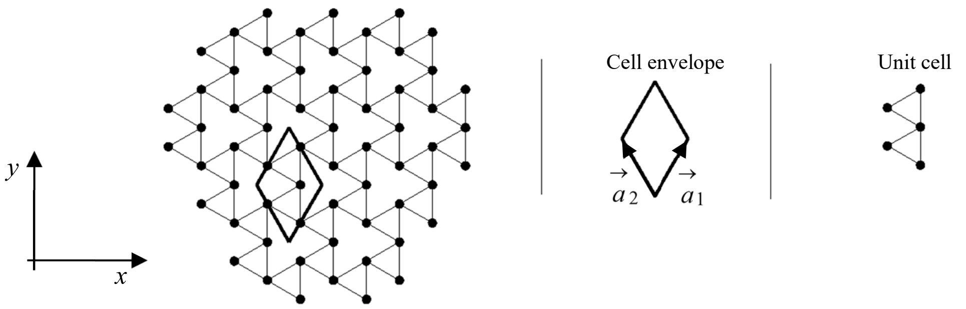

Figure (S8) Microstructure of the TT lattice material

$$\bar{K}_L = \frac{K_L}{E} = \bar{\rho}_L \begin{bmatrix} 0.0937 & -0.0937 & 0 \\ -0.0937 & 0.0937 & 0 \\ 0 & 0 & 0.0938 \end{bmatrix} + \left(\bar{\rho}_L\right)^3 \begin{bmatrix} 0.0053 & -0.0053 & -0.003 \\ -0.0053 & 0.0053 & 0.003 \\ -0.003 & 0.003 & 0.0018 \end{bmatrix}, \bar{\rho}_L = 2.3094\left(\frac{H}{L}\right)$$

$$(\bar{E}_L)_{xx} = (\bar{E}_L)_{yy} \approx 0.3956\bar{\rho}_L, \bar{G}_L = 0.0954\bar{\rho}_L, (\nu_L)_{xy} = (\nu_L)_{yx} = 1$$

9) Semi-Double Braced Square (SDBS) Lattice Material

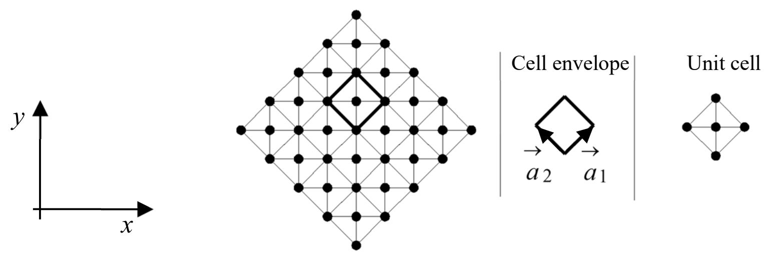

Figure (S9) Microstructure of the SDBS lattice material

$$\bar{K}_L = \frac{K_L}{E} = \bar{\rho}_L \begin{bmatrix} 0.3964 & 0.1036 & 0 \\ 0.1036 & 0.3964 & 0 \\ 0 & 0 & 0.1036 \end{bmatrix} + \left(\bar{\rho}_L\right)^3 \begin{bmatrix} 0.0022 & -0.0022 & 0 \\ -0.0022 & 0.0022 & 0 \\ 0 & 0 & 0.0063 \end{bmatrix}, \bar{\rho}_L = 3.4142\left(\frac{H}{L}\right)$$

$$(\bar{E}_L)_{xx} = (\bar{E}_L)_{yy} = \bar{\rho}_L\left(\frac{11\bar{\rho}_L^2 + 732}{11\bar{\rho}_L^2 + 1982}\right), \bar{G}_L = \frac{7\bar{\rho}_L}{10^4}\left(148 + 9\bar{\rho}_L^2\right), (\nu_L)_{xy} = (\nu_L)_{yx} = \left(\frac{518 - 11\bar{\rho}_L^2}{11\bar{\rho}_L^2 + 1982}\right)$$

10) Uni- Braced Square (UBS) Lattice Material

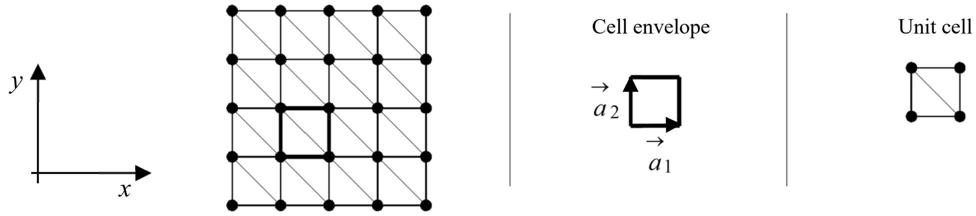

Figure (S-10) Microstructure of the UBS lattice material

$$\bar{K}_L = \frac{K_L}{E} = \bar{\rho}_L \begin{bmatrix} 0.3964 & 0.1036 & -0.1036 \\ 0.1036 & 0.3964 & -0.1036 \\ -0.1036 & -0.1036 & 0.1036 \end{bmatrix} + \left(\bar{\rho}_L\right)^3 \begin{bmatrix} 0.0016 & -0.0016 & 0 \\ -0.0016 & 0.0016 & 0 \\ 0 & 0 & 0.0063 \end{bmatrix}, \bar{\rho}_L = 3.4142\left(\frac{H}{L}\right)$$

$$(\bar{E}_L)_{xx} = (\bar{E}_L)_{yy} = \bar{\rho}_L \left(\frac{881.1 + 101.1\bar{\rho}_L^2 + \bar{\rho}_L^4}{3009.3 + 264.2\bar{\rho}_L^2 + \bar{\rho}_L^4}\right), \bar{G}_L = \bar{\rho}_L\left(0.06 + 0.0063\bar{\rho}_L^2\right),$$

$$(\nu_L)_{xy} = (\nu_L)_{yx} = \bar{\rho}_L^2 \left(\frac{48.3 - \bar{\rho}_L^2}{3009.3 + 264.2\bar{\rho}_L^2 + \bar{\rho}_L^4}\right)$$

11) Double-Braced Square (DBS) Lattice Material

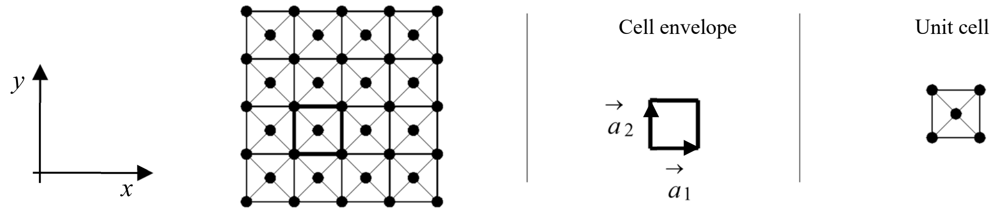

Figure (S11) Microstructure of the DBS lattice material

$$\bar{K}_L = \frac{K_L}{E} = \bar{\rho}_L \begin{bmatrix} 0.3536 & 0.1464 & 0 \\ 0.1464 & 0.3536 & 0 \\ 0 & 0 & 0.1464 \end{bmatrix} + \left(\bar{\rho}_L\right)^3 \begin{bmatrix} 0.0063 & -0.0063 & 0 \\ -0.0063 & 0.0063 & 0 \\ 0 & 0 & 0.0022 \end{bmatrix}, \bar{\rho}_L = 4.8284\left(\frac{H}{L}\right)$$

$$(\bar{E}_L)_{xx} = (\bar{E}_L)_{yy} = 7\bar{\rho}_L \left(\frac{148 + 9\bar{\rho}_L^2}{3536 + 63\bar{\rho}_L^2}\right), \bar{G}_L = \frac{\bar{\rho}_L}{5000}\left(732 + 11\bar{\rho}_L^2\right), (\nu_L)_{xy} = (\nu_L)_{yx} = 3\left(\frac{488 - 21\bar{\rho}_L^2 -}{3536 + 63\bar{\rho}_L^2}\right)$$

12) Patched Kagome' Lattice Material

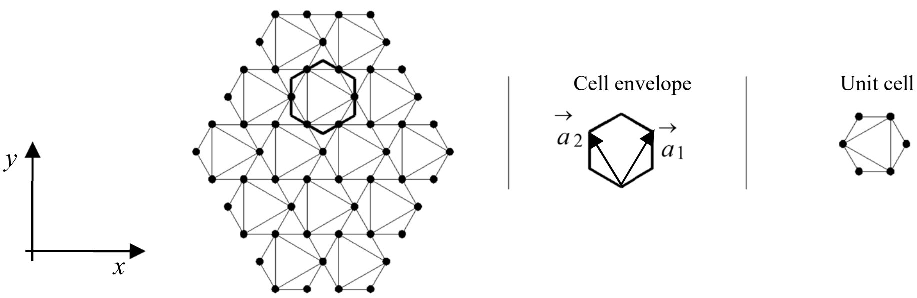

Figure (S12) Microstructure of the patched Kagome' lattice material

$$\bar{K}_L = \frac{K_L}{E} = \bar{\rho}_L \begin{bmatrix} 0.246 & 0.022 & 0 \\ 0.022 & 0.246 & 0 \\ 0 & 0 & 0.112 \end{bmatrix} + \left(\bar{\rho}_L\right)^3 \begin{bmatrix} 0.0037 & -0.0037 & -0.0003 \\ -0.0037 & 0.0037 & 0.0003 \\ -0.0003 & 0.0003 & 0.0034 \end{bmatrix}, \bar{\rho}_L = 3.2321\left(\frac{H}{L}\right)$$

$$(\bar{E}_L)_{xx} = (\bar{E}_L)_{yy} = \bar{\rho}_L\left(\frac{538.3 + 34.1\bar{\rho}_L^2 + \bar{\rho}_L^4}{2206 + 100.1\bar{\rho}_L^2 + \bar{\rho}_L^4}\right), \bar{G}_L = \bar{\rho}_L\left(\frac{125.44 + 7.952\bar{\rho}_L^2 + 0.1249\bar{\rho}_L^4}{1120 + 37\bar{\rho}_L^2}\right),$$

$$(\nu_L)_{xy} = (\nu_L)_{yx} = \left(\frac{197.3 - 27.2\bar{\rho}_L^2 - \bar{\rho}_L^4}{2206 + 100.1\bar{\rho}_L^2 + \bar{\rho}_L^4}\right)$$

13) Semi-Hexagonal Triangulation (SHT) Lattice Material

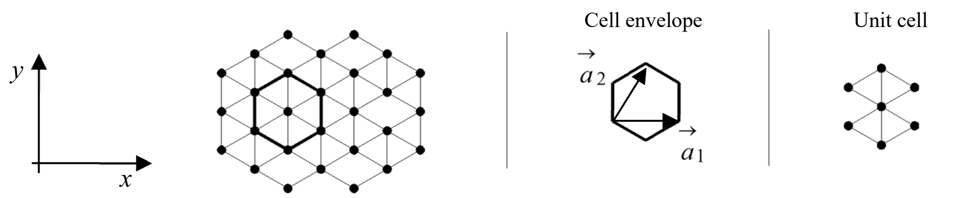

Figure (S13) Microstructure of the SHT lattice material

$$\bar{K}_L = \frac{K_L}{E} = \bar{\rho}_L \begin{bmatrix} 0.4219 & 0.1406 & 0 \\ 0.1406 & 0.1969 & 0 \\ 0 & 0 & 0.1406 \end{bmatrix} + \left(\bar{\rho}_L\right)^3 \begin{bmatrix} 0.005 & -0.005 & 0.0002 \\ -0.005 & 0.005 & -0.0002 \\ 0.0002 & -0.0002 & 0.013 \end{bmatrix}, \bar{\rho}_L = 3.0792\left(\frac{H}{L}\right)$$

$$(\bar{E}_L)_{xx} = \bar{\rho}_L\left(\frac{137 + 22.4\bar{\rho}_L^2 + 0.9\bar{\rho}_L^4}{426.2 + 50.2\bar{\rho}_L^2 + \bar{\rho}_L^4}\right), (\bar{E}_L)_{yy} = \bar{\rho}_L\left(\frac{137 + 22.4\bar{\rho}_L^2 + 0.9\bar{\rho}_L^4}{913.1 + 95.254\bar{\rho}_L^2 + \bar{\rho}_L^4}\right),$$

$$\bar{G}_L = \bar{\rho}_L\left(\frac{1.98 + 0.323\bar{\rho}_L^2 + 0.013\bar{\rho}_L^4}{14.06 + \bar{\rho}_L^2}\right), (\nu_L)_{xy} = \left(\frac{304.3 + 17.3\bar{\rho}_L^2 - \bar{\rho}_L^4}{913.1 + 95.254\bar{\rho}_L^2 + \bar{\rho}_L^4}\right),$$

$$(\nu_L)_{yx} = \left(\frac{304.3 + 17.3\bar{\rho}_L^2 - \bar{\rho}_L^4}{426.2 + 50.2\bar{\rho}_L^2 + \bar{\rho}_L^4}\right)$$