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Determination of the Probabilistic Properties of the Critical Fracture Energy of Concrete Integrating Scale Effect Aspects

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Supplementary Material

This supplementary material presents in detail the algebraic manipulations used to deduce Equation (8), which correlates the coefficient of variation of critical fracture energy ($\frac{\sigma}{\mu}(G_{IC})$) with the coefficient of variation of the material's tensile strength ($\frac{\sigma}{\mu}(f_t)$). With this aim, the degree of material heterogeneity (r_e) is isolated from the equation for estimating $\frac{\sigma}{\mu}(f_t)$. Subsequently, the same procedure is applied to the equation estimating $\frac{\sigma}{\mu}(G_{IC})$. Finally, by equating the obtained equations, Equation (8).

S1. *Coefficient of Variation of Tensile Strength:*

The coefficient of variation for tensile strength is represented by Equation S1. This value is determined by considering the ratio r_e, the compressive strength of concrete (f_c), and the estimated mean value of tensile strength ($\mu(f_t)$) and its standard deviation ($\sigma(f_t)$).

$$\frac{\sigma}{\mu}(f_t(r_e)) = c(r_e)^{-d} \tag{S1}$$

where, $c = 0.35$ and d is given by Equation (S2), being the constant $\alpha = 1\, MPa$.

$$d = 4.5 \times 10^{-2} + 4.5 \times 10^{-3}\left(\frac{f_c}{\alpha}\right) - 1.8 \times 10^{-5}\left(\frac{f_c}{\alpha}\right)^2 \tag{S2}$$

Therefore, by considering Equation (S1), the following algebraic manipulations can be done and the following can be written:

$$\ln\left(\frac{\sigma(f_t)}{c\mu(f_t)}\right) = \ln(r_e)^{-d} \tag{S3}$$

$$\ln(\sigma(f_t)) - (\ln(\mu(f_t)) + \ln(c)) = -d\ln(r_e) \tag{S4}$$

$$-\frac{1}{d}[\ln(\sigma(f_t)) - (\ln(\mu(f_t)) + \ln(c))] = \ln(r_e) \tag{S5}$$

$$\exp\left(-\frac{1}{d}[\ln(\sigma(f_t)) - (\ln(\mu(f_t)) + \ln(c))]\right) = \exp(\ln(r_e)) \tag{S6}$$

$$r_e = \exp\left(\frac{1}{d}[-\ln(\sigma(f_t)) + (\ln(\mu(f_t)) + \ln(c))]\right) \tag{S7}$$

$$r_e = \exp\left(\frac{1}{d}\ln\left(\frac{\mu(f_t)c}{\sigma(f_t)}\right)\right) = \left(\frac{\mu(f_t)c}{\sigma(f_t)}\right)^{\frac{1}{d}} \tag{S8}$$

S2. Coefficient of Variation of Fracture Energy:

The coefficient of variation of fracture energy is determined by Equation (S9). As observed, its calculation also involves the consideration of the ratio r_e.

$$\frac{\sigma(G_{IC})}{\mu(G_{IC}}=(A\ln(r_e)+B)) \tag{S9}$$

where, $A=-8.538$ and $B=70.88$ and $\mu(G_{IC})$ is the mean value of critical fracture energy.

$$\frac{\sigma(G_{IC})}{\mu(G_{IC})}-B=A\ln(r_e) \tag{S10}$$

$$\frac{\sigma(G_{IC})-B\mu(G_{IC})}{A\mu(G_{IC})}=\ln(r_e) \tag{S11}$$

$$\exp\left(\frac{\sigma(G_{IC})-B\mu(G_{IC})}{A\mu(G_{IC})}\right)=\exp(\ln(r_e)) \tag{S12}$$

$$r_e=\exp\left(\frac{\sigma(G_{IC})-B\mu(G_{IC})}{A\mu(G_{IC})}\right) \tag{S13}$$

S3. Deduction of the analytical relation between $\frac{\sigma}{\mu}(G_{IC})$ and $\frac{\sigma}{\mu}(f_t)$:

Finally, considering the last form presented in Equation (S8) and Equation (S13), the following can be written:

$$\left(\frac{\mu(f_t)c}{\sigma(f_t)}\right)^{\frac{1}{d}}=\exp\left(\frac{\sigma(G_{IC})-B\mu(G_{IC})}{A\mu(G_{IC})}\right) \tag{S14}$$

$$\ln\left(\frac{\mu(f_t)c}{\sigma(f_t)}\right)^{\frac{1}{d}}=\ln\left(\exp\left(\frac{\sigma(G_{IC})-B\mu(G_{IC})}{A\mu(G_{IC})}\right)\right) \tag{S15}$$

$$\frac{1}{d}\ln\left(\frac{c\mu(f_t)}{\sigma(f_t)}\right)=\frac{\sigma(G_{IC})-B\mu(G_{IC})}{A\mu(G_{IC})} \tag{S16}$$

$$\frac{1}{d}\ln\left(\frac{\mu_{f_t}c}{\sigma(f_t)}\right)=\frac{\sigma(G_{IC})}{A\mu(G_{IC})}-\frac{B}{A} \tag{S17}$$

$$\frac{\sigma(G_{IC})}{\mu(G_{IC})}=A\left[\frac{1}{d}\ln\left(\frac{\mu_{f_t}c}{\sigma_{f_t}}\right)+\frac{B}{A}\right] \tag{S18}$$